HOMEWORK #8s (15.3)

Due on May 17

1) In this problem, find the Fourier integral representation of the given function.

$$f(x)=\begin{cases} 0, & x<-1 \\ -1, & -1<x<0 \\ 2, & 0<x<1 \\ 0, & x>1 \end{cases} \quad \text{(Problem 1, page 750)}$$

ANS From formulas (5) and (6) in the text,

$$A(\alpha)=\int_{-1}^{0}(-1)\cos\alpha x dx+\int_{0}^{1}(2)\cos\alpha x dx=-\frac{\sin\alpha}{\alpha}+2\frac{\sin\alpha}{\alpha}=\frac{\sin\alpha}{\alpha}$$

and

$$B(\alpha)=\int_{-1}^{0}(-1)\sin\alpha x dx+\int_{0}^{1}(2)\sin\alpha x dx=\frac{1-\cos\alpha}{\alpha}-2\frac{\cos\alpha-1}{\alpha}=\frac{3(1-\cos\alpha)}{\alpha}$$

Hence

$$f(x)=\frac{1}{\pi}\int_{0}^{\infty}\frac{\sin\alpha\cos\alpha x+3(1-\cos\alpha)\sin\alpha x}{\alpha}d\alpha$$

2) In this problem, find the Fourier integral representation of the given function.

$$f(x)=\begin{cases} 0, & x<0 \\ e^{-x}, & x>0 \end{cases} \quad \text{(Problem 5, page 750)}$$

ANS From formula (5) in the text,

$$A(\alpha)=\int_{0}^{\infty}e^{-x}\cos\alpha x dx$$

$$\cos\alpha x=\frac{e^{i\alpha x}+e^{-i\alpha x}}{2}$$

$$A(\alpha)=\int_{0}^{\infty}e^{-x}\frac{e^{i\alpha x}+e^{-i\alpha x}}{2}dx$$

➔

$$=\int_{0}^{\infty}\frac{e^{i\alpha x-x}+e^{-i\alpha x-x}}{2}dx$$

$$=\frac{1}{2}\left[\frac{e^{i\alpha x}e^{-x}}{i\alpha-1}+\frac{e^{-i\alpha x}e^{-x}}{-i\alpha-1}\right]\Bigg|_{0}^{\infty}$$

$$=\frac{1}{2}\left[-\frac{1}{i\alpha-1}-\frac{1}{-i\alpha-1}\right]=-\frac{1}{2}\frac{-2}{-(i\alpha)^2+1}=\frac{1}{\alpha^2+1}$$

$$B(\alpha)=\int_{0}^{\infty}e^{-x}\sin\alpha x dx$$

similarly, we can obtain $B(\alpha)=\dfrac{\alpha}{1+\alpha^2}$

Hence $f(x)=\dfrac{1}{\pi}\int_0^\infty \dfrac{\cos\alpha x+\alpha\sin\alpha x}{1+\alpha^2}d\alpha$

3) In this problem, represent the given function by an appropriate cosine or sine integral

$$f(x)=\begin{cases}|x|, & |x|<\pi\\ 0, & |x|>\pi\end{cases}$$ (Problem 9, page 750)

ANS The function is even. Thus from formula (9) in the text

$$A(\alpha)=\int_0^\pi x\cos\alpha x dx=\frac{x\sin\alpha x}{\alpha}\Big|_0^\pi-\frac{1}{\alpha}\int_0^\pi\sin\alpha x dx$$

$$=\frac{\pi\alpha\sin\pi\alpha}{\alpha}+\frac{1}{\alpha^2}\cos\alpha x\Big|_0^\pi=\frac{\pi\alpha\sin\pi\alpha+\cos\pi\alpha-1}{\alpha^2}$$

Hence from formula (8) in the text,

$$f(x)=\frac{2}{\pi}\int_0^\infty\frac{(\pi\alpha\sin\pi\alpha+\cos\pi\alpha-1)\cos\alpha x}{\alpha^2}d\alpha$$

4) In this problem, find the cosine and sine integral representations of the given function

$f(x)=xe^{-2x},\quad x>0$ (Problem 15, page 750)

ANS For the cosine integral,

$$A(\alpha)=\int_0^\infty xe^{-2x}\cos\alpha x dx$$

Similar to Problem 2 ➔ $A(\alpha)=\dfrac{4-\alpha^2}{(4+\alpha^2)^2}$

Hence $f(x)=\dfrac{2}{\pi}\int_0^\infty\dfrac{(4-\alpha^2)\cos\alpha x}{(4+\alpha^2)^2}d\alpha$

For the sine integral, $B(\alpha)=\int_0^\infty xe^{-2x}\sin\alpha x dx$

we obtain $B(\alpha)=\dfrac{4\alpha}{(4+\alpha^2)^2}$

Hence

$$f(x)=\frac{8}{\pi}\int_0^\infty\frac{\alpha\sin\alpha x}{(4+\alpha^2)^2}d\alpha$$