Engineering Mathematics Ⅱ QUIZ-1th 日河工 2B

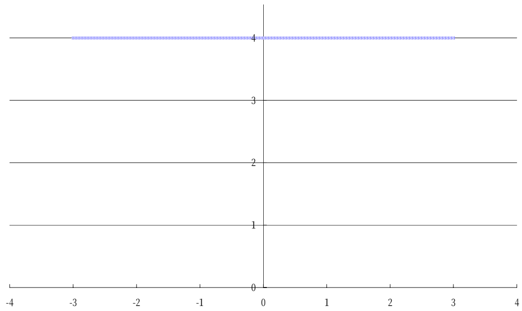
16 March 2005

1) Let $f(x)=4, \quad -3\le x\le 3$, plot the function and find its Fourier series on $[-3, \quad 3]$

(Section 13.2 Problems 1.)

ANS

(1)



Graph of $f(x)=4, \quad -3\le x\le 3$

(2)

The Fourier series of $f(x)=4$ on $[-3, \quad 3]$ is

$$\frac{a_0}{2}+\sum_{n=1}^{\infty}\left[a_n\cos\left(\frac{n\pi x}{3}\right)+b_n\sin\left(\frac{n\pi x}{3}\right)\right]$$

Step1: $f(-x)=4=f(x)$ so $f(x)$ is even.

Step2: Since $f(x)$ is even and $\sin\left(\frac{n\pi x}{3}\right)$ is odd, $f(x)\sin\left(\frac{n\pi x}{3}\right)$ is odd and

hence

$$b_n=\frac{1}{3}\int_{-3}^{3} f(x)\sin\left(\frac{n\pi x}{3}\right)=0$$

Step3: Since $f(x)$ is even and $\cos\left(\frac{n\pi x}{3}\right)$ is even, $f(x)\cos\left(\frac{n\pi x}{3}\right)$ is even and

hence

$$a_n=\frac{1}{3}\int_{-3}^{3} f(x)\cos\left(\frac{n\pi x}{3}\right)dx=\frac{2}{3}\int_{0}^{3} f(x)\cos\left(\frac{n\pi x}{3}\right)dx$$

Therefore,

$$a_0=\frac{2}{3}\int_{0}^{3} 4dx=8, \quad a_n=\frac{2}{3}\int_{0}^{3} 4\cos\left(\frac{n\pi x}{3}\right)dx=0$$

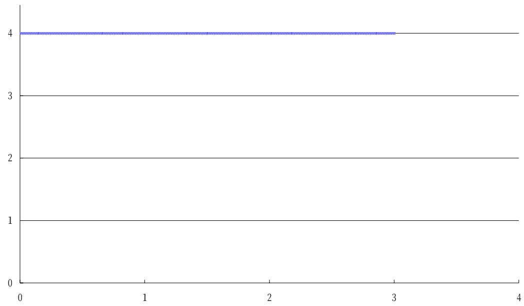
Step4: Therefore, the Fourier series is then 4.

2) Let $f(x)=4, \quad 0\le x\le 3$, plot the function and find its Fourier cosine series and Fourier sine series, respectively.

(Section 13.4 Problems 1.)

ANS

(1)



Graph of $f(x)=4, \quad 0\le x\le 3$

(2)

a) $f(x)=4, \quad 0\le x\le 3$, then $f_e(x)=\begin{cases} f(x)=4 & \text{for } 0\le x\le 3 \\ f(-x)=4 & \text{for } -3\le x\le 0 \end{cases}$

$f_e(x)$ is the even extension of $f(x)$ to $[-3, \quad 3]$. Because $f_e(x)$ is an even function on $[-3, \quad 3]$, its Fourier series on $[-3, \quad 3]$ is $\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{3}\right)$

Thus,

$$a_0=\frac{2}{3}\int_0^3 4dx=8, \quad a_n=\frac{2}{3}\int_0^3 4\cos\left(\frac{n\pi x}{3}\right)dx=0$$

Therefore, the Fourier series is then 4. This function is its own Fourier cosine expansion converging to 4 for $0\le x\le 3$.

b) $f(x)=4, \quad 0\le x\le 3$, then $f_o(x)=\begin{cases} f(x)=4 & \text{for } 0\le x\le 3 \\ -f(-x)=-4 & \text{for } -3\le x\le 0 \end{cases}$

$f_o(x)$ is the odd extension of $f(x)$ to $[-3, \quad 3]$. Because $f_o(x)$ is an odd function on $[-3, \quad 3]$, its Fourier series on $[-3, \quad 3]$ is $\sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{3}\right)$

Thus,

$$b_n=\frac{1}{3}\int_{-3}^3 f_o(x)\sin\left(\frac{n\pi x}{3}\right)=\frac{2}{3}\int_0^3 4\sin\left(\frac{n\pi x}{3}\right)=-\frac{8}{3}\left[\frac{3}{n\pi}\cos\left(\frac{n\pi x}{3}\right)\right]_0^3$$

$$= -\frac{8}{3}\left[\frac{3}{n\pi}\left((-1)^n - 1\right)\right]$$

If n is even (i.e. 2n)➔ $b_n = 0$

If n is odd (i.e. 2n-1)➔ $b_n = \frac{16}{\pi}\frac{1}{2n-1}$

Therefore, the Fourier series is then $\frac{16}{\pi}\sum_{n=1}^{\infty}\frac{1}{2n-1}\sin\left(\frac{(2n-1)\pi x}{3}\right)$. Note that, it converges to 0 if x=0 or x=3, and to 4 for 0<x<3.