

工程數學—Vector operators for radial basis functions

1. Given the radial position vector and radial basis function,

$$\mathbf{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$$

$$r = \sqrt{x^2 + y^2 + z^2}$$

$$\nabla\phi(r) = \frac{d\phi}{dr}\hat{\mathbf{r}}$$

$$\nabla \cdot [\phi(r)\mathbf{r}] = 3\phi(r) + r\frac{d\phi(r)}{dr}$$

$$\nabla^2\phi(r) = \frac{d^2\phi(r)}{dr^2} + \frac{2}{r}\frac{d\phi(r)}{dr}$$

$$\nabla \times [\phi(r)\mathbf{r}] = \mathbf{0}$$

$$\nabla r = \hat{\mathbf{r}}$$

$$\nabla \cdot \mathbf{r} = 3$$

$$\nabla(1/r) = -\frac{\hat{\mathbf{r}}}{r^2}$$

$$\nabla \cdot \left(\frac{\hat{\mathbf{r}}}{r^2}\right) = -\nabla^2(1/r) = 4\pi\delta(r)$$