HOMEWORK #3s (9.6 ~9.8)

Due on March 15

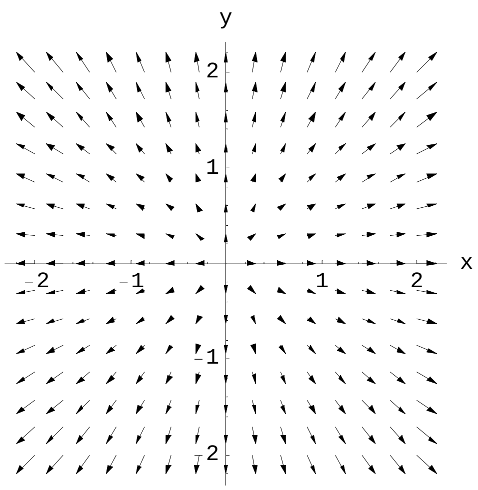
1) Find an equation of the tangent plane to the graph of the given equation at the indicated point $x^2+y^2+z^2=9$; $(-2,2,1)$ (Problem 15, page 480).

ANS $F(x,y,z)=x^2+y^2+z^2$; $\nabla F=2x\vec{i}+2y\vec{j}+2z\vec{k}$. $\nabla F(-2,2,1)=-4\vec{i}+4\vec{j}+2\vec{k}$.

The equation of the tangent plane is $-4(x+2)+4(y-2)+2(z-1)=0$
or $-2x+2y+z=9$.

2) Graph some representative vectors in the given vector field, and find the curl and the divergence of the given vector field. Also, what can you say about a source or sink based on the divergent result ? $\vec{F}(x,y)=x\vec{i}+y\vec{j}$ (Problem 1, page 484).

ANS $div(\vec{F})=\nabla\cdot\vec{F}=1+1=2$, $curl(\vec{F})=\nabla\times\vec{F}=0$; a source



3) Find the curl and the divergence of the given vector field $\vec{F}(x,y,z)=(x-y)^3\vec{i}+e^{-yz}\vec{j}+xye^{2y}\vec{k}$ (Problem 10, page 484).

ANS $curl\ \vec{F}=(xe^{2y}+ye^{-yz}+2xye^{2y})\vec{i}-ye^{2y}\vec{j}+3(x-y)^2\vec{k}$; $div\ \vec{F}=3(x-y)^2-ze^{-yz}$

4) For a differentiable function $f(x,y,z)$

(1) compute $\nabla f(x,y,z)$

(2) is $\nabla f(x,y,z)$ a scalar or vector ?

(3) Show that $\nabla \cdot \nabla f = \dfrac{\partial^2 f}{\partial x^2} + \dfrac{\partial^2 f}{\partial y^2} + \dfrac{\partial^2 f}{\partial z^2}$

This is known as the Laplacian and is also written $\nabla^2 f$ (Problem 33, page 485).

ANS (1) $\nabla f(x,y,z) = \dfrac{\partial f}{\partial x}\vec{i} + \dfrac{\partial f}{\partial y}\vec{j} + \dfrac{\partial f}{\partial z}\vec{k}$

(2) a vector

(3) $\nabla . \nabla f = \nabla .(f_x \vec{i} + f_y \vec{j} + f_z \vec{k}) = f_{xx} + f_{yy} + f_{zz}$

5) Evaluate $\int_C G(x,y)dx$, $\int_C G(x,y)dy$ and $\int_C G(x,y)ds$ on the indicated curve C

$G(x,y) = 2xy$; $x = 5\cos(t)$, $y = 5\sin(t)$, $0 \le t \le \pi/4$ (Problem 1, page 493).

ANS $\int_C 2xydx = \int_0^{\pi/4} 2(5\cos t)(5\sin t)(-5\sin t)dt = -250\int_0^{\pi/4} \sin^2 t\cos t dt$

$= -250(\frac{1}{3}\sin^3 t)\Big|_0^{\pi/4} = -\dfrac{125\sqrt{2}}{6}$

$\int_C 2xydy = \int_0^{\pi/4} 2(5\cos t)(5\sin t)(5\cos t)dt = 250\int_0^{\pi/4} \cos^2 t\sin t dt = 250(-\frac{1}{3}\cos^3 t)\Big|_0^{\pi/4}$

$= \dfrac{250}{3}(1 - \dfrac{\sqrt{2}}{4}) = \dfrac{125}{6}(4 - \sqrt{2})$

$\int_C 2xyds = \int_0^{\pi/4} 2(5\cos t)(5\sin t)\sqrt{25\sin^2 t + 25\cos^2 t}\,dt = 250\int_0^{\pi/4} \sin t\cos t dt$

$= 250(\frac{1}{2}\sin^2 t)\Big|_0^{\pi/4} = \dfrac{125}{2}$