工程數學—空間曲線描述法

Given

$$\left\{\begin{array}{c}\dot{\tau}\\ \dot{\nu}\\ \dot{\beta}\end{array}\right\}=\left[\begin{array}{ccc}0 & \frac{1}{\rho} & 0\\ \frac{-1}{\rho} & 0 & \frac{1}{\sigma}\\ 0 & \frac{-1}{\sigma} & 0\end{array}\right]\left\{\begin{array}{c}\tau\\ \nu\\ \beta\end{array}\right\} \tag{1}$$

(1) Set $\mathbf{x}=\left\{\begin{array}{c}\tau\\ \nu\\ \beta\end{array}\right\}$,

$$\left\{\begin{array}{c}\dot{\tau}_1\\ \dot{\tau}_2\\ \dot{\tau}_3\\ \dot{\nu}_1\\ \dot{\nu}_2\\ \dot{\nu}_3\\ \dot{\beta}_1\\ \dot{\beta}_2\\ \dot{\beta}_3\end{array}\right\}=\left[\begin{array}{ccccccccc}0&0&0&\frac{1}{\rho}&\frac{1}{\rho}&\frac{1}{\rho}&0&0&0\\ 0&0&0&\frac{1}{\rho}&\frac{1}{\rho}&\frac{1}{\rho}&0&0&0\\ 0&0&0&\frac{1}{\rho}&\frac{1}{\rho}&\frac{1}{\rho}&0&0&0\\ \frac{-1}{\rho}&\frac{-1}{\rho}&\frac{-1}{\rho}&0&0&0&\frac{1}{\sigma}&\frac{1}{\sigma}&\frac{1}{\sigma}\\ \frac{-1}{\rho}&\frac{-1}{\rho}&\frac{-1}{\rho}&0&0&0&\frac{1}{\sigma}&\frac{1}{\sigma}&\frac{1}{\sigma}\\ \frac{-1}{\rho}&\frac{-1}{\rho}&\frac{-1}{\rho}&0&0&0&\frac{1}{\sigma}&\frac{1}{\sigma}&\frac{1}{\sigma}\\ 0&0&0&\frac{-1}{\sigma}&\frac{-1}{\sigma}&\frac{-1}{\sigma}&0&0&0\\ 0&0&0&\frac{-1}{\sigma}&\frac{-1}{\sigma}&\frac{-1}{\sigma}&0&0&0\\ 0&0&0&\frac{-1}{\sigma}&\frac{-1}{\sigma}&\frac{-1}{\sigma}&0&0&0\end{array}\right]\left\{\begin{array}{c}\tau_1\\ \tau_2\\ \tau_3\\ \nu_1\\ \nu_2\\ \nu_3\\ \beta_1\\ \beta_2\\ \beta_3\end{array}\right\} \tag{2}$$

Determine ρ and σ for the following cases.

(1). Line — $(s,0,0)$

(2). Circle — $(cos(s),sin(s),0)$

(3). Circle — $(0,cos(s),sin(s))$

(4). Circular helix— $(cos(s/\sqrt{2}),sin(s/\sqrt{2}),s/\sqrt{2})$

(5). Helix— $(a\,cos(s\alpha/a),a\,sin(s\alpha/a),ssin(\alpha))$

(6). Curve — $(e^t cos(2t),e^t sin(2t),e^t)$

(7). Curve— $(1,t,t^2)$

(8). Plane curve by $xy=1$ at (1, 1).

(9). Plane curve by $y=x^2$ at (1, 1).

(10). Plane curve by $y=\sqrt{1-x^2}$ at (0, 1).