Derivation of simultaneous five ODEs:

Cauchy problem

$$F(x, y, u, u_x, u_y) = 0$$

Given strip data

$(x(s), y(s), z(s), p(s), q(s))$

where $p(s)$ and $q(s)$ can be determined by

$$\begin{cases} F(x(s), y(s), z(s), p, q) = 0 \\ \dfrac{dh(s)}{ds} = p\dfrac{dx}{ds} + q\dfrac{dy}{ds} \end{cases}$$

$$z - z_0 = (x - x_0)p_0 + (y - y_0)q_0$$

$$0 = (x - x_0)p'_\lambda + (y - y_0)q'_\lambda$$

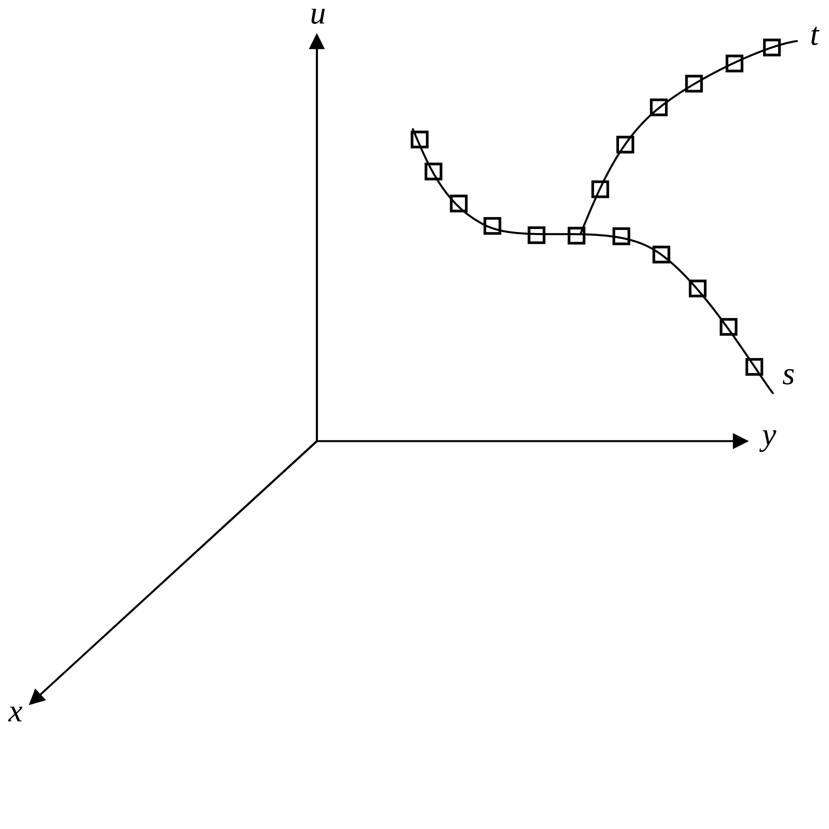
$$F(x, y, z, p(\lambda), q(\lambda)) = 0$$

$$\frac{\partial F}{\partial p}\frac{dp}{d\lambda} + \frac{\partial F}{\partial q}\frac{dq}{d\lambda} = 0$$

$$\frac{dx}{dt} = F_p$$

$$\frac{dy}{dt} = F_q$$

$$\frac{dz}{dt} = pF_p + qF_q$$



$$F(x, y, u, p, q) = 0$$

$$F_x + F_u u_x + \underline{F_p u_{xx} + F_q u_{yx}} = 0$$

$$F_y + F_u u_y + \underline{F_p u_{xy} + F_q u_{yy}} = 0$$

$$\frac{dp(t)}{dt} = u_{xx} F_p + u_{xy} F_q = -F_x - F_u u_x$$

$$\frac{dq(t)}{dt} = u_{yx} F_p + u_{yy} F_q = -F_y - F_u u_y$$

$$\begin{cases} \dfrac{dx}{dt} = F_p, & x(0,s) = x_0(s) \\ \dfrac{dy}{dt} = F_q, & y(0,s) = y_0(s) \\ \dfrac{du}{dt} = pF_p + qF_q, & u(0,s) = u_0(s) \\ \dfrac{dp}{dt} = -F_x - F_u p, & p(0,s) = p_0(s) \\ \dfrac{dq}{dt} = -F_y - F_u q, & q(0,s) = q_0(s) \end{cases}$$