ATTENUATION FOR THE SIMPLE EXPANSION CHAMBER MUFFLER WITH A RIGHT ANGLE INLET

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ABSTRACT

The simple expansion chamber muffler is a base control device whose use is to attenuate sound power, and has been known for a long time. Some of the most compelling research has focused on the mufflers with a straight inlet and outlet. To date, a muffler with a right angle inlet has never been studied. Therefore, the purpose of this work is to analyze the simple expansion chamber muffler with a right angle inlet. The numerical results show a comparable agreement between the experiment and other numerical approaches. A discussion is also presented in this work on the muffler with a different radius to that of the inlet/outlet pipe, a different inlet part, *etc*. The result clearly shows that the attenuation of the muffler with a right angle inlet is better than that with a straight inlet at particular frequencies. In addition, the mufflers do not have any absorbent linings attached to the inside of them, unlike the right angle-inlets which do. However, the propagation of sound in the muffler with a right angle inlet can lead its sound power to be attenuated about 10 ~ 40dB at those particular frequencies.

Keywords : Muffler, Silencer, Boundary element method, Transfer function.

1. INTRODUCTION

An acoustic filter is a widely employed control device for sound attenuation in the automobile industry. Its performance has also been calculated by different numerical simulations, such as the finite element method (FEM) [1-5], the boundary element method (BEM) [6-11] and plane wave theory [12-15]. The plane wave theory is a convenient and fast approach, but it is limited in that it can only predict the muffler attenuation below the cut-off frequency. It is essential to take into account the higher order effects in the muffler, such as the fact that noise pollution includes various frequencies. In addition, FEM in particular requires high computational and memory resources for three dimensional acoustic problems or at high frequencies. For those reasons, the BEM is adopted to analyze the performance of mufflers in the present work.

Generally speaking, one could make use of the dissipative mufflers [6,10,15,16-18] to make the sound power weaker, especially over a broad mid to high frequency range. Indeed, this could lead to better attenuation; however, the absorbent linings can actually decrease performance under conditions such as the fiber being clogged with particles, wear and tear and so forth. A reactive muffler [8,14,15,9-21] with broadband sound attenuation is more suitable in a motor vehicle in order to avoid the aforementioned defects. It is worth mentioning that a muffler without absorbent linings but with broadband sound attenuation has been successfully demonstrated in this work.

This study focused on simple expansion mufflers with right angle inlets. The numerical results have been compared with experiments and the modal meshing approach [22-25]. The results clearly show an accurate correlation. At particular frequencies, the attenuation of the muffler with a right angle inlet is more effective than the muffler with a straight inlet. It is significant that the mufflers do not have any absorbent linings attached to the inside of them, and only the inlet part has been customized (from a straight angle to a right angle). The propagation of sound in this muffler could be attenuated by about 10 ~ 40dB in particular frequencies.

2. BASIC FORMULATION

The geometric of the mufflers with right angle inlet is shown in Fig. 1(a). In addition, the parameters are described as the following requirements: All of radiuses of the inlet pipe and outlet pipe are r_1 cm, the radius of the expansion chamber is r_2 cm, and the length of expansion chamber is L cm.

For an interior acoustic problem in the condition of stationary medium, acoustic pressure p should be determined by means of the Helmholtz equation and the governing equation is given by the following expression:

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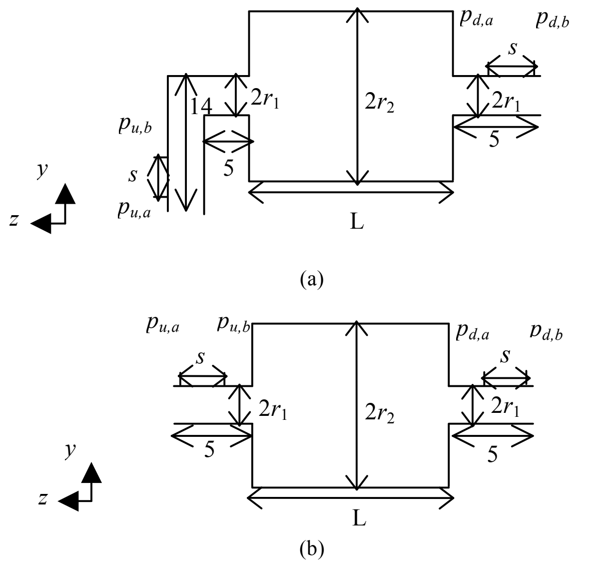


Fig. 1 (a) The setup of the simple expansion muffler with a right angle inlet. (b) The setup of the simple expansion muffler with a straight inlet, unit: cm

$$\nabla^2 p + k^2 p = 0 \tag{1}$$

where ∇^2 means the Laplacian operator, k is the wave number. Making use of the Green second identity and introducing the fundamental solution of the Helmholz equation, the boundary integral formulation can be derived as [26,27]:

$$\begin{aligned} &C(\vec{x})p(\vec{x}) \\ &= \int_S G(\vec{x}, \vec{y}) \frac{\partial p(\vec{y})}{\partial n(\vec{y})}\, dS(\vec{y}) \\ &\quad - \int_S \frac{\partial G(\vec{x}, \vec{y})}{\partial n(\vec{y})} p(\vec{y})\, dS(\vec{y})\ , \ \vec{x} \in S\ , \end{aligned} \tag{2}$$

where $\vec{x}$ means the field point, $\vec{y}$ is the source point, S indicates the surface of the domain, $\partial / \partial n(\vec{y})$ denotes an outward normal derivative at the source point, and $C(\vec{x})$ represents the coefficient of a solid angle of the field point. The free space fundamental solution of the Helmholtz equation is written in Eq. (3) and satisfies Eq. (4).

$$G(\vec{x}, \vec{y}) = \frac{e^{-jkr(\vec{x}, \vec{y})}}{4\pi r(\vec{x}, \vec{y})}\ , \tag{3}$$

where $r(\vec{x}, \vec{y})$ is the distance between the field and source point, and j is the square root of -1.

$$\nabla^2 G(\vec{x}, \vec{y}) + k^2 G(\vec{x}, \vec{y}) = -\delta(\vec{x}, \vec{y})\ , \tag{4}$$

where $\delta(\vec{x}, \vec{y})$ is called the Dirac delta function.

To solve the problem, the surface is divided into N elements. The state variables of pressure and pressure gradient are assumed to be constant and equal to the value on each center of element surface S_β. The coefficient $C(\vec{x})$ can be replaced with the value of 1/2, inasmuch as the node is located on a smooth part of the boundary. Accordingly, the boundary integral formulation Eq. (2) can be rewritten as:

$$\begin{aligned} \frac{1}{2} p(\vec{x}) &= \sum_{\beta=1}^{N} \int_{S_\beta} G(\vec{x}, \vec{y}) \frac{\partial p(\vec{y})}{\partial n(\vec{y})}\, dS_\beta(\vec{y}) \\ &\quad - \sum_{\beta=1}^{N} \int_{S_\beta} \frac{\partial G(\vec{x}, \vec{y})}{\partial n(\vec{y})} p(\vec{y})\, dS_\beta(\vec{y})\ , \end{aligned} \tag{5}$$

By making use of Eq. (5) to each node and integrating along each element, the linear algebraic system equations could be expressed in matrix form:

$$[A_{\text{in}} \ \ A_{\text{out}} \ \ A_{\text{other}}] \begin{Bmatrix} p_{\text{in}} \\ p_{\text{out}} \\ p_{\text{other}} \end{Bmatrix} = [B_{\text{in}} \ \ B_{\text{out}} \ \ B_{\text{other}}] \begin{Bmatrix} p_{\text{n,in}} \\ p_{\text{n,out}} \\ p_{\text{n,other}} \end{Bmatrix}\ , \tag{6}$$

where p and p_n are column vectors of pressure and the pressure gradient, respectively. The suffixes, in, out and other, indicate the inlet, the outlet and the wall of the muffler, respectively. The matrix coefficients in $[\boldsymbol{A}]$ and $[\boldsymbol{B}]$ are calculated by $A_{\alpha\beta} = \frac{1}{2}\delta_{\alpha\beta} + \int_{S_\beta} \frac{\partial G(\vec{x}, \vec{y})}{\partial n(\vec{y})}\, dS_\beta$ and $B_{\alpha\beta} = \int_{S_\beta} G(\vec{x}, \vec{y})\, dS_\beta$, where subscripts α and β are the labels of the collocation element and integration element, respectively. When $\alpha = \beta$, the constant $\delta_{\alpha\beta} = 1$; otherwise it is zero.

Furthermore, boundary conditions of this problem should be delivered to predict the attenuation of those mufflers. The inlet pressure is given uniform unit pressure and propagation of sound in downstream is under the assumption of non-reflection. The pressure gradient can be expressed with respect to normal velocity distribution u_n by means of the linearized Euler's equation.

$$p_n = -j\ k\ \rho\ c u_n\ , \tag{7}$$

where ρ and c are the density and sound speed. In addition, the wall of muffler is under the assumption of rigid. In other words, the pressure gradient $p_{\text{n,other}}$ is equal to zero. Therefore, the matrix form in Eq. (6) can be rearranged as:

$$[jkB_{\text{in}} \ \ A_{\text{out}} + jkB_{\text{out}} \ \ A_{\text{other}}] \begin{Bmatrix} \rho c u_{\text{n,in}} \\ p_{\text{out}} \\ p_{\text{other}} \end{Bmatrix} = [-A_{\text{in}}]\{1\}\ . \tag{8}$$

In Eq. (8), all unknowns are passed to the left-hand side. The matrix can now be solved for those unknown boundary values.

3. TRANSMISSION LOSS

According to the concept of the transfer function [28,29], the reflection coefficient R_u in the upstream tube can be defined:

$$R_u = \frac{H_u - e^{-jks}}{e^{jks} - H_u} \ , \tag{9}$$

where the $H_u = p_{u,b} / p_{u,a}$ is the transfer function, $p_{u,a}$ and $p_{u,b}$ designate the acoustic pressures at two discrete points in the inlet tube, s is the distance between the two discrete points. By the same way, the reflection coefficients R_d at the outlets of the muffler can also be obtained by:

$$R_d = \frac{H_d - e^{-jks}}{e^{jks} - H_d} \tag{10}$$

The transfer functions H_d in the outlet pipes are defined as the ratio between the sound pressures at two discrete points, and displayed as $p_{d,b}$ / $p_{d,a}$. The relationship between the distance s and the maximum analysis frequency $f_{\max}$ is displayed in the following equation [28,29]:

$$s \le \frac{c_0}{2 f_{\max}} \ . \tag{11}$$

Therefore, the incident sound power is given as [28,29]:

$$W_{in} = \frac{A_u \, S_{uu}}{\rho_0 \, c_0 \, |1 + R_u|^2} \ , \tag{12}$$

where A_u is called the upstream cross-section area, S_{uu} is the auto-spectrum at the upstream. Similarly the transmitted sound power at the outlets can also be indicated using the reflection coefficient, the auto-spectrum at the downstream S_{dd} and the downstream cross-section area A_d. One can obtain:

$$W_t = \frac{A_d \, S_{dd}}{\rho_0 \, c_0 \, |1 + R_d|^2} \ . \tag{13}$$

Making use of the sound power, the transmission loss can then be rearranged as:

$$TL = 10 \log\left(\frac{W_{in}}{W_t}\right) = 10 \log\left(\frac{S_{uu}}{S_{dd}}\right) + 10 \log\left(\frac{A_u}{A_d}\right) + 20 \log\left(\frac{|1 + R_d|}{|1 + R_u|}\right) \ . \tag{14}$$

4. NUMERICAL RESULTS

For attesting to the authenticity of this research, two cases were chosen to verify the present method. The first case, a single-inlet and double-outlet cylindrical expansion chamber muffler was analyzed and is displayed in Fig. 2. The parameters of the silencer are described as follows: The radius of the expansion chamber is 7.5cm, while the radii of the inlet pipe and outlet pipe are 2.5cm. In addition, there is a suggestion that one wavelength should include six to eight nodes for the convergence. There is considerable validity in this study as compared with the research of Wu *et al* [22-25].

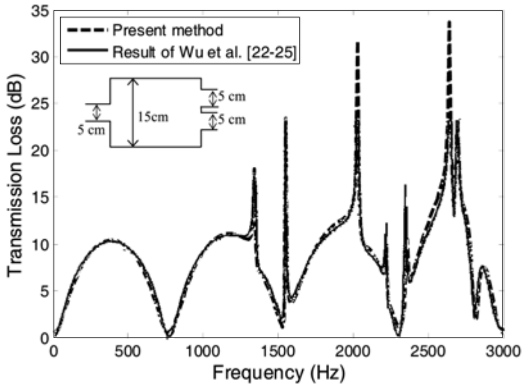
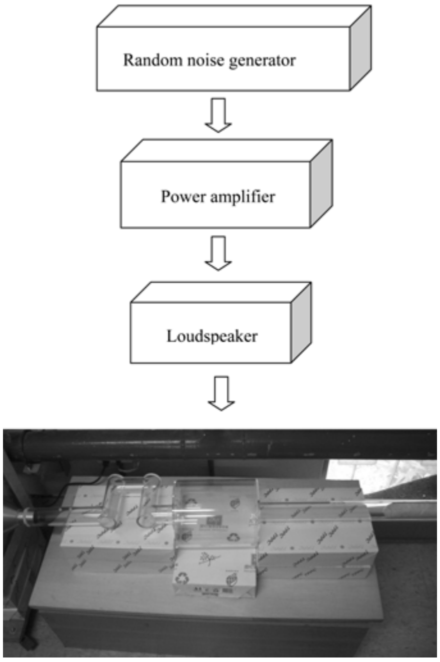


Fig. 2 The geometry of a single-inlet/double-inlet muffler and its transmission loss by Wu *et al.* and the present method

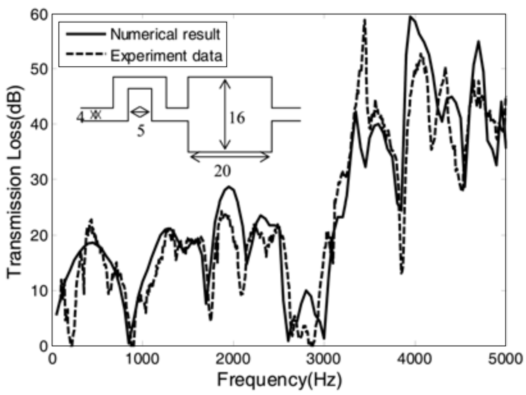
The apparatus of the experiment for the simple expansion chamber muffler with right angle inlet are shown in Fig. 3(a). A random noise, which is produced from the noise generator, propagates through the power amplifier, loudspeaker and then into the acrylic muffler with the wall thickness of 0.5cm. Two microphones located at the discrete points of the inlet/outlet pipe to capture the auto and cross-spectrum density function and finally obtained the sound power. The experiment is conducted according to the transfer function method, and more details refer to the concepts of Chung *et al* [28,29]. The radii of the inlet pipe and outlet pipe are 2cm, while the radius and length of the expansion chambers are 8cm and 20cm, respectively. The attenuation of this muffler via the present method and experiment is exhibited in Fig. 3(b). The results clearly show that the agreement is good when comparing the numerical results to the experiment. Therefore, one can conclude that the present method is suitable and reliable for studying the attenuation of mufflers.

The setups of the muffler with a right angle inlet/straight inlet are displayed in Fig. 1. Its dimensions can be described as follows: The radii of the inlet pipe and outlet pipe are 2cm, while the radius and length of the expansion chamber are 8cm and 20cm, respectively. The dimensions of the muffler with a straight inlet are the same as those of the one with a right angle inlet, only the inlet part is different. The attenuations of the two mufflers are exhibited in Fig. 4. It is noteworthy that the performance of the muffler with a right angle inlet is significantly better than the one with a straight inlet at the frequencies of about 3500 ~ 5000Hz. The results clearly show that the propagation of the sound in the muffler with a right angle inlet could attenuate its sound power by about 10 ~ 40dB. In addition, it is worth stressing that the mufflers have no absorbent linings attached to the inside of them.

The contours of the amplitude of the sound pressure for the two mufflers are also illustrated in Figs. 5(a), 5(b) in order to clarify the phenomenon of the attenuation of the two mufflers. From the two contours, one can find an interesting occurrence: The amplitude of the sound is less at the outlet of the muffler with a right angle inlet than it is at outlet of the muffler with a straight inlet. The inlet parts (only the right angle pipe



(a)



(b)

Fig. 3 (a) The apparatus of the experiment. (b) The dimension of the simple expansion chamber muffler with right angle inlet and its transmission loss obtained via present method and experiment

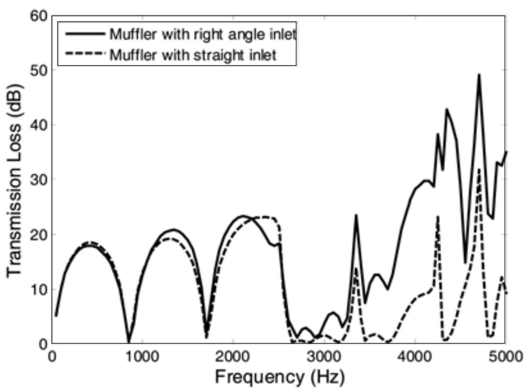
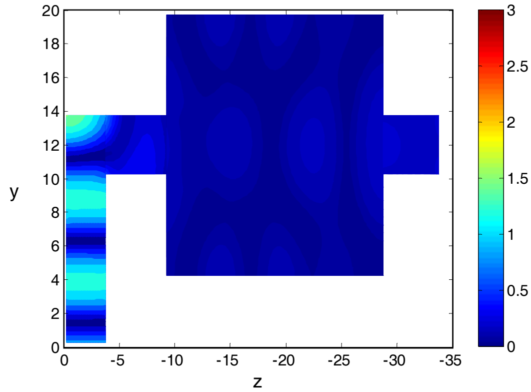
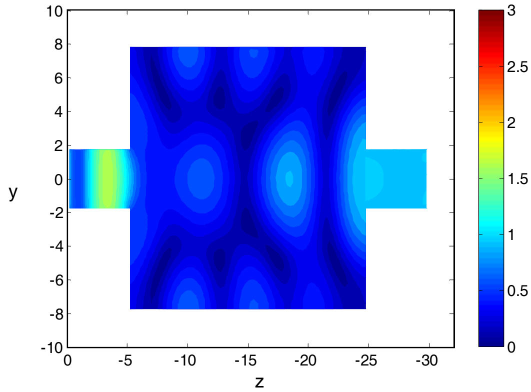


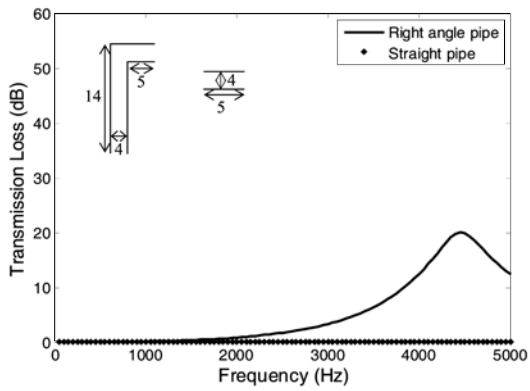
Fig. 4 The transmission losses for the simple expansion chamber muffler with a right angle and a straight inlet ($r_1 = 2$cm, $r_2 = 8$cm, $L = 20$cm)



(a)



(b)



(c)

Fig. 5 (a) The contour of the amplitude of the sound pressure for the muffler with a right angle inlet at the frequency of 3500Hz. (b) The contour of the amplitude of the sound pressure for the muffler with a straight inlet at the frequency of 3500Hz. (c) The transmission loss for the right angle pipe and straight pipe, unit: cm

and the straight pipe) were verified for their effects and contributions before the sound propagated into the expansion chamber. Their dimensions and transmission losses are shown in Fig. 5(c). It should be noted that the dimensions of those pipes are exactly the same as the inlet parts of Figs. 5(a), 5(b). The result clearly shows that the propagation of sound in the right angle pipe attenuates its sound power in the range of 2000 ~ 5000Hz. Conversely, the transmission loss of the straight pipe is almost zero. In other words, the effects could be ignored before the sound propagates into the expansion chamber of the muffler with a straight inlet. It can be concluded from the explanations mentioned above that the performance of a muffler with a right angle inlet is superior to that with a straight inlet at particular frequencies.

Furthermore, the mufflers with the same radius of the inlet/outlet pipe and the same length of the expansion chamber, but with different amounts of corner inlet, are analyzed in this section in order to determine the effects of the inlet pipe. The radii of the inlet pipe and outlet pipe are 2cm, while the length of the expansion chamber is 20cm. The setups and the transmission losses are exhibited in Fig. 6. The performances are improved in the range of 3000 ~ 4500Hz and are directly dependent on the increasing amount of the corner inlet. Taking the frequency of about 3600Hz as an example, it is obvious that the attenuation could be increased substantially via an elaborate inlet part.

The radii of the inlet and outlet pipes will be regarded as a parameter to elicit the effects of the performance. The transmission losses for the mufflers with a right angle and straight inlet are shown in Fig. 7. The radius and length of the expansion chambers are 8cm and 20cm for present cases; the radii of inlet and outlet pipes are 1 and 3cm for Figs. 7(a) and 7(b), respectively. The results of Fig. 7(a) show that the performance of the muffler with a right angle inlet is better than that with a straight inlet between the frequency of 6000Hz and 8000Hz. In addition, the performance of the muffler with a right angle inlet is significantly better than the one with a straight inlet at the frequencies of about 2200 ~ 4500Hz, while the radii of inlet and outlet pipes are 3cm. Furthermore, attention should be focused on the fact that the better performance can be shifted from high frequency to low frequency by increasing the radii of the inlet and out pipes in comparison with Fig. 4 and Fig. 7. This is an interesting phenomenon of the muffler design.

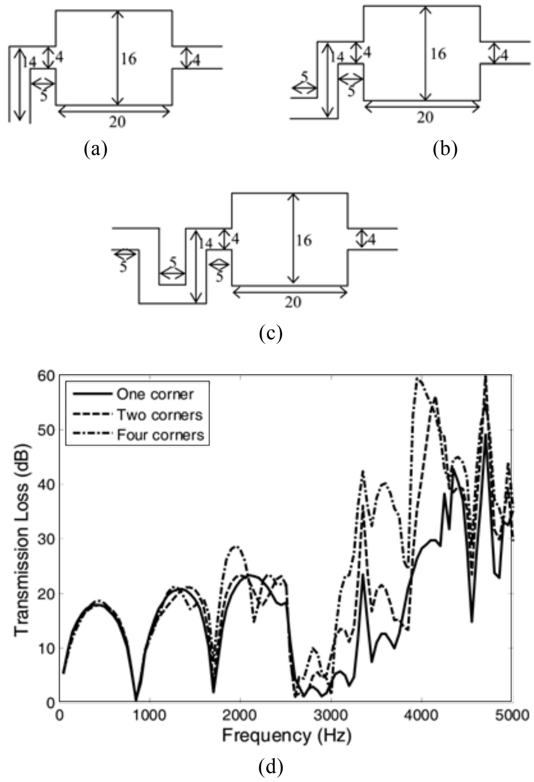
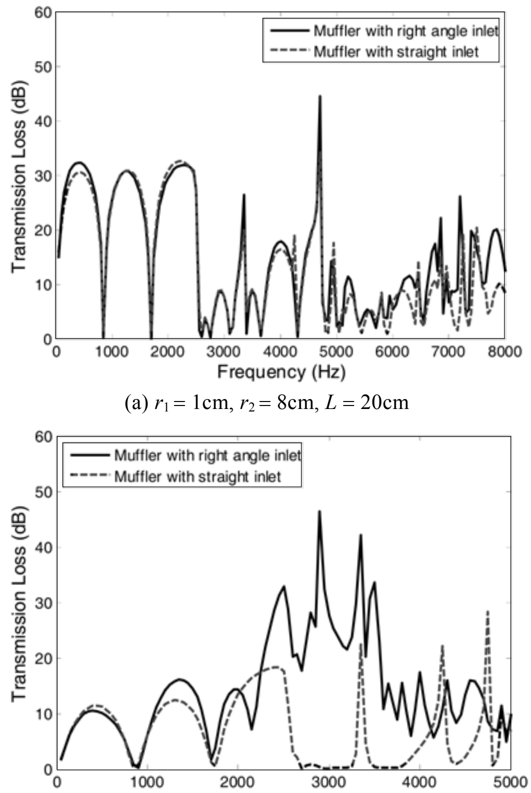


Fig. 6 (a) The muffler with one corner inlet, unit: cm. (b) The muffler with two corners inlet. (c) The muffler with four corners inlet. (d) The transmission loss for those mufflers with different corners inlet



(a) $r_1 = 1$cm, $r_2 = 8$cm, $L = 20$cm

(b) $r_1 = 3$cm, $r_2 = 8$cm, $L = 20$cm

Fig. 7 The transmission losses for the simple expansion chamber muffler with a right angle and a straight inlet.

5. CONCLUSIONS

Mufflers with right angle inlets have been studied under the assumption of stationary medium in this work, and the numerical results have been verified by making a comparison between the experiment and the modal meshing approach. However, it is worth mentioning that a muffler without absorbent linings but with broadband sound attenuation has also been successfully demonstrated in this work. The result shows that the performance of the muffler with a right angle inlet is better than that with a straight inlet, and that the propagation of the sound in this muffler can attenuate its sound power about 10 ~ 40dB in particular frequencies. Above all, the better performance can be shifted from high frequency to low frequency by increasing the radii of the inlet and out pipes. Therefore, one can design a broadband sound attenuated silencer via an appropriate expansion ratio and right angle inlet. A muffler with a right angle inlet could be studied under the condition of moving medium in the future.

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