

考試科目	開課系級	考試日期	印製份數	答案紙	命題教師	備註
工程數學二	二 A, B	6月2日	110	■ 需 □ 不需	陳桂鴻 呂學育	第三次大考

1. (1) Proof $F[f(t) * g(t)] = F(\omega)G(\omega)$, where $*$ is convolution operator. (10%)

(2) $f(t) = e^{-at}$, $a > 0$, compute $F[f(t)]$. (3%)

(3) $g(t) = \delta(t-1)$, compute $F[g(t)]$. (4%)

(4) Compute $F[f(t) * g(t)] = ?$ (3%)

(5) Solve y_p of $y''(t) + 2y(t) = f(t) * g(t)$. $\left(F^{-1}\left[\frac{1}{2-w^2}\right] = \frac{\pi \sin \sqrt{2}t}{\sqrt{2}}$ by using Residue theorem) (10%)

2. (1) Derive the Fourier transform formulation by using the complex Fourier series. (10%)

(2) Write the formulations of transform pairs according to your knowledge. (5%)

(3) Proof the relations of $F(\omega)$ and $A(\omega), B(\omega)$, where $F(\omega)$ is the Fourier transform of $f(t)$, and $A(\omega), B(\omega)$ are the Fourier integral coefficients of $f(t)$. (15%)

3. (1) The Heaviside function $H(t)$ is given by $H(t) = \begin{cases} 1 & t \geq 0 \\ 0 & t < 0 \end{cases}$

Find the Fourier transform of $f(t) = H(t)e^{-at}$ with a a positive constant. (4 scores)

(2) Determine the value of $\frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{e^{3i\omega}}{4+i\omega} e^{i\omega t} d\omega$ (5 scores)

(3) Find the Fourier transform of the Dirac delta function $\delta(t)$, $F\{\delta(t)\}$ (4 scores)

(4) Find the Fourier transform of $F\{\delta(t)\}$, $F\{F\{\delta(t)\}\}$ (5 scores)

(5) Find the Fourier transform of $\cos(5t)$, $F\{\cos(5t)\}$ (5 scores)

4. (1) Solve $\frac{d^2 y(t)}{dt^2} + 4y(t) = \cos(\omega_0 t)$ using Fourier transform with ω_0 a real number (15 scores)

(2) Discuss your result obtained in (1) if $\omega_0 = 2$ (5 scores)

5. (1) Find the Fourier sine transform of $f(t) = e^{-at}$ with a a positive constant. (5 scores)

(2) If we assume that $f(t) \rightarrow 0, f'(t) \rightarrow 0$ as $t \rightarrow \infty$, solve the boundary-value problem $y''(t) - 2y(t) = e^{-t}$, $0 < t < \infty$, with $y(0) = y_0$. (12 scores)