HOMEWORK #1s (Chapter 7 VectorS、9.1 Vector Functions)

1) Find a vector $\vec{b}$ that is parallel to the given vector and has the indicated magnitude $\vec{a} = 3\vec{i} + 7\vec{j}, \quad |\vec{b}| = 2$ (Problem 31, page 302).

ANS $|\vec{a}| = \sqrt{9+49} = \sqrt{58}; \quad \vec{b} = 2(\frac{1}{\sqrt{58}})(3\vec{i} + 7\vec{j}) = \frac{6}{\sqrt{58}}\vec{i} + \frac{14}{\sqrt{58}}\vec{j}$

2) $\vec{a} = \langle 1, -3, 2 \rangle$, $\vec{b} = \langle -1, 1, 1 \rangle$ and $\vec{c} = \langle 2, 6, 9 \rangle$. Find the indicated vector or scalar $\left| \frac{\vec{a}}{|\vec{a}|} \right| + 5 \left| \frac{\vec{b}}{|\vec{b}|} \right|$ (Problem 47, page 309).

ANS $\left| \frac{\vec{a}}{|\vec{a}|} \right| + 5 \left| \frac{\vec{b}}{|\vec{b}|} \right| = \frac{1}{|\vec{a}|}|\vec{a}| + 5\frac{1}{|\vec{b}|}|\vec{b}| = 1 + 5 = 6$

3) Find parametric and symmetric equations for the line through the given point parallel to the given vector $(4, 6, -7), \quad \vec{a} = \langle 3, 1/2, -3/2 \rangle$ (Problem 19, page 329).

ANS $parametric: x = 4 + 3t, \quad y = 6 + t/2, \quad z = -7 - 3t/2$

$symmetric: \frac{x-4}{3} = \frac{y-6}{1/2} = \frac{z+7}{-3/2}$

4) Find the parametric equation of the tangent line to the given curve at the indicated value of t. $x = t, \ y = t^2/2, \ z = t^3/3; \ t = 2$ (Problem 25, page 454).

ANS $\vec{r}(t) = t\vec{i} + \frac{1}{2}t^2\vec{j} + \frac{1}{3}t^3\vec{k}; \quad \vec{r}(2) = 2\vec{i} + 2\vec{j} + \frac{8}{3}\vec{k}; \quad \vec{r}'(t) = \vec{i} + t\vec{j} + t^2\vec{k}; \quad \vec{r}'(2) = \vec{i} + 2\vec{j} + 4\vec{k}$

Using the point $(2, 2, 8/3)$ and the direction vector $\vec{r}'(2)$, we have

$x = 2 + t, \quad y = 2 + 2t, \quad z = 8/3 + 4t$

5) Find the length of the curve traced by the given vector function on the indicated interval $\vec{r}(t) = a\cos(t)\vec{i} + a\sin(t)\vec{j} + ct\vec{k}; \ 0 \le t \le 2\pi$ (Problem 41, page 454).

ANS $\vec{r}'(t) = -a\sin t\vec{i} + a\cos t\vec{j} + c\vec{k}; |\vec{r}'(t)| = \sqrt{(-a\sin t)^2 + (a\cos t)^2 + c^2} = \sqrt{a^2 + c^2}$

$$s = \int_0^{2\pi} \sqrt{a^2 + c^2}\, dt = \sqrt{a^2 + c^2}\, t \Big|_0^{2\pi} = 2\pi\sqrt{a^2 + c^2}$$