班級：_____學號：_____姓名：_____

海洋大學河海工程學系 2005 工程數學(四)第六次作業

1. Derivation of an analytical solution for string vibration.

<step 1> Guess $u(x,t)=X(x)T(t)$

<step 2> Satisfy the PDE $c^2X''(x)T(t)=X(x)\ddot{T}(t), \dfrac{X''(x)}{X(x)}=\dfrac{\ddot{T}(t)}{c^2T(t)}=k$

<step 3> Satisfy B.C $X(x)=X(l)=0$

<step 4> Find eigenvalues k_n and eigenfuctions $X_n(x)$ ，

$$X''(x)=-\lambda_n{}^2X(x), X(0)=X(l)=0$$
$$k=-\lambda_n{}^2, X(x)=X_n(x), n=1,2,3.....$$

<step 5> Find $T_n(t)$ ，
$$\ddot{T}(t)=-c^2\lambda_n{}^2T(t)$$
$$T_n(t)=P_n\cos(c\lambda_n t)+Q_n\sin(c\lambda_n t)$$

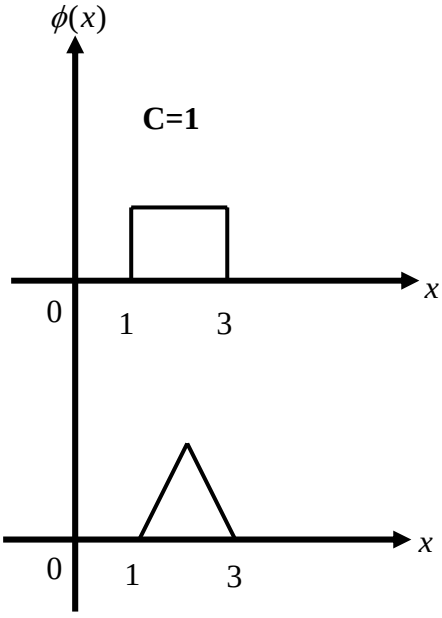
$$u(x,t)=\sum_{n=1}^{\infty}X_n(x)\left[P_n\cos(c\lambda_n t)+Q_n\sin(c\lambda_n t)\right]$$

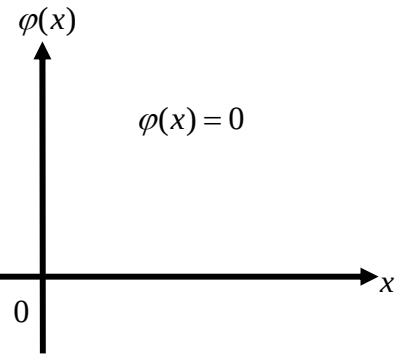
<step 6> Find P_n and Q_n

$$u(x,0)=\phi(x)=\sum_{n=1}^{\infty}P_nX_n(x)\rightarrow P_n$$

$$u'(x,0)=\varphi(x)=\sum_{n=1}^{\infty}cQ_n\lambda_nX_n(x)\rightarrow Q_n$$

Given





$$c^2\frac{\partial^2u}{\partial x^2}=\frac{\partial^2u}{\partial t^2}$$
$$u(x,t)=u(l,t)=0$$
$$u(x,0)=\phi(x)$$
$$\dot{u}(x,0)=\varphi(x)$$

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