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海洋大學河海工程學系 2005 工程數學(四)第一次作業

1. 課堂上老師教過求解 $\nabla\phi = 0$ 的可能解

猜: $\phi(x, y) = f(x + cy)$

$$\therefore \phi_{xx} = f''(x + cy) \cdot 1^2, \phi_{yy} = f''(x + cy) \cdot c^2$$

$$f''(x + cy)(1 + c^2) = 0$$

$$\therefore c = \pm i$$

$$\therefore \text{令 } \phi(x, y) = f(x + yi)$$

$$f(x) = 1 \rightarrow \phi = 1$$

$$f(x) = x \rightarrow \phi = x + yi$$

$$f(x) = x^2 \rightarrow \phi = x^2 - y^2 + 2xyi$$

$$f(x) = e^x \rightarrow \phi = e^x \cos y + ie^x \sin y$$

實虛部均滿足 Laplace 方程式

2. 仿照上述處理手法: 求函數滿足 $\nabla^2(\nabla^2\phi) = 0$ 亦即 $\phi_{xxxx} + 2\phi_{xxyy} + \phi_{yyyy} = 0$ 之解可能有哪些?