Review exercise 2

1) A real-valued function f is said to be **periodic** with period T if $f(x+T)=f(x)$. For example, 4π is a period of $\sin x$ since $\sin(x+4\pi)=\sin x$. The smallest value of T for which $f(x+T)=f(x)$ holds is called the **fundamental period** of f. For example, the fundamental period of $f(x)=\sin x$ is $T=2\pi$. What is the fundamental period of each of the following functions ? (Exercise 12.1, problem 21)

   (a) $f(x)=\cos 2\pi x$ (b) $f(x)=\sin\frac{4}{L}x$

   (c) $f(x)=\sin x+\sin 2x$ (d) $f(x)=\sin 2x+\cos 4x$

   (e) $f(x)=\sin 3x+\cos 2x$ (f) $f(x)=A_0+\sum_{n=1}^{\infty}(A_n\cos\frac{n\pi}{p}x+B_n\sin\frac{n\pi}{p}x)$, A_n and B_n depend only on n

2) In this problem, find the Fourier series of f on the given interval

$$f(x)=\begin{cases} 0, & -2<x<-1 \\ -2, & -1\le x<0 \\ 1, & 0\le x<1 \\ 0, & 1\le x<2 \end{cases}$$ (Exercise 12.2, Problem 11)

3) In this problem, expand the given function in an appropriate cosine or sine series

   $f(x)=|x|$, $-\pi<x<\pi$ (Exercise 12.3, Problem 13)

4) In this problem, expand the given function in an appropriate cosine or sine series $f(x)=x^2$, $-1<x<1$ (Exercise 12.3, Problem 15)

5) In this problem, find the half-range cosine and sine expansions of the given function $f(x)=x^2+x$, $0<x<1$ (Exercise 12.3, Problem 33)

6) In this problem, expand the given function in a Fourier series $f(x)=x^2$, $0<x<2\pi$ (Exercise 12.3, Problem 35)

7) Find the frequency spectrum of the periodic wave that is the periodic extension of the function f in Problem 1 (Exercise 12.4, problem 7)

   In Problem 1, find the complex Fourier series of f on the given interval

$$f(x)=\begin{cases}-1, & -2<x<0\\ 1, & 0<x<2\end{cases}$$

8) Suppose the function $f(x)=x^2+1$, $0<x<3$, is expanded in a Fourier series , a cosine series, and a sine series. Give the value to which each series will converge at $x=0$ (Chapter 12 Review Exercises, problem 7)