Method of complete integral

I. PDE for $u(x,y)$:

$$F(x,y,u,p,q)=0$$

II. Assume a two-parameter α abd β, we have

$$u=u(x,y,\alpha,\beta)$$

satisfy $F=0$. Such family of solution is called a complete integral of the solution.
III. Given a relation

$$\alpha=\alpha(s)$$

$$\beta=\beta(s)$$

$u=u(x,y,\alpha(s),\beta(s))$ given a one-parameter family solution. The group of the solution has a envelope function. This envelope is also a solution (singular solution).
IV. Example:

$$u_x^2+u_y^2=1$$

Two parameters solution is

$$u(x,y)=cos(\alpha)x+sin(\alpha)y+\beta$$

Choose specially

$$\beta=cos(\alpha)$$

Then

$$u=(x+1)cos(\alpha)+ysin(\alpha)$$

Differentiating with respect to α, we have

$$0=-(x+1)sin(\alpha)+ycos(\alpha)$$

Cancelling α, we have

$$u^2=(x+1)^2+y^2$$

V. To match the Cauchy data $(-1+cos(s),sin(s),1)$, we have

$$(-1+cos(s))cos(\alpha)+sin(s)sin(\alpha)+\beta=1 \quad (1)$$

$$-sin(s)cos(\alpha)+cos(s)sin(\alpha)=0 \quad (2)$$

Therefore,

$$\alpha=s \quad (3)$$

$$\beta=cos(s) \quad (4)$$

The solution is

$$u=cos(s)x+sin(s)y+cos(s)=cos(s)(1+x)+sin(s)y \quad (5)$$

$$0=sin(s)(x+1)+cos(s)y \quad (6)$$

The singular solution is

$$u^2=(x+1)^2+y^2$$