

Complex Numbers

The complex numbers are the [field](#) \mathbb{C} of numbers [of the form](#) $x + iy$, where x and y are [real numbers](#) and i is the [imaginary unit](#) equal to the [square root](#) of -1 , namely $i = \sqrt{-1}$.

We can apply a single letter $z = x + iy$ to denote a complex number. In component notation, $z = x + iy$ can be written (x, y) .

Through the [Euler formula](#), a complex number $z = x + iy$ may be written in "[polar](#)" form

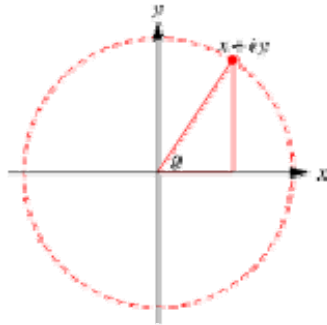
$$z = x + iy = r[\cos(\theta) + i \sin(\theta)] = re^{i\theta}$$

$r = |z| = |x + iy| = \sqrt{x^2 + y^2}$ is the magnitude, or modulus, of $z = x + iy$

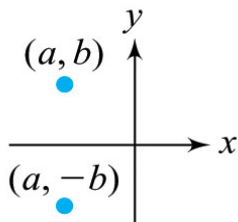
$\theta = \tan^{-1}\left(\frac{y}{x}\right)$ is known as the [argument](#) or [phase](#).

The plot below shows the point z , where the dashed circle represents the [complex modulus](#)

$r = |z| = |x + iy| = \sqrt{x^2 + y^2}$ of z and the angle θ represents its [complex argument](#).



Given a complex number $z = a + ib$, its conjugate is $\bar{z} = a - ib$



Complex conjugate as a reflection across the horizontal axis

Complex addition

$$(a + bi) + (c + di) = (a + c) + i(b + d),$$

complex subtraction

$$(a + bi) - (c + di) = (a - c) + i(b - d),$$

complex multiplication

$$(a + bi)(c + di) = (ac - bd) + i(ad + bc),$$

and complex division

$$\frac{a + bi}{c + di} = \frac{(ac + bd) + i(bc - ad)}{c^2 + d^2}$$

<http://mathworld.wolfram.com/ComplexNumber.html>

Equality of complex numbers

$$a + ib = c + id \text{ if and only if } a = c \quad \text{and} \quad b = d$$

$$e^{i\theta} = \cos \theta + i \sin \theta, \quad e^{-i\theta} = \cos \theta - i \sin \theta$$

$$\rightarrow \sin \theta = \frac{e^{i\theta} - e^{-i\theta}}{2i}, \quad \cos \theta = \frac{e^{i\theta} + e^{-i\theta}}{2}$$