Engineering Mathematics II---Quiz-3

April 28, 2006

1) Consider the function $f = x^2 y^4$. At (1,1) what is:

(1) The rate of change of $f = x^2 y^4$ in the direction of $\vec{i}$? (5 scores)

(2) The rate of change of $f = x^2 y^4$ in the direction of $\vec{i} + \vec{j}$? (5 scores)

2) Find the indicated expression for the vector field $\vec{F} = x^2 y\vec{i} + 2xyz\vec{k}$

(1) $\nabla \cdot \vec{F}$, (2) $\nabla \times \vec{F}$, (3) $\nabla \cdot (\nabla \times \vec{F})$, (4) $\nabla(\nabla \cdot \vec{F})$ (15 scores)

3) $\vec{F} = \dfrac{y}{x^2 + y^2}\vec{i} - \dfrac{x}{x^2 + y^2}\vec{j}$; Evaluate $\oint_C \vec{F} \cdot dr$

(1)C is shown as Fig1(a). (Hint: Using direct integral) (5 scores)

(2)C is shown as Fig1(b). (Hint: Using Green's theorem) (5 scores)

(3) C is shown as Fig1(c). (Hint: Using Green's theorem) (5 scores)

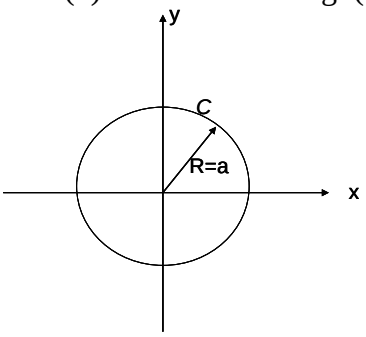

Fig. 1(a)

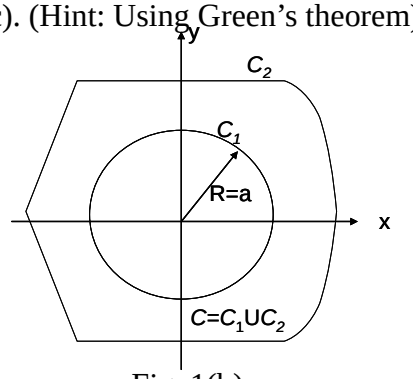

Fig. 1(b)

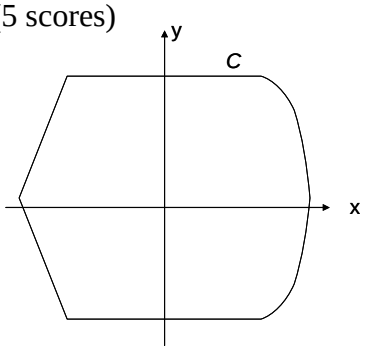

Fig. 1(c)

4) Suppose $\vec{r}(t) = 2t\vec{i} + (t^3 - 2t)\vec{j} + (t^2 - 5t)\vec{k}$ is the position vector of a moving particle. What are its speed, velocity, acceleration, curvature and tangent line at the point (0,0,0) ? (15 scores)

5) S is the portion of the plane $x + 2y + 3z = 12$ in the first octant.

(1) find the area of S (5 scores)

(2) find the upper unit normal of S (5 scores)

(3) Evaluate $\iint_S (3z^2 + 4yz)\, dS$ (10 scores)

6) If S is the surface of the region bounded by $x^2 + y^2 = 9$, $z = \sqrt{16 - x^2 - y^2}$, $z = 0$. $\vec{F} = -y^3\vec{i} - x^3\vec{j} + z^3\vec{k}$

(1) find the volume of the solid bounded by $x^2 + y^2 = 9$, $z = \sqrt{16 - x^2 - y^2}$, $z = 0$. (10 scores)

(2) use the divergence theorem to find the outward flux $\iint_S \left(\vec{F} \cdot \vec{n}\right) dS$ (15 scores)