海洋大學河海工程學系九十四學年度 第二學期 ■期中 □小考 考試命題紙

考 試 科 目	開課系級	考試日期	印製份數	答案紙	命題教師	備 註
工程數學二	二 A, B	6 月 23 日	110	■ 需 □不需	陳桂鴻 呂學育	第四次大考

1. Suppose a uniform beam of length L. If the concentrated load is given by $w(x)=w_0\delta(x-\frac{L}{2})$, $0<x<L$, and then the differential equation for the deflection $y(x)$ is $EI\frac{d^4y}{dx^4}=w_0\delta(x-\frac{L}{2})$, where E, I, and w_0 are constants. The B. C. is given as the following figure 1.

(20%)

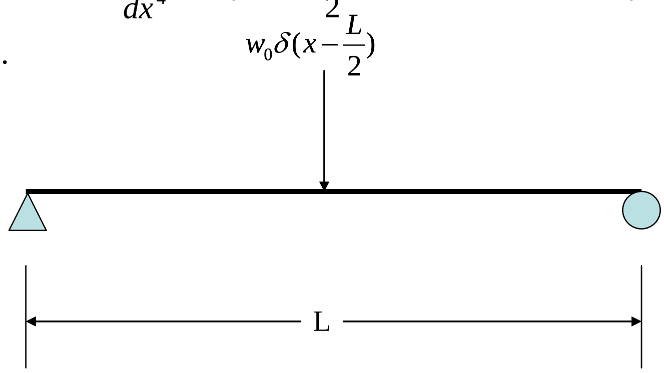

Figure 1

ANS

B.C: $y(0)=y''(0)=0$, $y(L)=y''(L)=0$

$$S^4Y(s)-S^3y(0)-S^2y'(0)-Sy''(0)-y'''(0)=\frac{w_0}{EI}e^{-(L/2)S}$$

$$Y(s)=\frac{w_0}{EI}e^{-(L/2)S}\frac{1}{S^4}+\frac{1}{S^2}c_1+\frac{1}{S^4}c_2$$

$$y(x)=\frac{w_0}{EI}\frac{(x-L/2)^3}{3!}H(x-\frac{L}{2})+xc_1+\frac{x^3}{3!}c_2$$

$$y''(x)=\frac{w_0}{EI}(x-\frac{L}{2})H(x-\frac{L}{2})+xc_2$$

$$c_1=\frac{L^2}{16}\frac{w_0}{EI},\quad c_2=\frac{-w_0}{2EI}$$

$$y(x)=\frac{w_0}{EI}\frac{(x-L/2)^3}{3!}H(x-\frac{L}{2})+\frac{L^2}{16}\frac{w_0}{EI}x-\frac{x^3}{3!}\frac{w_0}{2EI}$$

2. Use Laplace transform to solve the given system of ODE

$$\begin{cases}\frac{d^2x}{dt^2}+\frac{dx}{dt}+\frac{dy}{dt}=0\\ \frac{d^2y}{dt^2}+\frac{dy}{dt}-4\frac{dx}{dt}=0\end{cases}\quad \text{S.t.}\quad \begin{cases}x(0)=1, x'(0)=0\\ y(0)=-1, y'(0)=5\end{cases}\quad (25\%)$$

ANS
$$\begin{cases}(S+1)X(s)+Y(s)=1\\ 4X(s)-(S+1)Y(s)=1\end{cases}\rightarrow\begin{cases}X(s)=\frac{(S+2)}{S^2+2S+5}=\frac{(S+1)}{(S+1)^2+2^2}+\frac{1}{2}\frac{2}{(S+1)^2+2^2}\\ Y(s)=\frac{-S+3}{S^2+2S+5}=-\frac{S+1}{(S+1)^2+2^2}+2\frac{2}{(S+1)^2+2^2}\end{cases}$$

$$x(t)=e^{-t}\cos 2t+\frac{1}{2}e^{-t}\sin 2t,\quad y(t)=-e^{-t}\cos 2t+2e^{-t}\sin 2t.$$

3. Use Laplace transform to solve $\frac{d^2y}{dx^2}-2\frac{dy}{dx}-8y=f(t), y(0)=1, y'(0)=0$, where $f(t)$ is arbitrary.

(15%)

ANS $S^2Y(s)-Sy(0)-y'(0)-2SY(s)+2y(0)-8Y(s)=F(s)$

$(S^2-2S+8)Y(s)=F(s)+S-2$

$$Y(s)=\frac{S-2}{(S-4)(S+2)}+\frac{F(s)}{(S-4)(S+2)}=\frac{2/3}{(S+2)}+\frac{1/3}{(S-4)}+\frac{-1/6}{(S+2)}F(s)+\frac{1/6}{(S-4)}F(s)$$

$$y(t)=\frac{2}{3}e^{-2t}+\frac{1}{3}e^{4t}-\frac{1}{6}[e^{-2t}*f(t)]+\frac{1}{6}[e^{4t}*f(t)]$$

$$=\frac{2}{3}e^{-2t}+\frac{1}{3}e^{4t}-\frac{1}{6}\int_0^t e^{-2(t-\tau)}f(\tau)d\tau+\frac{1}{6}\int_0^t e^{4(t-\tau)}f(\tau)d\tau$$

4. (1) $f(t)=\cos(at)$, $g(t)=\sin(at)$, find $L\{f(t)+ig(t)\}$ with a any real number (3%)

(2) $f(t)=t\cos(at)$, $g(t)=t\sin(at)$, find $L\{f(t)+ig(t)\}$ with a any real number (5%)

(3) find $L^{-1}\left\{\frac{e^{-s}}{s(s+1)}\right\}$ (3%)

(4) $F(s)=\frac{2s+1}{s(s+1)(s^2+4s+6)}$, find $L^{-1}\{F(s)\}$ (5%)

(5) $f(t)=\begin{cases}0, & 0\le t<\pi\\ 1, & \pi\le t\le 2\pi\\ 0, & t\ge 2\pi\end{cases}$ find $L\{f(t)\}$ (4%)

ANS (1) $L\{f(t)+ig(t)\}=L\{\cos at+i\sin at\}=L\{e^{iat}\}=\frac{1}{s-ia}$

(2) $L\{f(t)+ig(t)\}=L\{t(\cos at+i\sin at)\}=L\{te^{iat}\}=\frac{1}{(s-ia)^2}$

(3) $L^{-1}\left\{\frac{e^{-s}}{s(s+1)}\right\}=L^{-1}\left\{\frac{e^{-s}}{s}-\frac{e^{-s}}{s+1}\right\}=U(t-1)-e^{-(t-1)}U(t-1)$

(4) $Y(s)=\frac{1/6}{s}+\frac{1/3}{s+1}-\frac{s/2+5/3}{s^2+4s+6}$

$$y(t)=\frac{1}{6}L^{-1}\left\{\frac{1}{s}\right\}+\frac{1}{3}L^{-1}\left\{\frac{1}{s+1}\right\}-\frac{1}{2}L^{-1}\left\{\frac{s+2}{(s+2)^2+2}\right\}-\frac{2}{3\sqrt{2}}L^{-1}\left\{\frac{\sqrt{2}}{(s+2)^2+2}\right\}$$

$$=\frac{1}{6}+\frac{1}{3}e^{-t}-\frac{1}{2}e^{-2t}\cos\sqrt{2}t-\frac{\sqrt{2}}{3}e^{-2t}\sin\sqrt{2}t$$

(5) $L\{f(t)\}=\int_\pi^{2\pi}e^{-st}(1)dt=\frac{e^{-\pi s}}{s}-\frac{e^{-2\pi s}}{s}$

5. (1)Solve $y'+y=\delta(t-a)$, $y(0)=y_0$ using Laplace transform with $a>0$ a real number (10%)

(2)Solve $y''+y=tu(t)$, $y(0)=y_0$, $y'(0)=y_1$ using Laplace transform with $u(t)$ the unit step function (or the Heaviside function) (15%)

(3) (a) Solve $y''+y=\delta(t)$, $y(0)=0$, $y'(0)=0$ using Laplace transform (10%)

(b) discuss your result considering the zero initial conditions and the action of $\delta(t)$ (5%)

ANS (1) $L\{y'+y\}(s)=sY(s)-y(0)+Y(s)$, $L\{\delta(t-a)\}(s)=e^{-as}$

$$(s+1)Y(s)-y_0=e^{-as},\quad Y(s)=\frac{e^{-as}+y_0}{s+1}$$

$$L^{-1}\{Y(s)\}(t)=L^{-1}\left\{\frac{e^{-as}+y_0}{s+1}\right\}(t)=y_0e^{-t}u(t)+e^{-(t-a)}u(t-a)$$

$$\therefore y(t) = y_0 e^{-t} u(t) + e^{-(t-a)} u(t-a)$$

(2) $L\{y'' + y\}(s) = s^2 Y(s) - sy(0) - y'(0) + Y(s)$; $\quad L\{tu(t)\}(s) = \dfrac{1}{s^2}$

$$Y(s) = \frac{\frac{1}{s^2} + sy_0 + y_1}{s^2 + 1}$$

$$L^{-1}\{Y(s)\}(t) = L^{-1}\left\{\frac{\frac{1}{s^2} + sy_0 + y_1}{s^2 + 1}\right\}(t) = (y_0 \cos t + y_1 \sin t + t - \sin t)u(t)$$

$$\therefore y(t) = (y_0 \cos t + y_1 \sin t + t - \sin t)u(t)$$

(3) (a) $y(t) = L^{-1}\{F(s)\} = L^{-1}\left\{\dfrac{Q(s)}{P(s)}\right\} + L^{-1}\left\{\dfrac{1}{P(s)}\right\}$; $\quad y(0) = 0,\ y'(0) = 0$ ➔ $Q(s) = 0$

$$y(t) = L^{-1}\{F(s)\} = L^{-1}\left\{\frac{1}{s^2 + 1}\right\} = \sin t$$

(b) ➔ $y'(t) = \cos t$ ➔ $y(0) = 0,\ y'(0) = 1$???

Note that, due to the action of the unit impulse on the system with zero initial conditions, there are new initial conditions just after impulse.

$\int_0^{\varepsilon} y'' dt + \int_0^{\varepsilon} y dt = \int_0^{\varepsilon} \delta(t) dt$ where ε ➔ 0 (just after the unit impulse)

➔ $y'(\varepsilon) - y'(0) = \int_0^{\varepsilon} \delta(t) dt = 1$ (注意衝量的作用在於造成速度改變,但位移的變化需要時間，當 ε ➔ 0 僅瞬間內，所以位移來不及變化)

➔ $y'(\varepsilon) = 1$, (ε ➔ 0 , just after the unit impulse).