國立臺灣海洋大學河海工程學系2004 工程數學 (三) 第二次小考

1. Given a spiral curve, we can describe by time-like parameter as follows:

$$x(t) = 4\,cos(t), y(t) = 4\,sin(t), z(t) = 3t \;\; \mathbf{r}(t) = (x(t), y(t), z(t))$$

Please describe the curve by using space-like parameter (arc length s).

2. Plot the curve from the starting point of $(4, 0, 0)$?

3. What is the distance of the arc length of the curve from $t = 0$ to $t = 2\pi$?

4. Please determine the radius of curvature for ρ and σ as shown below:

$$\begin{Bmatrix} \dot{\hat{\mathbf{t}}} \\ \dot{\hat{\mathbf{n}}} \\ \dot{\hat{\mathbf{b}}} \end{Bmatrix} = \begin{bmatrix} 0 & \frac{1}{\rho} & 0 \\ \frac{-1}{\rho} & 0 & \frac{1}{\sigma} \\ 0 & \frac{-1}{\sigma} & 0 \end{bmatrix} \begin{Bmatrix} \hat{\mathbf{t}} \\ \hat{\mathbf{n}} \\ \hat{\mathbf{b}} \end{Bmatrix} \tag{1}$$

5. Determine

$$(\frac{d\mathbf{r}}{ds} \times \frac{d^2\mathbf{r}}{ds^2}) \cdot \frac{d^3\mathbf{r}}{ds^3}$$