海洋大學河海工程學系 2005 工程數學(四)第一次作業小考解答
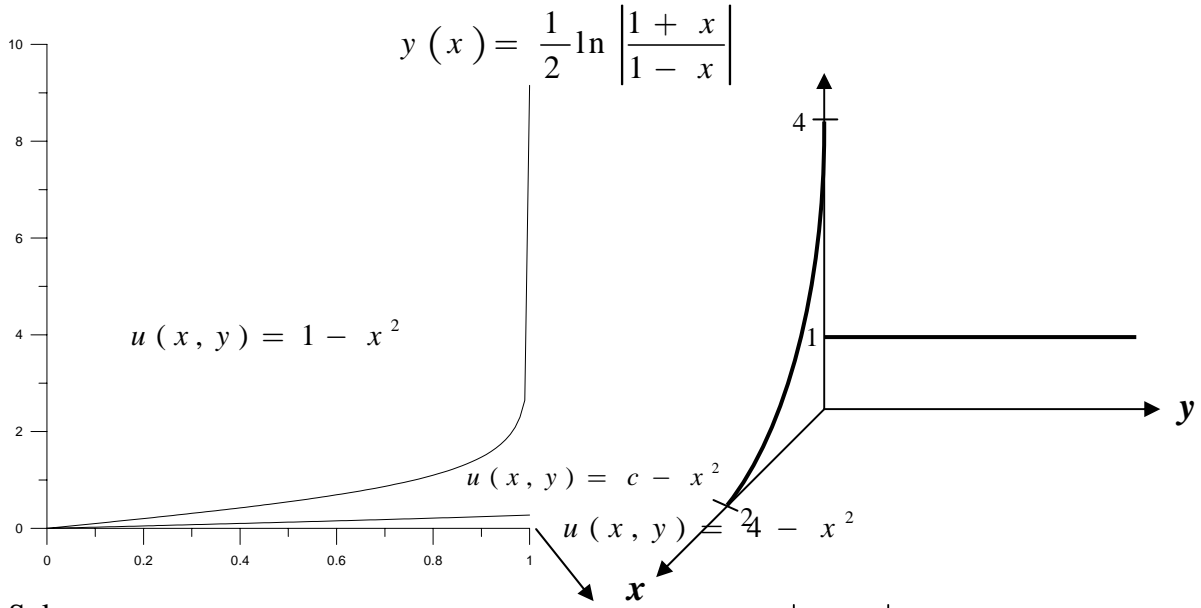
1. Solve $u(x, y)$ such that

$$\begin{cases} u_y + uu_x + 2ux = 0 & \text{for } y > 0 \text{ and } 0 < x < 1 \\ u(x,0) = 4 - x^2 & \text{for } 0 < x < 1 \\ u(0, y) = 1 & \text{for } y > 0 \end{cases}$$

$$y(x) = \frac{1}{2}\ln\left|\frac{1+x}{1-x}\right|$$

$$u(x, y) = 1 - x^2$$

$$u(x, y) = c - x^2$$

$$u(x, y) = 4 - x^2$$

$$y(x) = \frac{1}{4}\ln\left|\frac{2+x}{2-x}\right|$$

Sol:

(1)

dx/dt=u , x(0,s)=s

dy/dt=1 , y(0,s)=0

du/dt= -2ux , u(0,s)= 4-s²

→ y=t+g(s) , y(0,s)= 0+g(s)=0 , g(s)=0

∴y(t,s)=t

→ du/dx= -2x , u(x,y)= -x²+f(s) , u(0,s)= -s²+f(s)= 4-s² , f(s)=4

∴u(x,y)= 4-x²

(2)

dx/dt=u , x(0,s)=0

dy/dt=1 , y(0,s)=s

du/dt= -2ux , u(0,s)=1

→ y=t+g(s) , y(0,s)= 0+ g(s)=s , g(s)=s

∴y(t,s)=t+s

→ du/dx= -2x , u(x,y)= -x²+ f(s) , u(0,s)=0+f(s)=1 , f(s)=1

∴u(x,y)= 1-x²