Fourier coefficients

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Function decomposition: discrete form:

any time function, $f(t)$, with a period $2p$, we have

$$f(t) = \frac{1}{2}a_0 + \sum_{n=1}^{\infty}\{a_n cos(\frac{n\pi t}{p}) + b_n sin(\frac{n\pi t}{p})\}$$

Fourier coefficients:

$$a_n = \frac{1}{p}\int_0^{2p} f(t)cos(\frac{n\pi t}{p})dt$$
$$b_n = \frac{1}{p}\int_0^{2p} f(t)sin(\frac{n\pi t}{p})dt$$

Orthogonal relation for the bases:

$$p\delta_{ij} = \int_0^{2p} cos(\frac{i\pi t}{p})cos(\frac{j\pi t}{p})dt$$
$$p\delta_{ij} = \int_0^{2p} sin(\frac{i\pi t}{p})sin(\frac{j\pi t}{p})dt$$
$$0 = \int_0^{2p} sin(\frac{i\pi t}{p})cos(\frac{j\pi t}{p})dt$$

Minimize the distance, D, between $f(t)$ and the Fourier series:

$$D = \int_0^{2p} \mid \{f(t) - [\frac{1}{2}a_0 + \sum_{n=1}^{\infty}\{a_n cos(\frac{n\pi t}{p}) + b_n sin(\frac{n\pi t}{p})\}]\} \mid^2 dt$$

optimal a_n and b_n:

$$\frac{\partial D}{\partial a_n} = 0 \;\rightarrow\; a_n = \frac{1}{p}\int_0^{2p} f(t)cos(\frac{n\pi t}{p})dt$$
$$\frac{\partial D}{\partial b_n} = 0 \;\rightarrow\; b_n = \frac{1}{p}\int_0^{2p} f(t)sin(\frac{n\pi t}{p})dt$$

【存檔：c:/ctex/course/math2/cof1.te】【建檔:Mar./3/'97】