工程數學—空間曲線描述法

Given

$$\left\{ \begin{array}{c} \dot{\tau} \\ \dot{\nu} \\ \dot{\beta} \end{array} \right\} = \left[\begin{array}{ccc} 0 & \frac{1}{\rho} & 0 \\ \frac{-1}{\rho} & 0 & \frac{1}{\sigma} \\ 0 & \frac{-1}{\sigma} & 0 \end{array} \right] \left\{ \begin{array}{c} \tau \\ \nu \\ \beta \end{array} \right\} \tag{1}$$

(1) Set $\mathbf{x} = \left\{ \begin{array}{c} \tau \\ \nu \\ \beta \end{array} \right\}$,

$$\left\{ \begin{array}{c} \dot{\tau}_1 \\ \dot{\tau}_2 \\ \dot{\tau}_3 \\ \dot{\nu}_1 \\ \dot{\nu}_2 \\ \dot{\nu}_3 \\ \dot{\beta}_1 \\ \dot{\beta}_2 \\ \dot{\beta}_3 \end{array} \right\} = \left[\begin{array}{ccccccccc} 0 & 0 & 0 & \frac{1}{\rho} & \frac{1}{\rho} & \frac{1}{\rho} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{\rho} & \frac{1}{\rho} & \frac{1}{\rho} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{\rho} & \frac{1}{\rho} & \frac{1}{\rho} & 0 & 0 & 0 \\ \frac{-1}{\rho} & \frac{-1}{\rho} & \frac{-1}{\rho} & 0 & 0 & 0 & \frac{1}{\sigma} & \frac{1}{\sigma} & \frac{1}{\sigma} \\ \frac{-1}{\rho} & \frac{-1}{\rho} & \frac{-1}{\rho} & 0 & 0 & 0 & \frac{1}{\sigma} & \frac{1}{\sigma} & \frac{1}{\sigma} \\ \frac{-1}{\rho} & \frac{-1}{\rho} & \frac{-1}{\rho} & 0 & 0 & 0 & \frac{1}{\sigma} & \frac{1}{\sigma} & \frac{1}{\sigma} \\ 0 & 0 & 0 & \frac{-1}{\sigma} & \frac{-1}{\sigma} & \frac{-1}{\sigma} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{-1}{\sigma} & \frac{-1}{\sigma} & \frac{-1}{\sigma} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{-1}{\sigma} & \frac{-1}{\sigma} & \frac{-1}{\sigma} & 0 & 0 & 0 \end{array} \right] \left\{ \begin{array}{c} \tau_1 \\ \tau_2 \\ \tau_3 \\ \nu_1 \\ \nu_2 \\ \nu_3 \\ \beta_1 \\ \beta_2 \\ \beta_3 \end{array} \right\} \tag{2}$$

Reduce the Eq. (2) to simple case

$$\left\{ \begin{array}{c} \dot{\tau}_1 \\ \dot{\nu}_1 \\ \dot{\beta}_1 \end{array} \right\} = \left[\begin{array}{ccc} 0 & \frac{1}{\rho} & 0 \\ \frac{-1}{\rho} & 0 & \frac{1}{\sigma} \\ 0 & \frac{-1}{\sigma} & 0 \end{array} \right] \left\{ \begin{array}{c} \tau_1 \\ \nu_1 \\ \beta_1 \end{array} \right\} \tag{3}$$

$$\left\{ \begin{array}{c} \tau_1 \\ \nu_1 \\ \beta_1 \end{array} \right\} = e^{As} \left\{ \begin{array}{c} \tau_1(0) \\ \nu_1(0) \\ \beta_1(0) \end{array} \right\} \tag{4}$$

If ρ and σ are constants and initial conditions $\left\{ \begin{array}{c} \tau(s) \\ \nu(s) \\ \beta(s) \end{array} \right\} = \left\{ \begin{array}{c} 1 \\ 0 \\ 0 \end{array} \right\}$, what is the curve determined by Eq. (3)?

Eq.(3) can be reformulated to $\dot{\mathbf{x}}$=W $\mathbf{x}$=$\omega\times\mathbf{x}$, find matrix of W and vector of ω.