HOMEWORK #10s (Laplace Transform)

Due on June 14

1) Find the Laplace transform $L\left\{\left(1-e^{t}+3e^{-4t}\right)\cos 5t\right\}$ (Problem 9, page 212)

ANS $L\{(1-e^{t}+3e^{-4t})\cos 5t\}=L\{\cos 5t-e^{t}\cos 5t+3e^{-4t}\cos 5t\}$

$$=\frac{s}{s^2+25}-\frac{s-1}{(s-1)^2+25}+\frac{3(s+4)}{(s+4)^2+25}$$

2) Find the inverse Laplace transform $L^{-1}\left\{\frac{2s-1}{s^2(s+1)^3}\right\}$ (Problem 19, page 212)

ANS $L^{-1}\{\frac{2s-1}{s^2(s+1)^3}\}=L^{-1}\{\frac{5}{s}-\frac{1}{s^2}-\frac{5}{s+1}-\frac{4}{(s+1)^2}-\frac{3}{2}\frac{2}{(s+1)^3}\}$

$$=5-t-5e^{-t}-4te^{-t}-\frac{3}{2}t^2e^{-t}$$

3) Use the Laplace transform to Solve $y''-y'=e^{t}\cos t,\quad y(0)=0, y'(0)=0$ (Problem 29, page 212)

ANS The Laplace transform of the differential equation is

$$s^2L\{y\}-sy(0)-y'(0)-[sL\{y\}-y(0)]=\frac{s-1}{(s-1)^2+1}$$

Solving for $L\{y\}$ we obtain

$$L\{y\}=\frac{1}{s(s^2-2s+2)}=\frac{1}{2}\frac{1}{s}-\frac{1}{2}\frac{s-1}{(s-1)^2+1}+\frac{1}{2}\frac{1}{(s-1)^2+1}$$

Thus

$$y=\frac{1}{2}-\frac{1}{2}e^{t}\cos t+\frac{1}{2}e^{t}\sin t$$

4) Find the Laplace transform $L\left\{\cos(2t)\ \mathcal{U}\ (t-\pi)\right\}$ (Problem 41, page 213)

ANS $L\{\cos 2t\,\mathcal{U}\ (t-\pi)\}=L\{\cos 2(t-\pi)\mathcal{U}\ (t-\pi)\}=\frac{se^{-\pi s}}{s^2+4}$

5) Find the inverse Laplace transform $L^{-1}\left\{\frac{e^{-s}}{s(s+1)}\right\}$ (Problem 47, page 213)

ANS $L^{-1}\{\frac{e^{-s}}{s(s+1)}\}=L^{-1}\{\frac{e^{-s}}{s}-\frac{e^{-s}}{s+1}\}=\mathcal{U}\ (t-1)-e^{-(t-1)}\mathcal{U}\ (t-1)$

6) Use the Laplace transform to Solve $y'' + y = f(t), \quad y(0) = 0, y'(0) = 1$ where

$$f(t) = \begin{cases} 0, & 0 \le t < \pi \\ 1, & \pi \le t \le 2\pi \\ 0, & t \ge 2\pi \end{cases} \qquad \text{(Problem 69, page 214)}$$

ANS The Laplace transform of the differential equation is

$$s^2 L\{y\} - sy(0) - y'(0) + L\{y\} = \frac{e^{-\pi s}}{s} - \frac{e^{-2\pi s}}{s}$$

Solving for $L\{y\}$ we obtain

$$L\{y\} = e^{-\pi s}[\frac{1}{s} - \frac{s}{s^2+1}] - e^{-2\pi s}[\frac{1}{s} - \frac{s}{s^2+1}] + \frac{1}{s^2+1}$$

Thus

$$y = [1 - \cos(t-\pi)]\mathcal{U}(t-\pi) - [1 - \cos(t-2\pi)]\mathcal{U}(t-2\pi) + \sin t$$