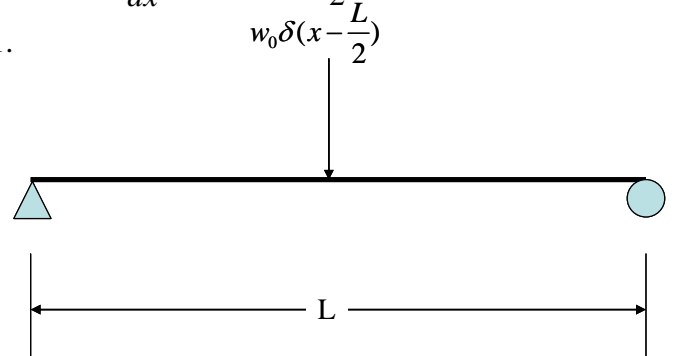


考試科目	開課系級	考試日期	印製份數	答案紙	命題教師	備註
工程數學二	二 A, B	6月23日	110	■ 需 □ 不需	陳桂鴻 呂學育	第四次大考

1. Suppose a uniform beam of length L . If the concentrated load is given by $w(x) = w_0\delta(x - \frac{L}{2})$, $0 < x < L$,

and then the differential equation for the deflection $y(x)$ is $EI \frac{d^4 y}{dx^4} = w_0\delta(x - \frac{L}{2})$, where E , I , and w_0 are constants. The B. C. is given as the following figure 1.

(20%)



2. Use Laplace transform to solve the given system of ODE

$$\begin{cases} \frac{d^2 x}{dt^2} + \frac{dx}{dt} + \frac{dy}{dt} = 0 \\ \frac{d^2 y}{dt^2} + \frac{dy}{dt} - 4 \frac{dx}{dt} = 0 \end{cases} \quad \text{S.t.} \quad \begin{cases} x(0) = 1, x'(0) = 0 \\ y(0) = -1, y'(0) = 5 \end{cases} \quad (25\%)$$

3. Use Laplace transform to solve $\frac{d^2 y}{dx^2} - 2 \frac{dy}{dx} - 8y = f(t)$, $y(0) = 1, y'(0) = 0$, where $f(t)$ is arbitrary.

(15%)

4. (1) $f(t) = \cos(at)$, $g(t) = \sin(at)$, find $L\{f(t) + ig(t)\}$ with a any real number (3%)

(2) $f(t) = t \cos(at)$, $g(t) = t \sin(at)$, find $L\{f(t) + ig(t)\}$ with a any real number (5%)

(3) find $L^{-1}\left\{\frac{e^{-s}}{s(s+1)}\right\}$ (3%)

(4) $F(s) = \frac{2s+1}{s(s+1)(s^2+4s+6)}$, find $L^{-1}\{F(s)\}$ (5%)

(5) $f(t) = \begin{cases} 0, & 0 \leq t < \pi \\ 1, & \pi \leq t \leq 2\pi \\ 0, & t \geq 2\pi \end{cases}$ find $L\{f(t)\}$ (4%)

5. (1) Solve $y' + y = \delta(t-a)$, $y(0) = y_0$ using Laplace transform with $a > 0$ a real number (10%)

(2) Solve $y'' + y = tu(t)$, $y(0) = y_0$, $y'(0) = y_1$ using Laplace transform with $u(t)$ the unit step function (or the Heaviside function) (15%)

(3) (a) Solve $y'' + y = \delta(t)$, $y(0) = 0$, $y'(0) = 0$ using Laplace transform (10%)

(b) discuss your result considering the zero initial conditions and the action of $\delta(t)$ (5%)