THE MODE-III FRACTURE ANALYSIS OF AN INCLINED CRACK EMBEDDED IN THIN LAYER SYSTEMS BY ANALYTICAL ALTERNATING METHODS

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ABSTRACT

This paper analyzes the mode-III stress intensity factor of an inclined crack, embedded in a thin layer, bonded to a half plane, subjected to arbitrary distributed anti-plane loads. Special alternating procedures are presented to evaluate the mode-III S.I.F. and the numerical results confirm the validity of the proposed alternating procedure. The solution of a bi-material problem in an infinite plane with an inclined crack and the analytical solution of a thin layer, without crack, bonded to a half plane, subjected to an anti-plane point force applied on the boundary are referred to as fundamental solutions. By using these fundamental solutions and alternating procedures, the stress intensity factors of a crack in a thin layer bonded to a half plane are evaluated. The numerical results of some reduced problems are computed and excellent agreements with existing solutions are obtained.

Key Words: alternating method, stress intensity factor, thin layer.

I. INTRODUCTION

The purpose of this work is to investigate the mode-III fracture analysis of an inclined crack embedded in a finite layer, bonded to a semi-infinite substrate. In the application of components of coated materials such as seals on semiconductors, it is desirable that the coated layer provide environmental protection, against corrosion and oxidation. It is important to analyze the fracture characteristic of a crack in the layer or in the substrate for the thin-layer problem. Among the published articles on the cracked thin-layer problem, Blanco *et al*. (1995) developed a mathematical model to deal with the general anisotropic case of a straight crack lying in the thin layer coated to a semi-infinite substrate. The crack was modeled as a continuous distribution of dislocation and the numerical results for the monoclinic case were presented in graphic form. Chao and Kao (1997) proposed a method that was based upon the complex potential theory and iteration of Möbius transformation that allowed them to express the solution as a rapidly convergent series. Their numerical results include the stress intensity factor, K_{III}, of a crack in a thin-layer, bonded to a half-plane and the reduced problem of a crack in a strip. Chang (1999) considered a cracked layer bonded to a viscoelastic substrate, subjected to an anti-plane point force on the boundary. The results show that the time-dependent stress intensity factor may increase or decrease with time evolution. Concerning literature on the reduced problem, Irwin (1957) obtained a theoretical solution of a normal crack in a strip subjected to crack face loadings. By a use of Fourier transform, Ma (1988) presented the theoretical solution of a mode-III central crack in a rectangular sheet, subjected to arbitrary anti-plane load on boundaries. The special case of a parallel crack in a strip was also obtained. Bassani and Erdogan (1979) considered two bonded dissimilar half planes containing an inclined crack and subjected to anti-plane crack face loads. A similar problem was considered by Wang and Meguid (1996), they applied the technique of SCI (Self-Consistent Iterative) to obtain the numerical solution and obvious difference from the theoretical solution provided

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by Bassani and Erdogan (1979). Choi and Earmme (1996) provided an alternation technique for solving sub-interface crack problems in anisotropic bi-materials, only the numerical results of the orthotropic case were computed.

Instead of analyzing the complicated crack problem with a finite boundary, the Schwartz-Neumann alternating method proposed the solution of a crack in a finite body by iteration between the solution for the noncrack body and the solution for an infinite body with a crack. Various methods used to solve the sub-problem of a noncrack body lead to various characteristics of Schwartz-Neumann alternating methods. The analytical alternating method utilized analytical solutions for the noncrack finite body, as well as for a crack in an infinite plane. Early applications of the analytical alternating method involved solving the edge crack problem in a semi-infinite plane by Hartranft and Sih (1973), and a surface crack in a 3-D body by Shah and Kobayashi (1973). The method was used later by O'Donoghue *et al.* (1985) to solve multiple embedded elliptical cracks in an infinite body. Zhang and Hasebe (1993) studied the interactions between rectilinear and circumferential cracks in an infinite domain. Recently, Chang and Ma (2001) extended the analytical alternating method to complicated geometric configurations. They studied the transient heat conduction problem of a rectangular plate with multiple insulated cracks. Chang and Ma (2002) analyzed the mixed-mode S.I.F. of multiple cracks in a semi-infinite plane. The finite element alternating method (FEAM) (Raju and Fichter 1989, Chen and Chang 1989, Krishnamurthy and Raju 1990) and boundary element alternating method (BEAM) (Rajiyah and Atluri 1989, Raju and Krishnamurthy 1992) extended the utilizable ability of the alternating technique to complicated cracked geometries. In the absence of cracks, FEM or BEM can easily obtain the numerical solution of the noncrack finite body with a coarser mesh.

In this study, we consider the mode-III fracture problem of cracked layer systems where the inclined crack is located at the thin layer with arbitrary angles. The alternating procedure for a bi-material problem with an inclined crack is developed first. The solution of the bi-material problem with an inclined crack and the analytical solution of a thin layer without crack bonded to a half plane subjected to an anti-plane point force applied on the boundary are referred to as fundamental solutions. By using these two fundamental solutions and proposed alternating procedures, the mode-III S.I.F. of an inclined crack in a thin layer bonded to a half plane is easily computed. The alternating procedure of an inclined crack in a strip is also presented in this paper. Numerical solutions of various cases are obtained and excellent agreement with existing results are obtained. Finally, the S.I.F. of an inclined crack in a thin layer bonded to a semi-infinite plane are presented. The numerical solutions of reduced problems, in which the shear modulus of the semi-infinite plane is infinite or infinitesimal are compared to the solutions of a crack in a strip with free or fixed boundary, and excellent agreements are obtained.

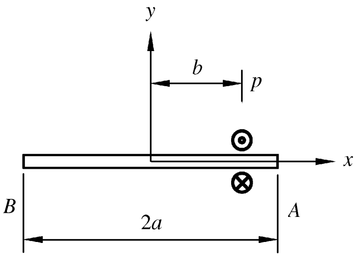

Fig. 1 A finite crack in an infinite plane subjected to a pair of anti-plane point forces

II. ANALYTICAL FUNDAMENTAL SOLUTIONS

In the literature on Schwartz-Neumann alternating methods, various types of analytical solutions have been used to analyze different problems. Polynomial and Chebyshev polynomial distributions have been mostly employed to simulate continuously distributed loads. For discontinuously distributed loads, the point loads or piecewise distributed loads could more precisely describe the large variation of the crack face loads. For simplicity and versatility, all the analytical fundamental solutions presented in this section are point load solutions.

1. A Finite Crack in an Infinite Plane Subjected to a Pair of Point Forces on the Crack Faces

Figure 1 shows an infinite plane containing a finite crack lying along the x-axis. The center of the crack is located at the origin, and the crack length is $2a$. The crack face is subjected to a pair of anti-plane point forces p at $x=b$ and μ is the shear modulus of the material. From the well-known complex variable method, the complex stress functions are found as

$$Z_{III} = \frac{p}{\pi}\frac{\sqrt{a^2-b^2}}{(z-b)\sqrt{z^2-a^2}} \tag{1a}$$

$$\overline{Z}_{III} = \frac{p}{\pi}\sin^{-1}[\frac{bz-a^2}{a(z-b)}] \tag{1b}$$

where $z=x+iy$. The full field solutions of displacement and stresses can be expressed as

$$W = \frac{1}{\mu}\text{Im}\,\overline{Z}_{III} = \frac{p}{\mu\pi}\,\text{Im}\{\sin^{-1}[\frac{bz-a^2}{a(z-b)}]\} \tag{2}$$

$$\tau_{xz} = \mathrm{Im}\, Z_{III} = \frac{p}{\pi}\frac{\sqrt{a^2-b^2}}{r_3\sqrt{r_1 r_2}}\sin(-\frac{\phi_1+\phi_2}{2}-\phi_3) \tag{3a}$$

$$\tau_{yz} = \mathrm{Re}\, Z_{III} = \frac{p}{\pi}\frac{\sqrt{a^2-b^2}}{r_3\sqrt{r_1 r_2}}\cos(-\frac{\phi_1+\phi_2}{2}-\phi_3) \tag{3b}$$

where $r_1=|z-a|$, $r_2=|z+a|$, $r_3=|z-b|$, $\phi_1=\tan^{-1}(\frac{y}{x-a})$, $\phi_2=\tan^{-1}(\frac{y}{x+a})$, $\phi_3=\tan^{-1}(\frac{y}{x-b})$ and the stress intensity factors of crack tips A and B are

$$K_{IIIA} = \frac{p}{\sqrt{\pi a}}\sqrt{\frac{a+b}{a-b}} \tag{4a}$$

$$K_{IIIB} = \frac{p}{\sqrt{\pi a}}\sqrt{\frac{a-b}{a+b}} \tag{4b}$$

2. The Bi-material Problem Subjected to an Anti-plane Point Force

Figure 2 shows a bi-material problem subjected to an anti-plane point force with magnitude p at (ξ, h) and μ_1 and μ_2 are the shear moduli of material 1 and 2, respectively. The governing equations could be expressed as

$$\frac{\partial^2 W_{1A}}{\partial x^2}+\frac{\partial^2 W_{1A}}{\partial y^2}=0 \quad y\geq h \tag{5a}$$

$$\frac{\partial^2 W_{1B}}{\partial x^2}+\frac{\partial^2 W_{1B}}{\partial y^2}=0 \quad 0\leq y<h \tag{5b}$$

$$\frac{\partial^2 W_{2}}{\partial x^2}+\frac{\partial^2 W_{2}}{\partial y^2}=0 \quad y<0 \tag{5c}$$

The continuity conditions are

$$W_{1A}(x, h)=W_{1B}(x, h) \tag{6a}$$

$$W_{1B}(x, 0)=W_2(x, 0) \tag{6b}$$

$$\frac{\partial W_{1B}(x,0)}{\partial y}=\frac{\partial W_2(x,0)}{\partial y} \tag{6c}$$

and the jumped condition is

$$\frac{p\delta(x-\xi)}{\mu_1}+\frac{\partial W_{1A}(x,h)}{\partial y}=\frac{\partial W_{1B}(x,h)}{\partial y} \tag{7}$$

The Fourier transformation pair is

$$F(\omega)=\int_{-\infty}^{\infty} f(x)e^{-i\omega x}dx \tag{8a}$$

$$f(x)=\frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega)e^{i\omega x}d\omega \tag{8b}$$

The solutions of displacement field in the Fourier transform domain are

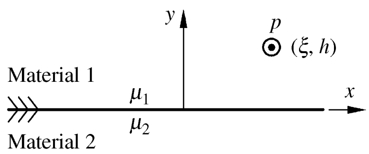

Fig. 2 The bi-material problem subjected to an anti-plane point force

$$\overline{W}_{1A}=\frac{p}{|\omega|}[\frac{1}{2\mu_1}e^{|\omega|(h-y)}+\frac{\mu_1-\mu_2}{2\mu_1(\mu_1+\mu_2)}e^{-|\omega|(h+y)}] \tag{9a}$$

$$\overline{W}_{1B}=\frac{p}{|\omega|}[\frac{1}{2\mu_1}e^{-|\omega|(h-y)}+\frac{\mu_1-\mu_2}{2\mu_1(\mu_1+\mu_2)}e^{-|\omega|(h+y)}] \tag{9b}$$

$$\overline{W}_{2}=\frac{p}{|\omega|}\frac{1}{(\mu_1+\mu_2)}e^{-|\omega|(h-y)} \tag{9c}$$

After the inverse transformation procedure, the stress field of materials 1 and 2 could be expressed as

$$\tau_{xz1}=-\frac{p}{2\pi}\frac{x-\xi}{(x-\xi)^2+(y-h)^2}-\frac{p}{2\pi}\frac{(\mu_1-\mu_2)}{(\mu_1+\mu_2)}\frac{x-\xi}{(x-\xi)^2+(y+h)^2} \tag{10a}$$

$$\tau_{yz1}=-\frac{p}{2\pi}\frac{y-h}{(x-\xi)^2+(y-h)^2}-\frac{p}{2\pi}\frac{\mu_1-\mu_2}{(\mu_1+\mu_2)}\frac{y+h}{(x-\xi)^2+(y+h)^2} \tag{10b}$$

$$\tau_{xz2}=-\frac{p}{2\pi}\frac{2\mu_2}{(\mu_1+\mu_2)}\frac{x-\xi}{(x-\xi)^2+(y-h)^2} \tag{10c}$$

$$\tau_{yz2}=-\frac{p}{2\pi}\frac{2\mu_2}{(\mu_1+\mu_2)}\frac{y-h}{(x-\xi)^2+(y-h)^2} \tag{10d}$$

From the solution presented in Eq. (10), the solution of a bi-material problem subjected to an anti-plane point force can be represented a simple form as

$$\boldsymbol{S}(z,z_0)=\begin{cases}\boldsymbol{S}_0(z,z_0)+\frac{\mu_1-\mu_2}{\mu_1+\mu_2}\boldsymbol{S}_0(z,z_0'), & y\geq 0\\ \frac{2\mu_2}{\mu_1+\mu_2}\boldsymbol{S}_0(z,z_0), & y<0\end{cases} \tag{11}$$

where $\boldsymbol{S}_0(z, z_0)$ denotes the solution of an infinite plane subjected to an anti-plane force p at $z_0(\xi, h)$ and $\boldsymbol{S}_0(z, z_0')$ denotes the solution of an infinite plane subjected to an anti-plane force p at $z_0'(\xi, -h)$. The physical representation of this fundamental solution is shown in Fig. 3.

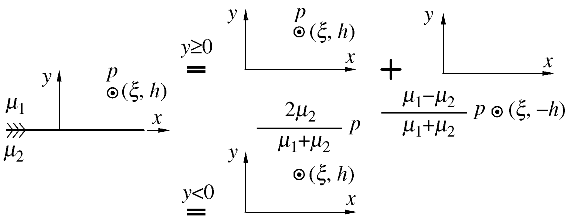

Fig. 3 The physical representation of the solution of bi-material problem subjected to an anti-plane point force

3. A Thin Layer Bonded to a Half Plane Subjected to a Point Force on the Boundary

Figure 4 shows a layer with finite thickness bonded to a half plane subjected to a point force on the boundary. The origin of the coordinate is located at the interface and the thickness of the layer is h. An anti-plane concentrated force with magnitude p is applied on the boundary at $x=\xi$. The governing equations are

$$\frac{\partial^2 W_1}{\partial x^2} + \frac{\partial^2 W_1}{\partial y^2} = 0 \qquad 0 \le y < h \tag{12a}$$

$$\frac{\partial^2 W_2}{\partial x^2} + \frac{\partial^2 W_2}{\partial y^2} = 0 \qquad y < 0 \tag{12b}$$

The boundary conditions and continuity equations are

$$\mu_1 \frac{\partial W_1}{\partial y} = p\delta(x-\xi)\,, \quad y=h \tag{13a}$$

$$W_1 = W_2, \qquad y=0 \tag{13b}$$

$$\mu_1 \frac{\partial W_1}{\partial y} = \mu_2 \frac{\partial W_2}{\partial y}\,, \qquad y=0 \tag{13c}$$

The solutions of displacement in the transform domain are

$$\overline{W}_1 = \frac{p}{\mu_1|\omega|} \cdot \sum_{n=0}^{\infty} \{K^{n+1} e^{-|\omega|[(2n+1)h+y]} + K^n e^{-|\omega|[(2n+1)h-y]}\} \tag{14a}$$

$$\overline{W}_2 = \frac{2}{(\mu_1+\mu_2)} \frac{p}{|\omega|} \sum_{n=0}^{\infty} K^n e^{-|\omega|[(2n+1)h-y]} \tag{14b}$$

where $K = \frac{\mu_1 - \mu_2}{\mu_1 + \mu_2}$. The solutions of stress field are expressed in series forms as

$$\tau_{xz1} = \frac{-p}{\pi} \sum_{n=0}^{\infty} \left[K^{n+1} \frac{x-\xi}{(x-\xi)^2 + [(2n+1)h+y]^2} + K^n \frac{x-\xi}{(x-\xi)^2 + [(2n+1)h-y]^2} \right] \tag{15a}$$

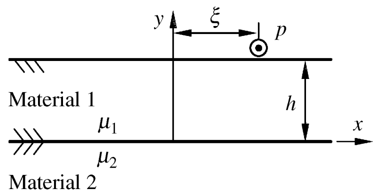

Fig. 4 A thin layer bonded to a half plane subjected to a point force on the boundary

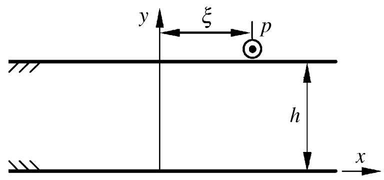

Fig. 5 A strip subjected to a point force applied on the upper boundary

$$\tau_{yz1} = \frac{p}{\pi} \sum_{n=0}^{\infty} \left[-K^{n+1} \frac{(2n+1)h+y}{(x-\xi)^2 + [(2n+1)h+y]^2} + K^n \frac{(2n+1)h-y}{(x-\xi)^2 + [(2n+1)h-y]^2} \right] \tag{15b}$$

$$\tau_{xz2} = \frac{2\mu_2}{\mu_1+\mu_2} \frac{-p}{\pi} \sum_{n=0}^{\infty} \left[K^n \frac{x-\xi}{(x-\xi)^2 + [(2n+1)h-y]^2} \right] \tag{15c}$$

$$\tau_{yz2} = \frac{2\mu_2}{\mu_1+\mu_2} - \frac{p}{\pi} \sum_{n=0}^{\infty} \left[K^n \frac{(2n+1)h-y}{(x-\xi)^2 + [(2n+1)h-y]^2} \right] \tag{15d}$$

4. A Strip Subjected to a Point Force on the Boundary

Figure 5 shows a strip subjected to a point force p on the upper boundary at $x=\xi$. The governing equation is

$$\frac{\partial^2 W}{\partial x^2} + \frac{\partial^2 W}{\partial y^2} = 0 \tag{16}$$

and the boundary conditions are

$$\mu \frac{\partial W}{\partial y} = 0\,, \qquad y=0 \tag{17a}$$

$$\mu \frac{\partial W}{\partial y} = p\delta(x-\xi)\,, \qquad y=\mathrm{h} \tag{17b}$$

The displacement and stress in the transform domain are

$$\overline{W} = \frac{p}{\mu\omega} \frac{\cosh\omega y}{\sinh\omega h} \tag{18a}$$

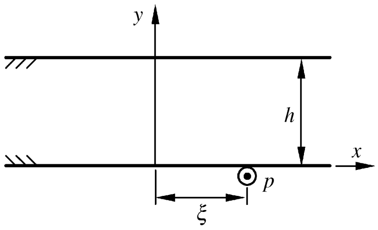

Fig. 6 A strip subjected to a point force applied on the lower boundary

$$\bar{\tau}_{xz} = ip\frac{\cosh\omega y}{\sinh\omega h} \tag{18b}$$

$$\bar{\tau}_{yz} = p\frac{\sinh\omega y}{\sinh\omega h} \tag{18c}$$

The stress field solutions can be represented with explicit functional forms as

$$\tau_{xz} = \frac{p}{2h}\frac{-\sinh\frac{\pi(x-\xi)}{h}}{\cos\frac{\pi y}{h}+\cosh\frac{\pi(x-\xi)}{h}} \tag{19a}$$

$$\tau_{yz} = \frac{p}{2h}\frac{\sin\frac{\pi y}{h}}{\cos\frac{\pi y}{h}+\cosh\frac{\pi(x-\xi)}{h}} \tag{19b}$$

Similarly, the solutions of stresses in a strip subjected to a point force p on the lower boundary at $x=\xi$ as shown in Fig. 6 are

$$\tau_{yz} = \frac{p}{2h}\frac{\sin\frac{\pi(y-h)}{h}}{\cos\frac{\pi(y-h)}{h}+\cosh\frac{\pi(x-\xi)}{h}} \tag{20a}$$

$$\tau_{xz} = \frac{p}{2h}\frac{-\sinh\frac{\pi(x-\xi)}{h}}{\cos\frac{\pi(y-h)}{h}+\cosh\frac{\pi(x-\xi)}{h}} \tag{20b}$$

5. A Strip with a Concentrated Load Applied at the Upper Boundary and Lower Boundary is Fixed

Figure 7 shows the geometrical configuration of the problem. The boundary conditions are

$$W=0, \qquad y=0 \tag{21a}$$

$$\mu\frac{\partial W}{\partial x} = p\delta(x-\xi)\,, \qquad y=h \tag{21b}$$

The displacement and stress in the transform form are

$$\bar{W} = \frac{p}{\mu\omega}\frac{\sinh\omega y}{\cosh\omega h} \tag{22a}$$

$$\bar{\tau}_{xz} = ip\frac{\sinh\omega y}{\cosh\omega h} \tag{22b}$$

$$\bar{\tau}_{yz} = p\frac{\cosh\omega y}{\cosh\omega h} \tag{22c}$$

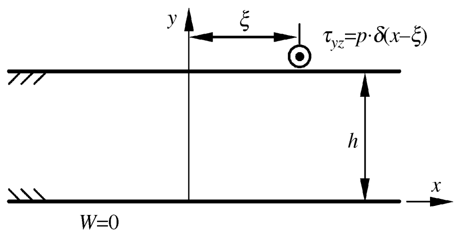

Fig. 7 A strip with fixed lower boundary subjected to a point force on the upper boundary

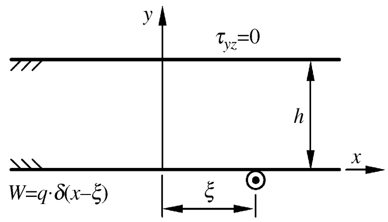

Fig. 8 A strip traction free on the upper boundary subjected to a point displacement on the lower boundary

The stress field solutions are

$$\tau_{xz} = \frac{p}{4h}\left[\frac{\sinh\frac{\pi(x-\xi)}{2h}}{\cosh\frac{\pi(x-\xi)}{2h}+\sin\frac{\pi y}{2h}} - \frac{\sinh\frac{\pi(x-\xi)}{2h}}{\cosh\frac{\pi(x-\xi)}{2h}-\sin\frac{\pi y}{2h}}\right] \tag{23a}$$

$$\tau_{yz} = \frac{2p}{h}\frac{\cos\frac{\pi y}{2h}\cdot\cosh\frac{\pi(x-\xi)}{2h}}{\cosh\frac{\pi(x-\xi)}{h}+\cos\frac{\pi y}{h}} \tag{23b}$$

Fig. 8 shows a strip traction free on the upper boundary and subjected to a point displacement $W=q\cdot\delta(x-\xi)$ applied on the lower boundary. The stress fields are

$$\tau_{xz} = \frac{\mu\pi q}{h^2}\left[\frac{\sinh\frac{\pi(x-\xi)}{2h}\sin\frac{\pi y}{2h}}{2(\cosh\frac{\pi(x-\xi)}{h}-\cos\frac{\pi y}{h})} - \frac{\cosh\frac{\pi(x-\xi)}{2h}\sinh\frac{\pi(x-\xi)}{h}\sin\frac{\pi y}{2h}}{(\cosh\frac{\pi(x-\xi)}{h}-\cos\frac{\pi y}{h})^2}\right] \tag{24a}$$

$$\tau_{yz} = \frac{\mu q}{h^2}\left[\frac{\cosh\frac{\pi(x-\xi)}{2h}\cos\frac{\pi y}{2h}}{2(\cosh\frac{\pi(x-\xi)}{h}-\cos\frac{\pi y}{h})} - \frac{\cosh\frac{\pi(x-\xi)}{2h}\sin\frac{\pi y}{2h}\sin\frac{\pi y}{h}}{(\cosh\frac{\pi(x-\xi)}{h}-\cos\frac{\pi y}{h})^2}\right] \tag{24b}$$

III. ALTERNATING PROCEDURE FOR AN INCLINED CRACK IN A THIN LAYER BONDED TO A HALF PLANE

In this section, analytical alternating procedures will be proposed to analyze the S.I.F. of an inclined crack in a thin layer bonded to a half plane. In order to illustrate the alternating procedure, a fracture problem is considered as shown in Fig. 9(a) in which an arbitrarily distributed loading is applied on the boundary of the thin layer with an embedded inclined crack bonded to a half plane. The iteration and superposition are performed with external and internal alternating procedures.

1. External Alternating Procedure

(i) Solve the non-crack problem of a thin layer bonded to a half plane under distributed loads $T_b^{(0)}$ as shown in Fig. 9(b) by integration of the solution given in Eq. (15). Evaluate the stress distribution $f_c^{(1)}$ in the non-crack body at the crack surfaces of the fictitious cracks. To satisfy the traction free condition on the crack faces, the sub-problem of Fig. 9(c) that is subjected to residual stress $f_c^{(1)}$ at the crack faces, should be superposed.

(ii) The sub-problem Fig. 9(c) could be further split into two sub-problems as indicated in Fig. 9(d) and Fig. 9(e). Fig. 9(d) shows a crack in a bi-material, subjected to the same traction $f_c^{(1)}$ on the crack faces as shown in Fig. 9(c). Evaluate the residual stress $T_b^{(1)}$ on the virtual boundary by the internal alternating procedure that will be described later. The problem indicated in Fig. 9 (e) is similar to that shown in Fig. 9(a), and the procedure will be iterated.

(iii) After several cycles of iteration, the residual traction $f_c^{(k)}$ on the crack faces of the k'th cycle of iteration will be very small; then the solution of Fig. 9(a) may be obtained by the superposition of Fig. 9(b) and Fig. 9(d) for k times. The S.I.F. of the embedded crack for the original problem as shown in Fig. 9(a) can be evaluated by considering the bi-material problem with a crack subjected to the crack face loads

$$f_c = \sum_{i=1}^{k} f_c^{(i)} \tag{25}$$

(iv) The convergence criterion for the iteration in this paper is taken as

$$\frac{\int f_c^{(k+1)}(\xi_c)d\xi_c - \int f_c^{(k)}(\xi_c)d\xi_c}{\int f_c^{(k)}(\xi_c)d\xi_c} < 10^{-5} \tag{26}$$

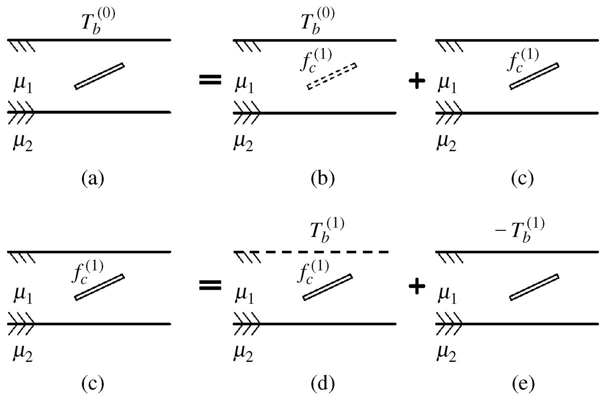

Fig. 9 The external alternating procedure of a crack in a thin layer bonded to a half plane

2. Internal Alternating Procedure

In the abovementioned external iteration procedure, the solution of a bi-material problem with a crack as shown in Fig. 9(d) can be constructed by the procedure indicated in Fig. 10, in which $f_c^{(k)}$ in Fig. 10(a) is the surface traction of the crack face in the k'th external alternating procedure. This alternating procedure is based on the solution of a bi-material problem subjected to an anti-plane force and the validity will be verified by numerical results. The solution of Fig. 10(a) may be obtained by the following internal alternating procedures.

(i) Consider a bi-material problem with a crack subjected to the traction $f_c^{(k)}$ on crack faces as shown in Fig. 10(a). The stress field for $y \geq 0$ can be split into two sub-problems as indicated in Fig. 10(b) and Fig. 10(c), that is an infinite plane with a crack. Evaluate the traction $g_c^{(1)}$ on the fictitious crack of the sub-problem of Fig. 10(c) under crack surface loads $\frac{\mu_1 - \mu_2}{\mu_1 + \mu_2} f_c^{(k)}$ on the image crack. The problem indicated in Fig. 10(d) is similar to that shown in Fig. 10(a), and the procedure will be iterated.

(ii) After several cycles of iteration, the residual stress $g_c^{(j)}$ approaches zero, the solution of Fig. 10(a) can be obtained by combining the solutions for the infinite plate with a single crack subjected to the crack face loads g_c as

$$g_c = f_c^{(k)} + \sum_{j=1}^{\infty} g_c^{(j)} \tag{27}$$

(iii) The convergence criterion of this iteration is taken as

$$\frac{\int g_c^{(j+1)}(\xi_c)d\xi_c - \int g_c^{(j)}(\xi_c)d\xi_c}{\int g_c^{(j)}(\xi_c)d\xi_c} < 10^{-5} \tag{28}$$

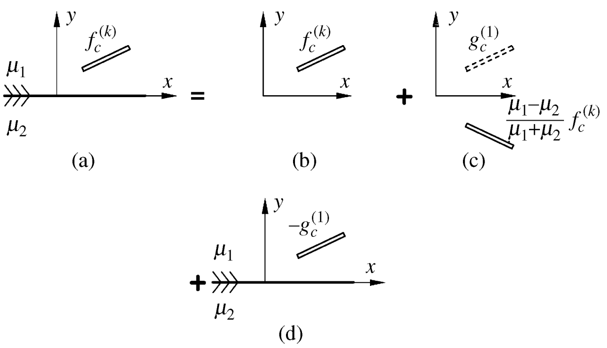

Fig. 10 The internal alternating procedure of a crack in a bi-material

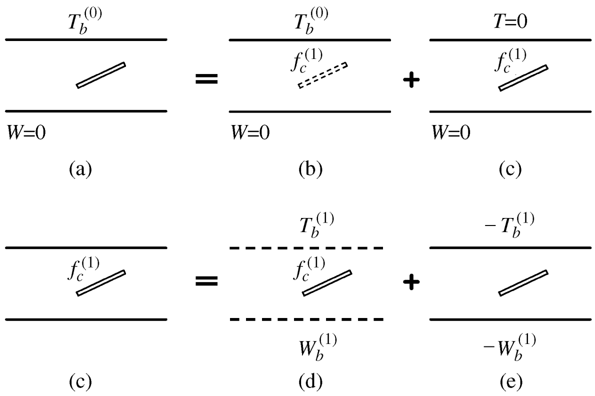

Fig. 11 Alternating procedure for a crack in a strip

IV. ALTERNATING PROCEDURE FOR A CRACK IN A STRIP

Consider an inclined crack in a strip subjected to a traction $T_b^{(0)}$ on upper boundary with the lower boundary fixed. The alternating procedure could be illustrated by Fig. 11 in which the original problem is shown in Fig. 11(a).

(i) Solve the non-crack problem of a strip under given loads $T_b^{(0)}$ as shown in Fig. 11(b) by integration of the solution given in Eq. (23). Evaluate the stress distribution $f_c^{(1)}$ in the non-crack body at the crack surfaces of the fictitious cracks. To satisfy the traction free condition on the crack faces, the sub-problem of Fig. 11(c) that is subjected to residual stress $f_c^{(1)}$ at the crack faces should be superposed.

(ii) The sub-problem Fig. 11(c) may be further split into two sub-problems as indicated in Fig. 11(d) and Fig. 11(e). Fig. 11(d) shows a crack in an infinite plane, subjected to the same traction $f_c^{(1)}$ on the crack faces as shown in Fig. 11(c). Evaluate the residual stress $T_b^{(1)}$ and residual displacement $W_b^{(1)}$ on the virtual boundary as the original strip. The problem indicated in Fig. 11(e) is similar to that shown in Fig. 11(a), and the procedure will be iterated.

V. NUMERICAL RESULTS AND DISCUSSIONS

To examine the validity of the proposed analytical alternating method, several configurations of cracked bodies and loading conditions are analyzed. First, the results of an inclined crack in a bi-material are checked with the existing solutions. The accuracy proves the applicability of the alternating procedure of bi-material problem with a crack. Next, an inclined crack in a strip with various loads and boundary conditions is considered and excellent agreements are found when compared with the results obtained from the literature. Finally, the solutions of an inclined crack in a thin layer bonded to a half plane are compared with the existing results. The reduced problems show that the shear moduli of the half plane are infinite or infinitesimal and are checked with the solutions of a crack in a strip with free or fixed lower boundary, and the excellent agreements prove the validity of the present method.

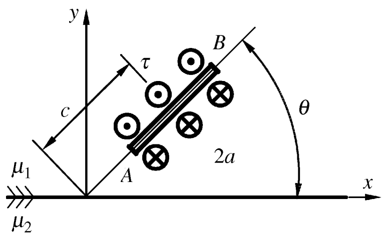

Fig. 12 An inclined crack in a bi-material subjected to uniform anti-plane shear loads τ at crack faces

1. An Inclined Crack in a Bi-material

Consider an inclined crack near the interface of bonded half planes subjected to uniformly distributed anti-plane shear loads τ on crack faces as shown in Fig. 12, the inclined angle is θ and the distance between the crack center and the interface is c. The normalized stress intensity factors of crack tips A and B are given as

$$F_{III} = K_{III} \cdot \frac{1}{\tau\sqrt{\pi a}} \tag{29}$$

The numerical results for different values of c/a and inclined angle θ are compared with those obtained by Bassani and Erdogan (1979). Excellent agreements are found as indicated in Tables 1~3, the only distinct value being symbolized by (*). The critical case μ_2=0 denotes the problem of an inclined crack in a semi-infinite plane with traction free condition. The numerical accuracy proves the practicability of the alternating procedure for a crack in a bi-material.

Table 1 Comparisons of normalized mode-III S.I.F. (F_{III}) for an inclined crack in a bi-material (μ_1/μ_2= 0.0433)

	μ_1/μ_2=0.0433, $\theta=\pi/8$				$\theta=\pi/4$				$\theta=\pi/2$			
	F_{IIIA}		F_{IIIB}		F_{IIIA}		F_{IIIB}		F_{IIIA}		F_{IIIB}	
c/a	Bassani & Erdogan (1979)	Present	Bassani & Erdogan (1979)	Present	Bassani & Erdogan (1979)	Present	Bassani & Erdogan (1979)	Present	Bassani & Erdogan (1979)	Present	Bassani & Erdogan (1979)	Present
1.05	0.659	0.660*	0.852	0.852	0.622	0.622	0.893	0.893	0.610	0.610	0.916	0.916
1.10	0.696	0.696	0.857	0.857	0.693	0.693	0.900	0.900	0.712	0.712	0.924	0.924
1.15	0.719	0.719	0.862	0.862	0.737	0.737	0.906	0.906	0.770	0.770	0.931	0.931
1.25	0.752	0.752	0.869	0.869	0.794	0.794	0.916	0.916	0.837	0.837	0.941	0.941
1.50	0.804	0.804	0.886	0.886	0.869	0.869	0.936	0.936	0.911	0.911	0.959	0.959
2.00	0.865	0.865	0.913	0.913	0.931	0.931	0.959	0.959	0.959	0.959	0.976	0.976
5.00	0.970	0.970	0.975	0.975	0.990	0.990	0.992	0.992	0.995	0.995	0.996	0.996
10.0	0.992	0.992	0.993	0.993	0.998	0.998	0.998	0.998	0.999	0.999	0.999	0.999
∞	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

Table 2 Comparisons of normalized mode-III S.I.F. (F_{III}) for an inclined crack in a bi-material (μ_1/μ_2= 23.08)

	μ_1/μ_2=23.08, $\theta=\pi/8$				$\theta=\pi/4$				$\theta=\pi/2$			
	F_{IIIA}		F_{IIIB}		F_{IIIA}		F_{IIIB}		FI_{IIIA}		F_{IIIB}	
c/a	Bassani & Erdogan (1979)	Present	Bassani & Erdogan (1979)	Present	Bassani & Erdogan (1979)	Present	Bassani & Erdogan (1979)	Present	Bassani & Erdogan (1979)	Present	Bassani & Erdogan (1979)	Present
1.05	2.22	2.21*	1.34	1.34	1.87	1.87	1.19	1.19	1.70	1.70	1.13	1.13
1.10	1.87	1.87	1.30	1.30	1.58	1.58	1.16	1.16	1.44	1.44	1.11	1.11
1.15	1.71	1.71	1.28	1.28	1.44	1.44	1.14	1.14	1.32	1.32	1.09	1.09
1.25	1.53	1.53	1.24	1.24	1.30	1.31	1.12	1.12	1.21	1.21	1.07	1.07
1.50	1.33	1.33	1.18	1.18	1.17	1.17	1.08	1.08	1.10	1.10	1.05	1.05
2.00	1.18	1.18	1.12	1.12	1.08	1.08	1.05	1.05	1.04	1.04	1.03	1.03
5.00	1.03	1.03	1.03	1.03	1.01	1.01	1.01	1.01	1.00	1.00	1.00	1.00
10.0	1.01	1.01	1.01	1.01	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
∞	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

2. An Inclined Crack in a Strip

(i) Uniform Anti-Plane Shear Stress Loads Applied at Boundaries for $-d<x<d$

Figure 13 shows a parallel crack in a strip subjected to an anti-plane uniformly distributed shear stress at boundaries for $-d<x<d$ Table 4 shows the dimensionless mode-III S.I.F. for different values of a/h. Excellent agreements are obtained if compared with the solutions of Ma (1988) who also analyzed the same problem.

(ii) Two Pairs of Anti-Plane Point Forces Applied at Boundaries

Consider a parallel crack in a strip subjected to two pairs of anti-plane point forces applied at boundaries as shown in Fig. 14. Table 5 shows the dimensionless mode-III S.I.F. for different values of a/h. Again, excellent agreements with the solutions of Ma (1988) are obtained.

(iii) A Pair of Anti-Plane Point Forces Applied at Boundaries

Consider an inclined crack in a strip subjected to a pair of anti-plane point forces $2p$ applied at boundaries as shown in Fig. 15. The inclined angle is θ. Table 6 shows the dimensionless mode-III S.I.F. for different values of a/h and θ. There are obvious differences between the present results and Chao and Kao (1997). However, for the special case of a/h=0.5 and θ=0°, the present result is the same as Ma (1988)

Table 3 Comparisons of normalized mode-III S.I.F. (F_{III}) for an inclined crack in a bi-material (μ_2=0)

	μ_2=0, $\theta=\pi/8$				$\theta=\pi/4$				$\theta=\pi/2$			
	F_{IIIA}		F_{IIIB}		F_{IIIA}		F_{IIIB}		FI_{IIIA}		F_{IIIB}	
c/a	Bassani & Erdogan (1979)	Present	Bassani & Erdogan (1979)	Present	Bassani & Erdogan (1979)	Present	Bassani & Erdogan (1979)	Present	Bassani & Erdogan (1979)	Present	Bassani & Erdogan (1979)	Present
1.05	2.48	2.48	1.40	1.40	2.01	2.01	1.22	1.22	1.79	1.79	1.15	1.15
1.10	2.03	2.03	1.35	1.35	1.66	1.66	1.18	1.18	1.49	1.49	1.12	1.12
1.15	1.82	1.82	1.32	1.32	1.50	1.50	1.16	1.16	1.36	1.36	1.10	1.10
1.25	1.60	1.60	1.27	1.27	1.34	1.34	1.13	1.13	1.23	1.23	1.08	1.08
1.50	1.37	1.37	1.20	1.20	1.18	1.18	1.09	1.09	1.11	1.11	1.05	1.05
2.00	1.20	1.20	1.13	1.13	1.09	1.09	1.05	1.05	1.05	1.05	1.03	1.03
5.00	1.04	1.04	1.03	1.03	1.01	1.01	1.01	1.01	1.01	1.01	1.00	1.00
10.0	1.01	1.01	1.01	1.01	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
∞	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

Table 4 Dimensionless mode-III S.I.F. of a parallel crack in a strip subjected to an anti-plane uniformly distributed shear stress at boundaries for $-d<x<d$

a/h	$K_{III}/\tau\sqrt{a}$ (d=a)		$K_{III}/\tau\sqrt{a}$ (d=∞)	
	Present	Ma (1988)	Present	Ma (1988)
4	1.548	1.548	1.772	1.772
2	1.061	1.061	1.377	1.377
1	0.702	0.702	1.149	1.149
0.5	0.431	0.431	1.047	1.047
0.25	0.239	0.239	1.013	1.013

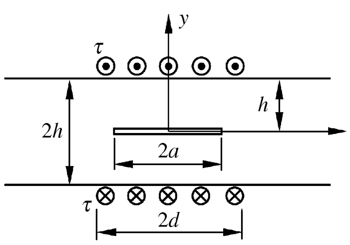

Fig. 13 A parallel crack in a strip subjected to an anti-plane uniformly distributed shear stress τ at boundaries for $-d<x<d$

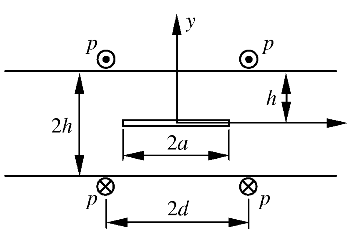

Fig. 14 A parallel crack in a strip subjected to two pairs of anti-plane point forces applied at boundaries

(iv) Uniform Anti-Plane Shear Stress Applied at Crack Faces

Consider an inclined crack in a strip subjected

Table 5 Dimensionless mode-III S.I.F. of a parallel crack in a strip subjected to two pairs of anti-plane point forces applied at boundaries

a/h	$K_{III}\sqrt{a}/p$ (d=0)		$K_{III}\sqrt{a}/p$ (d=a)	
	Present	Ma (1988)	Present	Ma (1988)
4	1.596	1.596	1.128	1.128
2	1.126	1.126	0.798	0.798
1	0.764	0.764	0.563	0.563
0.5	0.457	0.457	0.382	0.382
0.25	0.244	0.244	0.228	0.228

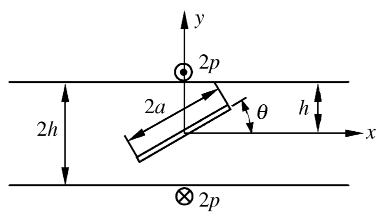

Fig. 15 An inclined crack in a strip subjected to a pair of anti-plane point forces $2p$ at boundary

to a uniformly distributed anti-plane shear stress τ applied at crack faces as shown in Fig. 16. Table 7 shows the dimensionless mode-III S.I.F. for different values of a/h and θ. For the case of θ=0° and a/h=0.5, the present result is the same as the solution obtained by Ma (1988). For the case of θ=90°, our solution is the same as that presented by Irwin (1957).

(v) An Anti-Plane Point Force Applied at Upper Boundary

Consider an inclined crack in a strip subjected to an anti-plane point force $2p$ applied at boundary as shown in Fig. 17. Table 8 shows the dimensionless mode-III S.I.F. for different values of a/h and θ.

Table 6 Dimensionless mode-III S.I.F. of an inclined crack in a strip subjected to a pair of anti-plane point forces 2*p* applied at boundary

θ	$K_{III}\sqrt{a}/p$														
	a/h=0.1			a/h=0.3			a/h=0.5			a/h=0.7			a/h=0.9		
	Present	Chao & Kao (1997)	ε (%)	Present	Chao & Kao (1997)	ε (%)	Present	Chao & Kao (1997)	ε (%)	Present	Chao & Kao (1997)	ε (%)	Present	Chao & Kao (1997)	ε (%)
0°	0.100	0.103	-2.9	0.290	0.295	-1.7	0.457 (0.457)*	0.455	0.4	0.597	0.575	3.8	0.713	0.660	8.0
10°	0.098	0.100	-2.0	0.287	0.292	-1.7	0.457	0.454	0.7	0.602	0.579	4.0	0.724	0.668	8.4
20°	0.094	0.096	-2.1	0.279	0.284	-1.8	0.455	0.452	0.7	0.615	0.589	4.4	0.759	0.691	9.8
30°	0.087	0.089	-2.2	0.265	0.270	-1.9	0.450	0.447	0.7	0.636	0.606	5.0	0.819	0.734	11.6
40°	0.077	0.080	-3.8	0.243	0.247	-1.6	0.438	0.434	0.9	0.662	0.626	5.8	0.912	0.802	13.7
50°	0.065	0.067	-3.0	0.213	0.217	-1.8	0.412	0.408	1.0	0.684	0.646	5.9	1.046	0.904	15.7
60°	0.051	0.052	-1.9	0.173	0.175	-1.1	0.363	0.360	0.8	0.684	0.650	5.2	1.226	1.054	16.3
70°	0.035	0.036	-2.8	0.122	0.124	-1.6	0.279	0.277	0.7	0.620	0.600	3.3	1.435	1.273	12.7
80°	0.018	0.019	-5.3	0.064	0.065	-1.5	0.154	0.154	0.0	0.410	0.404	1.5	1.504	1.458	3.2
90°	0.000	0.000	0.0	0.000	0.000	0.0	0.000	0.000	0.0	0.000	0.000	0.0	0.000	0.000	0.0

*MA (1988)

Table 7 Dimensionless mode-III S.I.F. of an inclined crack in a strip subjected to uniform anti-plane shear stress applied at crack faces

	$K_{III}/(\tau\sqrt{\pi a})$								
θ	a/h=0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0°	1.002	1.008	1.018	1.031	1.047 (1.047)**	1.064	1.084	1.105	1.127
10°	1.002	1.008	1.018	1.032	1.049	1.068	1.089	1.112	1.136
20°	1.002	1.009	1.020	1.035	1.054	1.077	1.103	1.131	1.163
30°	1.003	1.010	1.023	1.041	1.064	1.092	1.126	1.166	1.212
40°	1.003	1.012	1.027	1.048	1.076	1.113	1.159	1.216	1.289
50°	1.003	1.013	1.030	1.055	1.090	1.137	1.199	1.284	1.403
60°	1.003	1.015	1.034	1.063	1.104	1.162	1.245	1.367	1.565
70°	1.004	1.016	1.037	1.069	1.117	1.186	1.289	1.458	1.779
80°	1.004	1.017	1.039	1.074	1.125	1.202	1.323	1.534	2.005
90°	1.004 (1.004)*	1.017 (1.017)*	1.040 (1.040)*	1.075 (1.075)*	1.128 (1.128)*	1.208 (1.208)*	1.336 (1.336)*	1.565 (1.565)*	2.113 (2.113)*

*Irwin (1957) **MA (1988)

Table 8 Dimensionless mode-III S.I.F. of an inclined crack in a strip subjected to an anti-plane point force applied at upper boundary

	$K_{III}\sqrt{a}/p$									
θ	a/h=0.1		0.3		0.5		0.7		0.9	
	Tip A	Tip B	Tip A	Tip B	Tip A	Tip B	Tip A	Tip B	Tip A	Tip B
0°	0.0498	0.0498	0.1448	0.1448	0.2284	0.2284	0.2986	0.2986	0.3567	0.3567
10°	0.0478	0.0504	0.1326	0.1545	0.2022	0.2544	0.2586	0.3432	0.3047	0.4197
20°	0.0444	0.0494	0.1186	0.1604	0.1768	0.2783	0.2240	0.3912	0.2646	0.4940
30°	0.0400	0.0468	0.1035	0.1614	0.1525	0.2977	0.1953	0.4411	0.2381	0.5811
40°	0.0347	0.0424	0.0876	0.1557	0.1294	0.3086	0.1721	0.4899	0.2275	0.6848
50°	0.0286	0.0364	0.0711	0.1418	0.1068	0.3052	0.1529	0.5313	0.2348	0.8111
60°	0.0220	0.0288	0.0540	0.1186	0.0836	0.2791	0.1335	0.5510	0.2587	0.9675
70°	0.0149	0.0200	0.0364	0.0859	0.0583	0.2205	0.1065	0.5139	0.2845	1.1507
80°	0.0075	0.0102	0.0183	0.0453	0.0302	0.1242	0.0626	0.3473	0.2525	1.2518
90°	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

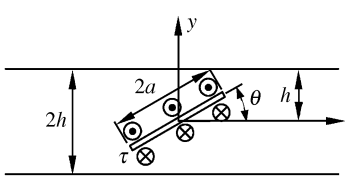

Fig. 16 An inclined crack in a strip subjected to uniform anti-plane shear stress τ at crack faces

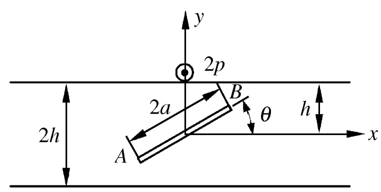

Fig. 17 An inclined crack in a strip subjected to an anti-plane point force at upper boundary

(vi) An Inclined Crack in a Strip with Fixed Lower Boundary Subjected to an Anti-Plane Point Force Applied at Upper Boundary

Consider an inclined crack in a strip with fixed lower boundary subjected to an anti-plane point force $2p$ applied at upper boundary as shown in Fig. 18. Table 9 shows the dimensionless mode-III S.I.F. for different values of a/h and θ.

3. An Inclined Crack in a Thin Layer Bonded to a Half Plane

(i) An Inclined Crack in a Thin Layer Bonded to a Half Plane Subjected to an Anti-Plane Point Force 2p Applied at the Boundary

Figure 19 shows an inclined crack in a thin layer bonded to a half plane subjected to an anti-plane point

Table 9 Dimensionless mode-III S.I.F. of an inclined crack in a strip with fixed lower boundary subjected to an anti-plane point force applied at upper boundary

θ	$K_{III}\sqrt{a}/p$									
	a/h=0.1		0.3		0.5		0.7		0.9	
	Tip A	Tip B	Tip A	Tip B	Tip A	Tip B	Tip A	Tip B	Tip A	Tip B
0°	0.0704	0.0704	0.2048	0.2048	0.3231	0.3231	0.4223	0.4223	0.5045	0.5045
10°	0.0685	0.0703	0.1951	0.2101	0.3044	0.3382	0.3969	0.4471	0.4757	0.5362
20°	0.0646	0.0681	0.1814	0.2103	0.2822	0.3493	0.3695	0.4725	0.4464	0.5745
30°	0.0589	0.0637	0.1640	0.2044	0.2559	0.3549	0.3384	0.4988	0.4133	0.6234
40°	0.0516	0.0571	0.1431	0.1914	0.2252	0.3523	0.3018	0.5248	0.3729	0.6879
50°	0.0430	0.0485	0.1191	0.1701	0.1895	0.3364	0.2583	0.5461	0.3222	0.7742
60°	0.0333	0.0381	0.0922	0.1395	0.1486	0.2995	0.2066	0.5498	0.2595	0.8898
70°	0.0227	0.0263	0.0629	0.0997	0.1027	0.2323	0.1462	0.5037	0.1857	1.0363
80°	0.0115	0.0134	0.0319	0.0521	0.0526	0.1295	0.0767	0.3377	0.1029	1.1342
90°	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

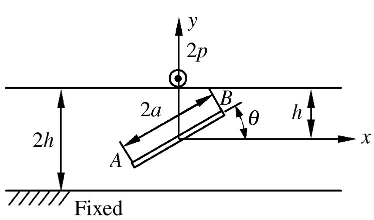

Fig. 18 An inclined crack in a strip with fixed lower boundary subjected to an anti-plane point force at upper boundary

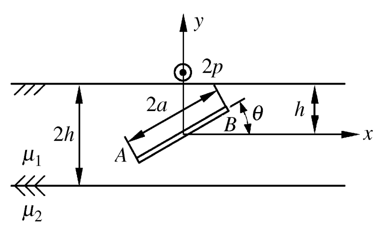

Fig. 19 An inclined crack in thin layer bonded to half plane subjected to an anti-plane point force at the boundary

force $2p$ applied at the boundary. First, the critical case for $\frac{\mu_1}{\mu_2}$=10000 and $\frac{\mu_1}{\mu_2}$=0.0001 are evaluated and are checked with the solutions of a crack in a strip with free or fixed lower boundary, and the results are presented in Table 10 and Table 11. The excellent agreements prove the validity of the present method in reduced problems. Then, the case investigated by Chao and Kao (1997) is considered, the shear modulus is μ_1=194.7Gpa for the thin layer and μ_2= 229.5Gpa for the half plane. The results are indicated in Table 12. Obvious differences between the present result and Chao and Kao (1997) are found.

(ii) An Inclined Crack in a Thin Layer Bonded to a Half Plane Subjected to Uniform Shear Stress Applied at Crack Faces

Figure 20 shows an inclined crack in a thin layer bonded to a half plane subjected to uniform shear stress τ applied at crack faces. First, the critical case for $\frac{\mu_1}{\mu_2}$=10000 is considered and the results are verified by the solution of a crack in a strip as presented in Table 13, the distinct values are symbolized by (*). The comparison shows that the solutions of thin layer for $\frac{\mu_1}{\mu_2}$=10000 are close to the solutions for a strip. Finally, the cases of $\frac{\mu_1}{\mu_2}$=2 and $\frac{\mu_1}{\mu_2}=\frac{1}{3}$ are considered, the results are shown in Table 14 and Table 15, respectively.

VI. CONCLUSION

In this study, we present alternating procedures to evaluate the mode-III S.I.F. of an inclined crack embedded in a thin layer which is bonded to a half plane. The numerical results confirm the validity of the proposed alternating procedure. The solutions of a bi-material problem and a thin layer bonded to a half plane subjected to an anti-plane point force are referred to as fundamental solutions. By using these fundamental solutions and alternating procedure, the stress intensity factors of an inclined crack in a thin layer bonded to a half plane are evaluated. The numerical results of critical cases for $\frac{\mu_1}{\mu_2}$=10000 and $\frac{\mu_1}{\mu_2}$= 0.0001 are checked with the solutions of a crack in a strip with free or fixed lower boundary and excellent agreements are obtained. The analytical alternating method proposed in this study is found to be a simple and accurate technique for the computation of mode-III S.I.F. of the cracked layer problem.

Table 10 Comparison of the critical case for $\frac{\mu_1}{\mu_2}$= 10000 and an inclined crack in a strip subjected to an anti-plane point force applied at upper boundary

θ	$K_{III}\sqrt{a}/p$ (a/h=0.5)					
	Tip A			Tip B		
	$\frac{\mu_1}{\mu_2}$=10000	Table 8	ε (%)	$\frac{\mu_1}{\mu_2}$=10000	Table 8	ε (%)
0°	0.2284	0.2284	0.00	0.2284	0.2284	0.00
10°	0.2022	0.2022	0.00	0.2543	0.2544	-0.04
20°	0.1768	0.1768	0.00	0.2782	0.2783	-0.04
30°	0.1525	0.1525	0.00	0.2976	0.2977	-0.03
40°	0.1294	0.1294	0.00	0.3085	0.3086	-0.03
50°	0.1068	0.1068	0.00	0.3051	0.3052	-0.03
60°	0.0835	0.0836	-0.12	0.2790	0.2791	-0.04
70°	0.0583	0.0583	0.00	0.2205	0.2205	0.00
80°	0.0302	0.0302	0.00	0.1242	0.1242	0.00
90°	0.0000	0.0000	0.00	0.0000	0.0000	0.00

Table 11 Comparison of the critical case for $\frac{\mu_1}{\mu_2}$= 0.0001 and a crack in a strip with lower fixed boundary subjected to an anti-plane point force at upper boundary

θ	$K_{III}\sqrt{a}/p$ (a/h=0.5)					
	Tip A			Tip B		
	$\frac{\mu_1}{\mu_2}$=0.0001	Table 9	ε (%)	$\frac{\mu_1}{\mu_2}$=0.0001	Table 9	ε (%)
0°	0.3231	0.3231	0.00	0.3231	0.3231	0.00
10°	0.3038	0.3044	-0.20	0.3382	0.3382	0.00
20°	0.2806	0.2822	-0.57	0.3487	0.3493	-0.17
30°	0.2533	0.2559	-1.02	0.3532	0.3549	-0.48
40°	0.2216	0.2252	-1.60	0.3492	0.3523	-0.88
50°	0.1854	0.1895	-2.16	0.3321	0.3364	-1.28
60°	0.1446	0.1486	-2.69	0.2947	0.2995	-1.60
70°	0.0995	0.1027	-3.12	0.2281	0.2323	-1.81
80°	0.0508	0.0526	-3.42	0.1270	0.1295	-1.93
90°	0.0000	0.0000	0.00	0.0000	0.0000	0.00

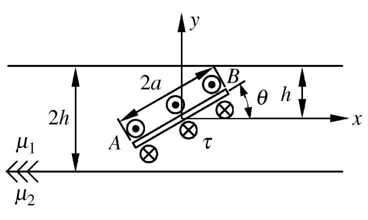

Fig. 20 An inclined crack in thin layer bonded to half plane subjected to uniform shear stress at crack face

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NOMENCLATURE

a	half length of the crack
$f_c^{(i)}$	residual traction on the crack faces of the i'th cycle of external alternating procedure
$g_c^{(j)}$	residual traction on the crack faces of the j'th cycle of internal alternating procedure
h	thickness of the strip; thickness of the layer
K_{IIIA}, K_{IIIB}	stress intensity factor of the crack tip A

Table 12(a) The dimensionless mode-III S.I.F. for a crack in a thin layer bonded to a half plane subjected to an anti-plane point force applied at the boundary (Tip A)

θ	$K_{III}\sqrt{a}/p$														
	a/h=0.1			a/h=0.3			a/h=0.5			a/h=0.7			a/h=0.9		
	Present	Chao & Kao (1997)	ε (%)	Present	Chao & Kao (1997)	ν (%)	Present	Chao & Kao (1997)	ε (%)	Present	Chao & Kao (1997)	ε (%)	Present	Chao & Kao (1997)	ε (%)
0°	0.064	0.066	-2.9	0.187	0.191	-2.1	0.296	0.299	-0.9	0.390	0.386	0.9	0.468	0.453	3.4
10°	0.062	0.064	-3.0	0.176	0.180	-2.1	0.275	0.276	-0.4	0.359	0.352	1.9	0.431	0.412	4.5
20°	0.058	0.060	-2.7	0.162	0.165	-1.6	0.251	0.250	0.4	0.328	0.318	3.1	0.396	0.373	6.0
30°	0.053	0.055	-3.5	0.145	0.147	-1.1	0.225	0.222	1.2	0.296	0.284	4.0	0.361	0.336	7.5
40°	0.046	0.051	-9.0	0.126	0.127	-0.9	0.195	0.192	1.8	0.261	0.248	5.4	0.327	0.298	9.7
50°	0.039	0.040	-3.5	0.104	0.105	-0.9	0.163	0.159	2.6	0.224	0.210	6.6	0.291	0.258	12.8
60°	0.030	0.031	-3.9	0.080	0.080	0.1	0.127	0.124	2.7	0.181	0.144	25.9	0.251	0.215	16.7
70°	0.020	0.021	-3.3	0.055	0.055	-0.9	0.088	0.085	3.5	0.131	0.092	42.5	0.202	0.146	38.4
80°	0.010	0.010	3.0	0.028	0.028	-1.4	0.045	0.043	4.9	0.070	0.063	11.7	0.132	0.098	34.2
90°	0.000	0.000	0.0	0.000	0.000	0.0	0.000	0.000	0.0	0.000	0.000	0.0	0.000	0.000	0.0

Table 12(b) The dimensionless mode-III S.I.F. for a crack in a thin layer bonded to a half plane subjected to an anti-plane point force applied at the boundary (Tip B)

θ	$K_{III}\sqrt{a}/p$														
	a/h=0.1			a/h=0.3			a/h=0.5			a/h=0.7			a/h=0.9		
	Present	Chao & Kao (1997)	ε (%)	Present	Chao & Kao (1997)	ε (%)	Present	Chao & Kao (1997)	ε (%)	Present	Chao & Kao (1997)	ε (%)	Present	Chao & Kao (1997)	ε (%)
0°	0.064	0.066	-2.9	0.187	0.191	-2.1	0.296	0.299	-0.9	0.390	0.386	0.9	0.468	0.453	3.4
10°	0.064	0.066	-2.7	0.194	0.198	-2.3	0.315	0.318	-1.1	0.420	0.418	0.6	0.510	0.495	2.9
20°	0.062	0.064	-2.5	0.195	0.200	-2.3	0.329	0.332	-0.8	0.452	0.446	1.2	0.557	0.534	4.3
30°	0.059	0.060	-2.5	0.191	0.195	-1.8	0.338	0.340	-0.5	0.483	0.474	1.9	0.613	0.581	5.5
40°	0.053	0.054	-2.6	0.181	0.184	-1.9	0.339	0.340	-0.4	0.513	0.502	2.1	0.682	0.637	7.1
50°	0.045	0.046	-2.6	0.161	0.164	-1.7	0.326	0.327	-0.5	0.537	0.524	2.4	0.771	0.709	8.7
60°	0.035	0.036	-1.9	0.133	0.136	-2.4	0.291	0.293	-0.6	0.542	0.531	2.1	0.887	0.803	10.4
70°	0.024	0.025	-2.4	0.095	0.097	-1.9	0.227	0.229	-1.0	0.498	0.492	1.3	1.034	0.931	11.1
80°	0.012	0.013	-4.6	0.050	0.051	-2.4	0.127	0.128	-1.0	0.335	0.335	-0.1	1.134	1.055	7.5
90°	0.000	0.000	0.0	0.000	0.000	0.0	0.000	0.000	0.0	0.000	0.000	0.0	0.000	0.000	0.0

Table 13 Comparison of the critical case for a crack in a thin layer $\frac{\mu_1}{\mu_2}$=10000 and a crack in a strip subjected to uniform anti-plane shear stress applied at crack faces

θ	$K_{III}(\tau\sqrt{\pi a})$									
	a/h=0.1		a/h=0.3		a/h=0.5		a/h=0.7		a/h=0.9	
	Thin Layer	Strip	Thin Layer	Strip	Thin Layer	Strip	Thin Layer	Strip	Thin Layer	Strip
0°	1.002	1.002	1.018	1.018	1.047	1.047	1.084	1.084	1.127	1.127
10°	1.002	1.002	1.018	1.018	1.049	1.049	1.089	1.089	1.136	1.136
20°	1.002	1.002	1.020	1.020	1.054	1.054	1.103	1.103	1.163	1.163
30°	1.003	1.003	1.023	1.023	1.064	1.064	1.126	1.126	1.212	1.212
40°	1.003	1.003	1.026	1.027	1.076	1.076	1.158	1.159	1.289	1.289
50°	1.003	1.003	1.030	1.030	1.090	1.090	1.199	1.199	1.402*	1.403
60°	1.003	1.003	1.034	1.034	1.104	1.104	1.244	1.245	1.564*	1.565
70°	1.004	1.004	1.037	1.037	1.116*	1.117	1.289	1.289	1.778*	1.779
80°	1.004	1.004	1.039	1.039	1.125	1.125	1.322*	1.323	2.003*	2.005
90°	1.004	1.004	1.039*	1.040	1.128	1.128	1.335*	1.336	2.111*	2.113

Table 14 Dimensionless mode-III S.I.F. of an inclined crack in a thin layer $\frac{\mu_1}{\mu_2}$=2 subjected to uniform anti-plane shear stress at crack faces

θ	$K_{III}(\tau\sqrt{\pi a})$									
	a/h=0.1		a/h=0.3		a/h=0.5		a/h=0.7		a/h=0.9	
	Tip A	Tip B	Tip A	Tip B	Tip A	Tip B	Tip A	Tip B	Tip A	Tip B
0°	1.002	1.002	1.013	1.013	1.034	1.034	1.061	1.061	1.092	1.092
10°	1.002	1.002	1.013	1.013	1.034	1.036	1.061	1.065	1.092	1.100
20°	1.002	1.002	1.013	1.014	1.035	1.038	1.064	1.072	1.098	1.114
30°	1.002	1.002	1.014	1.015	1.037	1.042	1.069	1.083	1.110	1.138
40°	1.002	1.002	1.015	1.016	1.040	1.047	1.077	1.097	1.130	1.175
50°	1.002	1.002	1.016	1.017	1.043	1.052	1.088	1.115	1.159	1.230
60°	1.002	1.002	1.016	1.018	1.047	1.058	1.100	1.136	1.199	1.308
70°	1.002	1.002	1.017	1.019	1.050	1.062	1.112	1.156	1.249	1.412
80°	1.002	1.002	1.017	1.020	1.052	1.065	1.120	1.171	1.297	1.521
90°	1.002	1.002	1.018	1.020	1.053	1.067	1.124	1.177	1.318	1.572

Table 15 Dimensionless mode-III S.I.F. of an inclined crack in a thin layer $\frac{\mu_1}{\mu_2}=\frac{1}{3}$ subjected to uniform anti-plane shear stress at crack faces

θ	$K_{III}(\tau\sqrt{\pi a})$									
	a/h=0.1		a/h=0.3		a/h=0.5		a/h=0.7		a/h=0.9	
	Tip A	Tip B	Tip A	Tip B	Tip A	Tip B	Tip A	Tip B	Tip A	Tip B
0°	1.001	1.001	1.008	1.008	1.020	1.020	1.037	1.037	1.057	1.057
10°	1.001	1.001	1.007	1.008	1.018	1.022	1.033	1.041	1.049	1.065
20°	1.001	1.001	1.006	1.008	1.016	1.023	1.027	1.045	1.040	1.074
30°	1.001	1.001	1.005	1.008	1.013	1.023	1.020	1.049	1.028	1.085
40°	1.001	1.001	1.004	1.007	1.009	1.024	1.013	1.053	1.013	1.102
50°	1.001	1.001	1.003	1.007	1.005	1.024	1.004	1.059	0.994	1.129
60°	1.000	1.001	1.002	1.007	1.002	1.024	0.994	1.066	0.968	1.171
70°	1.000	1.001	1.001	1.006	0.999	1.025	0.986	1.074	0.939	1.231
80°	1.000	1.000	1.001	1.006	0.997	1.025	0.979	1.080	0.910	1.297
90°	1.000	1.000	1.000	1.006	0.996	1.025	0.977	1.082	0.897	1.330

	and B, respectively
p	magnitude of the point force
q	magnitude of the point displacement
W	displacement
x, y	Cartesian coordinates
z	complex variable
Z	complex stress function
μ_1, μ_2	shear modulus of material 1 and 2, respectively
τ	stress
ω	variable in the Fourier transform domain

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