Flow-net problem

Another benchmark problem is the ground water flow under a coffer dam. By considering the equation of continuity of flow, Darcy law, it can be shown that a Laplace operator governs the potential of ground water, (φ). For an isotropic soil with constant hydraulic conductivity the problem can be formulated as

$$\frac{\partial^2\varphi}{\partial x^2}+\frac{\partial^2\varphi}{\partial y^2}=0 \qquad (24)$$

In Addition, boundary condition for this problem is illustrated in Figure 14.

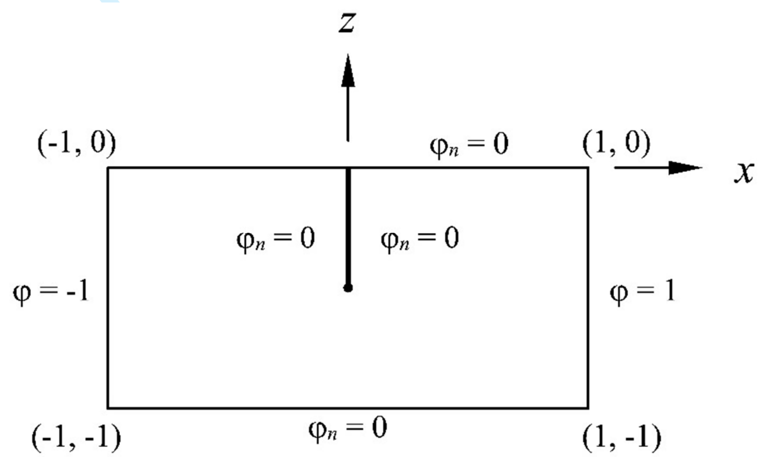

Figure 14. The flow-net problem.

It is noticeable a singularity at $(0,-1/2)$ roots from the defined boundary conditions. The final obtained result presented by XEBF method is shown in Figure 15.

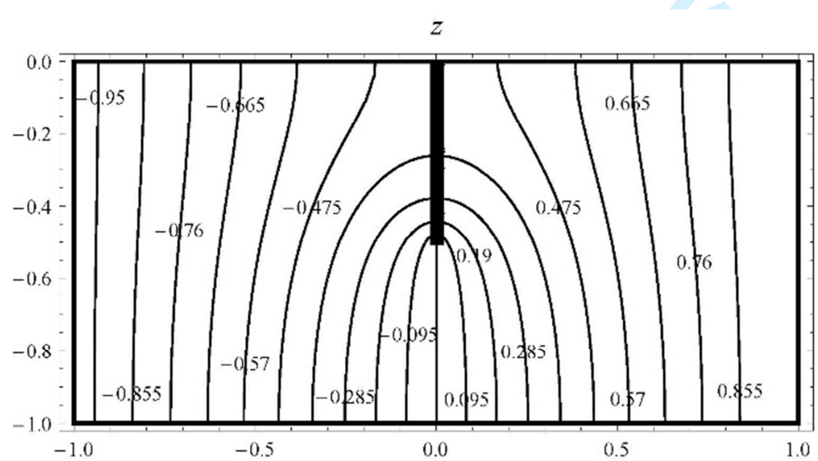

Figure 15. The results obtained by the XEBF method for flow-net problem.

For the sake of quantitative comparison, the results of three other methods are taken into account. MFS (Young et al. 2006), the dual boundary element method (J. T. Chen & Y. W. Chen 2000), the exact solution (Liggett & P. L.-F. Liu 1983) and the XEBF method results of potential value beneath the sheet pile are shown in Table 4.

Table 4. Comparison of flow potential beneath the sheet pile

Position of the nodes(z)	Values				Errors(%)		
	Exact	MFS	BEM	XEBF	MFS	BEM	XEBF
-0.48586	0.1096	0.1123	0.1005	0.1089	2.463	8.302	0.588
-0.45757	0.1869	0.1838	0.1803	0.1858	1.658	3.531	0.549
-0.35858	0.3225	0.3175	0.3117	0.3206	1.550	3.348	0.581
-0.35858	0.4057	0.4070	0.4119	0.4032	0.320	1.528	0.592
-0.2391	0.4159	0.4194	0.4029	0.4135	0.841	3.125	0.571
0.0	0.4614	0.4728	0.4555	0.4582	2.470	1.278	0.678

The percentage of errors shows clearly that XEBF outperforms the results of other methods.

Conclusions

We have presented a new approach based on extending the method of exponential basis functions (EBF) for the solution of boundary singularity problems. The proposed method incorporates the form of singularity by adding a few EBF-Like terms from the asymptotic expansion of the solution near the singular point, and the coefficients of these terms are obtained through the same method as the original EBF method. The remarkable feature of the proposed method which we referred to it as XEBF is that high convergence rates can be obtained by adding only a few of these influence functions, while no additional post-processing is needed. We focused on 2D Laplace problems with boundary singularities and applied the method for solution of some well-known benchmark problems. For the purpose of comparison, the results of some other boundary approximation methods were taken into account. The results demonstrated that the proposed method not only does perform robust but also it offers many advantages over some other boundary approximation methods.

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