Properties of Fourier transform

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1 Definition of Fourier transform($f(t)$ is real):

$$F(\omega) = \mathcal{F}\{f(t)\} = \int_{-\infty}^{\infty} f(t) e^{-i\omega t} dt, \quad F(-\omega) = F^*(\omega)$$

2 Definition of Inverse Fourier transform:

$$u(t) = \mathcal{F}^{-1}\{F(\omega)\} = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(\omega) e^{i\omega t} d\omega$$

3 Linear property:

$$\mathcal{F}\{a\ f(t) + b\ g(t)\} = a\ F(\omega) + b\ G(\omega)$$

4 Double Fourier transform:

$$\mathcal{F}\{\mathcal{F}\{f(t)\}\} = 2\pi f(-t)$$

5 Time shifting:

$$\mathcal{F}\{f(t-a)\} = e^{-i\omega a} F(\omega)$$

6 Frequency shifting:

$$\mathcal{F}\{e^{i\omega_0 t} f(t)\} = F(\omega - \omega_0)$$

7 Time derivative:

$$\mathcal{F}\{f'(t)\} = i\omega\, F(\omega)$$

8 Frequency derivative:

$$F'(\omega) = \mathcal{F}\{-itf(t)\}$$

9 Convolution:

$$\mathcal{F}\{f(t) * g(t)\} = F(\omega) G(\omega)$$

where

$$f(t) * g(t) = \int_{-\infty}^{\infty} f(u) g(t-u) du$$

10 Energy conservation:

$$\int_{-\infty}^{\infty} f^2(t) dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} \mid F(\omega) \mid^2 d\omega$$