班級：_____學號：_____姓名：_____

海洋大學河海工程學系2005工程數學(四)第一次作業參考解答

1.課堂上老師教過求解$\nabla\phi=0$的可能解

猜:$\phi(x,y)=f(x+cy)$

$\therefore \phi_{xx}=f''(x+cy)\cdot 1^2, \phi_{yy}=f''(x+cy)\cdot c^2$

$f''(x+cy)(1+c^2)=0$

$\therefore c=\pm i$

$\therefore$ 令$\phi(x,y)=f(x+yi)$

$f(x)=1\rightarrow\phi=1$

$f(x)=x\rightarrow\phi=x+yi$

$f(x)=x^2\rightarrow\phi=x^2-y^2+2xyi$

$f(x)=e^x\rightarrow\phi=e^x\cos y+ie^x\sin y$

實虛部均滿足Laplace方程式

2.仿照上述處理手法:求函數滿足$\nabla^2(\nabla^2\phi)=0$亦即$\phi_{xxxx}+2\phi_{xxyy}+\phi_{yyyy}=0$之解可能有哪些?

sol:

$$\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial y^2}=0$$

$$\phi(x,y)\Rightarrow U(z,\overline{z}), z=x+yi, z=x-yi$$

$$\frac{\partial^2 U(z,\overline{z})}{\partial z\partial\overline{z}}=0\Rightarrow f(z)+g(\overline{z})$$

$$\phi_{\xi\xi\xi\xi}+2\phi_{\xi\xi\psi\psi}+\phi_{\psi\psi\psi\psi}=0\Rightarrow\frac{\partial^2 U(z,\overline{z})}{\partial z^2\partial\overline{z}^2}=0\Rightarrow\overline{z}f(z)+\psi(z)$$

where $f(z)$ and $\psi(z)$ are analytical functions.