Series

A **series** is a <u>sum</u> of a <u>sequence</u> of <u>terms</u>. For example

$1 + 2 + 3 + 4 + 5 + ...$

Series may be <u>finite</u>, or *infinite.*

Infinite series

An **infinite series** is a sum of <u>infinitely</u> many terms. Such a sum can have a finite value; if it has, it is said to *converge*; if it does not, it is said to *diverge.*

An infinite series is formally written as

$$\sum_{n=0}^{\infty} a_n$$

where the elements a_n are real (or <u>complex</u>) numbers. We say that this series **converges towards** S, or that **its value is** S, if the <u>limit</u>

$$\lim_{N \to \infty} \sum_{n=0}^{N} a_n$$

exists and is equal to S. If there is no such number, then the series is said to *diverge.*

The <u>sequence</u> of **partial sums** is defined as the sequence

$$\sum_{n=0}^{N} a_n$$

indexed by N. Then, the definition of series convergence simply says that the sequence of partial sums has limit S, as N .

If the series $\sum a_n$ converges, then the sequence (a_n) converges to 0 for n ; the <u>converse</u> is in general not true.

Absolute convergence

The sum

$$\sum_{n=0}^{\infty} a_n$$

is said to **converge absolutely** if the series of <u>absolute values</u>

$$\sum_{n=0}^{\infty} |a_n|$$

converges. In this case, the original series, and all reorderings of it, converge, and converge towards the same sum.