DAMAGE-COUPLED VISCOPLASTIC CONSTITUTIVE MODELING FOR SOLDER MATERIALS

C. L. Chow* and Yong Wei

ABSTRACT

This paper presents a damage-coupled viscoplastic constitutive model for characterizing thermomechanical fatigue (TMF) life of solder materials. The model takes into account the effects of microstructure, temperature and strain rate in the development of damage evolution equations. The TMF model is applied to characterize hysteresis loops of two solder materials, 63Sn-37Pb and 95.5Sn-3.9Ag-0.6Cu, under both isothermal and thermomechanical cyclic loads. The TMF life prediction of 63Sn-37Pb solder has been found to be a successful model, based on the load-drop criterion, under fatigue loading.

Key Words: damage, viscoplasticity, thermomechanical fatigue, solder.

I. INTRODUCTION

Solder materials have been extensively used for electronic interconnections, including applications in surface mount assembly. It is well known that reliability of solder joints is critical during the service period. Several researchers have focused on experimental testing and constitutive modeling (Busso, *et al*., 1992; Guo, *et al*., 1992; McDowell, *et al*., 1994; Solomon, *et al*., 1986; Wei and Chow, 2001). The majority of the current methodologies for testing were developed for isothermal loading conditions. However, a key issue in the long term reliability of solder joints is failure due to thermal cycling. At present, most thermomechanical fatigue (TMF) predictive methodologies are based on empirical curve fitting techniques. New mechanics-based computational models and failure criteria are required for valid and robust durability analysis in thermomechanical fatigue loading. Through experimental observation, it is well known that the progression of failure in solder joints may involve coarsening of the microstructure, strain rate or temperature rate effects, and damage accumulation. A unified viscoplastic model is presented in this paper to characterize time-dependent, temperature-dependent, microstructure-dependent and damage-dependent mechanical behavior of solder materials. The constitutive model can be used to simulate load-drop curve, based on which isothermal and TMF fatigue life can be determined.

*Corresponding author. (clchow@umich.edu)

The authors are with the Department of Mechanical Engineering University of Michigan-Dearborn, MI, USA.

II. CONSTITUTIVE MODEL

A damage-coupled viscoplastic constitutive model is presented here for solder materials to characterize mechanical/damage behavior and to predict isothermal or TMF fatigue life. The model is established on the theory of damage mechanics with the introduction of several internal state variables.

The total strain is divided into the elastic strain and the inelastic strain. The damage-coupled elastic equation is introduced as

$$\varepsilon^e = \boldsymbol{C}^{-1} : \sigma \tag{1}$$

The damage-coupled inelastic equation is formulated as

$$\dot{\varepsilon}^{in} = \dot{p}^{in} \frac{3}{2} \frac{\boldsymbol{S} - \boldsymbol{X}}{J_2} \tag{2}$$

where $\dot{\varepsilon}^{in}$ is the inelastic strain rate tensor, $\boldsymbol{\sigma}$ is the Cauchy stress tensor, $\boldsymbol{C}$ is the damaged elasticity tensor. Its matrix form is (Chow and Wei, 1999)

$$\boldsymbol{C}^{-1} = \frac{1}{E}\begin{bmatrix} 1 & -v & -v & 0 & 0 & 0 \\ -v & 1 & -v & 0 & 0 & 0 \\ -v & -v & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2(1+v) & 0 & 0 \\ 0 & 0 & 0 & 0 & 2(1+v) & 0 \\ 0 & 0 & 0 & 0 & 0 & 2(1+v) \end{bmatrix}$$

$$E = \frac{E_0(1-D)^2}{1-4v_0\mu + 2(1-v_0)\mu^2}$$

$$v = \frac{v_0 - 2(1-v_0)\mu - (1-3v_0)\mu^2}{1-4v_0\mu + 2(1-v_0)\mu^2} \quad (3)$$

E_0 and v_0 are initial values of the Young's modulus and the Poisson's ratio for as-fabricated material, E and v are the effective Young's modulus and the effective Poisson's ratio for damaged material under load, and D and μ are two damage scalars. J_2 is a second invariant of stress difference defined as

$$J_2\{\frac{3}{2}(\boldsymbol{S}-\boldsymbol{X}):(\boldsymbol{S}-\boldsymbol{X})\}^{1/2} \quad (4)$$

$\boldsymbol{S}$ is the deviatoric stress tensor, $\boldsymbol{X}$ is the deviatoric back stress tensor, $\dot{p}^{in}$ is the equivalent inelastic strain rate. The equivalent inelastic strain rate $\dot{p}^{in}$ is expressed with the effects of grain/phase size, damage and temperature as

$$\dot{p}^{in} = \frac{1-\mu}{1-D} f \exp(\frac{-Q}{RT})(\frac{\lambda_0}{\lambda})^p \sinh^m(\frac{1-\mu}{1-D}\ \frac{J_2-\sigma_y}{c+\hat{c}}) \quad (5)$$

where f, m, p and Q are material parameters, R is the gas constant, T is the absolute temperature, λ is the current phase size, λ_0 is the initial phase size, c and $\hat{c}$ are state variables, and σ_y is the yield strength. The state variables c and $\hat{c}$ were given by Frear, *et al*. (1997)

$$\dot{c} = A_1\dot{p}^{in} - (A_2\dot{p}^{in} + A_3)(c-c_0)^2 \quad (6)$$

$$\hat{c} = A_7(\frac{\lambda_0}{\lambda})^{A_8} \quad (7)$$

where A_1, A_2, A_3, A_7, A_8 and c_0 are the material parameters, c_0 is the initial value of the state variable c.

The damage-coupled kinematic hardening equation is (Wei, *et al*., 2002)

$$\boldsymbol{\alpha}_k = \frac{3}{2}\ \frac{(1-\mu)^2}{(1-D)^2 A_4}\boldsymbol{X} \quad (8)$$

where $\boldsymbol{\alpha}_k$ is the back strain tensor, A_4 is the parameter for kinematic hardening. The evolution equation for the back strain tensor is formulated as

$$\dot{\boldsymbol{\alpha}}_k = \dot{\varepsilon}^{in} - A_4(A_5\dot{p}^{in} + A_6)\boldsymbol{\alpha}_k\sqrt{\frac{2}{3}\boldsymbol{\alpha}_k:\boldsymbol{\alpha}_k} \quad (9)$$

where c_0, A_1, A_2, A_3, A_4, A_5 and A_6 are material parameters. Then the evolution equation of back stress can be derived with Eq. (8) for isothermal loading

$$\dot{\boldsymbol{X}} = \frac{2}{3}A_4\frac{(1-D)^2}{(1-\mu)^2}\dot{\boldsymbol{\alpha}}_k - \frac{2\boldsymbol{X}}{1-D}\dot{D} + \frac{2\boldsymbol{X}}{1-\mu}\dot{\mu} \quad (10)$$

For thermomechanical cyclic loading, it is necessary to include the temperature rate term as an independent term. Therefore, a general form of the back stress evolution becomes

$$\dot{\boldsymbol{X}} = \frac{2}{3}A_4\frac{(1-D)^2}{(1-\mu)^2}\dot{\boldsymbol{\alpha}}_k - \frac{2\boldsymbol{X}}{1-D}\dot{D} + \frac{2\boldsymbol{X}}{1-\mu}\dot{\mu} + f(T, \boldsymbol{X}, \omega)\frac{3}{2}\ \frac{\boldsymbol{S}-\boldsymbol{X}}{J_2}\dot{T} \quad (11)$$

where $f(T, X, \omega)$ is a function of the absolute temperature T, the back stress $\boldsymbol{X}$, and a strain variable ω. Here, the strain variable ω is defined as the accumulated inelastic strain under monotonic loading. For cyclic loading, the value of ω is defined as the accumulated inelastic strain for each monotonic process (loading or reversed loading). In general, isothermal data are insufficient to prescribe a particular form for $f(T, X, \omega)$ under thermomechanical loading. From limited TMF testing data on solder materials, a temperature rate dependent function $f(T, X, \omega)$ is established for 63Sn-37Pb solder

$$f(T, \boldsymbol{X}, \omega) = 12.7\tanh(\frac{T-273}{150})\tanh(50\omega)\frac{\sigma - X}{|\sigma - X|} \quad (12)$$

and another form is proposed for 95.5Sn-3.9Ag-0.6Cu solder

$$f(T, X, \omega) = 1.2\tanh(50\omega) \quad (13)$$

Experimental tests are in progress to more precisely formulate the function.

Evolution equations for two damage variables D and μ are given as (Wei, *et al*., 2004)

$$\dot{D} = -\dot{w}\frac{Y_D}{2Y_d} \quad \dot{\mu} = -\dot{w}\frac{\gamma Y_\mu}{2Y_d} \quad (14)$$

where w is the equivalent damage variable, and Y_d is the equivalent damage energy release rate expressed as

$$Y_d = [\frac{1}{2}(Y_D^2 + \gamma Y_\mu^2]^{1/2} \quad (15)$$

γ is a damage-related material constant. Y_D and Y_μ are the damage energy release rates corresponding to the damage variables D and μ

$$Y_\mu = -\frac{1}{1-D}\boldsymbol{\sigma}: \boldsymbol{C}^{-1}: \boldsymbol{\sigma} - \frac{3}{2}\frac{(1-\mu)^2}{(1-D)^3 A_4}\boldsymbol{X}: \boldsymbol{X} \quad (16)$$

$$Y_\mu = -\frac{1}{1-D}\boldsymbol{\sigma}: \boldsymbol{Z}: \boldsymbol{\sigma} + \frac{3}{2}\frac{(1-\mu)}{(1-D)^2 A_4}\boldsymbol{X}: \boldsymbol{X} \quad (17)$$

The matrix form of the tensor $\boldsymbol{Z}$ is defined as (Chow and Wei, 1999)

$$\boldsymbol{Z} = \frac{1}{E_0(1-D)} \cdot \begin{bmatrix} z_1 & z_2 & z_2 & 0 & 0 & 0 \\ z_2 & z_1 & z_2 & 0 & 0 & 0 \\ z_2 & z_2 & z_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2(z_1 - z_2) & 0 & 0 \\ 0 & 0 & 0 & 0 & 2(z_1 - z_2) & 0 \\ 0 & 0 & 0 & 0 & 0 & 2(z_1 - z_2) \end{bmatrix}$$

$$z_1 = 2\mu(1-\nu_0) - 2\nu_0 \quad z_2 = (1+\mu)(1-\nu_0) - 2\mu\nu_0 \quad (18)$$

The evolution equation for the equivalent damage is formulated for isothermal loading (Wei, *et al.*, 2004)

$$\dot{w} = \begin{cases} \dot{p}^{in}\left(\frac{Y_d}{Y_h}\right)^{B_1}, & \text{if } F_d = 0 \text{ and } \frac{\partial \boldsymbol{F}_d}{\partial \boldsymbol{\varepsilon}^{in}}: \dot{\boldsymbol{\varepsilon}}^{in} > 0 \\ \dot{p}^{in}\left(\frac{Y_d}{Y_{hf}}\right), & \text{if } F_d < 0 \text{ or } F_d = 0 \text{ and } \frac{\partial \boldsymbol{F}_d}{\partial \boldsymbol{\varepsilon}^{in}}: \dot{\boldsymbol{\varepsilon}}^{in} \le 0 \end{cases} \quad (19)$$

where B_1 is a material constant, Y_h is the inelastic damage hardening/softening variable, and Y_{hf} is the fatigue damage hardening/softening variable. F_d is the damage surface that is defined to distinguish the damage accumulation under monotonic loading from that under cyclic loading. A damage surface in strain space is proposed to characterize the type of damage observed in the solder materials

$$F_d = \varepsilon_{eq}^{in} - \varepsilon_{eq,\max}^{in} = 0 \quad (20)$$

where ε_{eq}^{in} is the equivalent inelastic strain, a scalar measure of the inelastic strain, and $\varepsilon_{eq,\max}^{in}$ is the maximum value that this equivalent inelastic strain has achieved at its previous loading. The scalar measure of inelastic strain is defined as

$$\varepsilon_{eq}^{in} = \sqrt{\frac{2}{3}\boldsymbol{\varepsilon}^{in}: \boldsymbol{\varepsilon}^{in}} \quad (21)$$

$\boldsymbol{\varepsilon}^{in}$ in is the inelastic strain tensor. In principle, if the inelastic strain of material is located within the damage surface, the damage accumulated is defined as fatigue damage. If the strain of material is on the damage surface and attempts to expand the damage surface, the damage accumulated is defined as inelastic damage. The form of the inelastic damage hardening/softening variable is proposed as

$$Y_h = Y_0 e^{B_2 w + \frac{B_3}{T}} \quad (22)$$

The evolution equation for the fatigue damage hardening/softening variable is proposed as

$$Y_{hf} = Y_{0f}\{1 - \frac{1}{2}\tanh[20(w - 0.26)]\} e^{\frac{B_3}{T}} \quad (23)$$

$$\dot{Y}_{0f} = B_4\dot{w} - (B_5\dot{w} + B_6)(Y_{0f} - Y_{hf0})^2 \quad (24)$$

where Y_{hf0} is the initial value of the fatigue damage hardening/softening variable, B_2, B_3, B_4, B_5, B_6 and Y_{hf0} are temperature-dependent material constants.

However, the damage evolution Eq. (19) cannot be applied to TMF loading due to the experimental observation that in-phase thermal cycles reduced substantially the number of cycles to failure as compared with isothermal fatigue at a constant temperature (Guo, *et al.*, 1992). A temperature rate term should be introduced in the TMF damage evolution, and the fatigue damage part in Eq. (19) is accordingly revised as:

$$\dot{w} = \dot{p}^{in}\frac{Y_d}{Y_{hf}} + g(\varepsilon, Y_d, w, T)\left|\dot{T}\right| \quad (25)$$

where $\left|\dot{T}\right|$ is the absolute value of temperature rate (or accumulated temperature rate). According to Guo's data under TMF loading, a preliminary function is adopted for 63Sn-37Pb solder

$$g(\varepsilon, Y_d, w, T) = B_7\omega\frac{Y_d}{Y_{hf}} \quad (26)$$

where B_7 is a damage-related constant.

III. APPLICATIONS

Two solder materials, 63Sn-37Pb and 95.5Sn-3.9Ag-0.6Cu, were chosen to check the validity of the proposed TMF material model. The evolution of the phase size for 63Sn-37Pb was observed experimentally and formulated by Vianco et al (1999) as

$$\dot{\lambda} = \frac{1.05 \times 10^{-5} e^{-11023/T} + 4.00 \times 10^{-8} e^{-3123/T}\dot{p}^{in}}{(\lambda - \lambda_0)^{2.9}} \quad (27)$$

The change of the phase size for 95.5Sn-3.9Ag-0.6Cu was ignored in this study due to insufficient experimental data. The measured material constants are summarized in Tables 1 to 4. The validation analysis of the proposed constitutive model was conducted by comparing the hysteresis loops of the materials for the first few cycles that were measured.

Table 1 63Sn-37Pb solder material parameters (temperature independent)

Poisson's ratio v_0	0.4
Young's modulus E_0 (GPa)	33.26
A_1 (MPa)	0
A_2 (1/MPa)	0
A_3 (1/MPa-sec)	0
A_7 (MPa)	5.66
A_8	0.5
Flow rate f (1/sec)	1.802E+06
Sinh exponent m	3.04
Growth exponent p	3.00
Flow stress C_0 (MPa)	2.83
Phase size λ_0 (mm)	2.257E-03
Activation energy Q (cal/mol)	1.376E+4
Gas constant R (cal/mol-K)	1.987
Yield stress σ_y, MPa	0
B_1	0.4
B_2	5.0
B_3 (K)	3.34E+03
γ	-0.2
Y_0 (MPa)	6.0E-08

Table 2 63Sn-37Pb solder material parameters (temperature dependent)

Temperature (°K)	298	348	373
A_4 (GPa)	28.15	22.08	19.32
A_5 (1/MPa)	44.95	103.0	153.7
A_6 (1/MPa-sec)	8.42E-03	5.95E-02	0.2
B_4 (MPa)		$0.1^9 e^{B_3/T}$	
B_5 (1/MPa)		$1.1\times10^9 e^{-2B_3/T}$	
B_6 (1/MPa-sec)		$2.8\times10^5 e^{-2B_3/T}$	
Y_{hf0} (MPa)		$2.0\times10^{-6} e^{B_3/T}$	

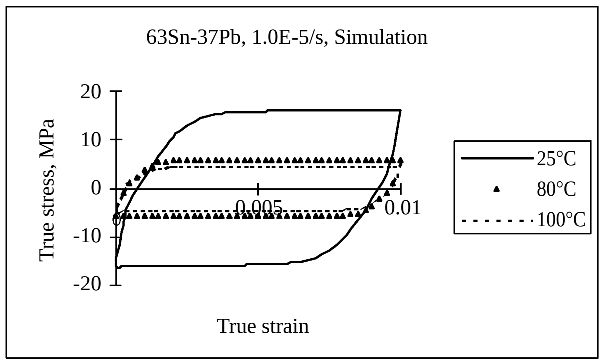
The proposed constitutive model was first applied to a typical tin-lead solder of 63Sn-37Pb. The cyclic strain range of 0 to 1% at strain rate of 10^{-5}/s was chosen. The predicted isothermal hysteresis loops are shown in Fig. 1(a), while the measured curves are shown in Fig. 1(b) for three temperatures, 25°C, 80°C and 100°C. It can be observed from the figures that the predicted maximum stress and stress range agree well with the experimental results under isothermal fatigue loading. The TMF hysteresis loops were studied next. The in-phase TMF loading was carried out under the strain range from 0 to 1% at the strain rate of 10^{-5}/s with cyclic temperature range from 25°C to 80°C. The predicted profiles of the in-phase TMF loops are found similar in shape and stress level as compared with the measurement, as depicted in Fig. 2. Both in-phase and out-of-phase (180° angle of phase lag) TMF loads were simulated under total

Table 3 95.5Sn-3.9Ag-0.6Cu solder parameters (temperature independent)

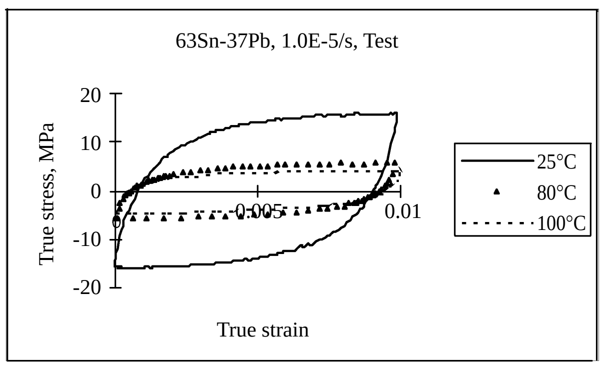
Poisson's ratio v_0	0.4
Young's modulus E_0 (GPa)	50
Yield strength σ_y (MPa)	6
A_1 (MPa)	10
A_2 (1/MPa)	0
A_3 (1/MPa-sec)	0
A_7 (MPa)	0
A_8	0
Flow rate f (1/sec)	4.35E+09
Sinh exponent m	3
Growth exponent p	0
Flow stress C_0 (MPa)	6
Activation energy Q (cal/mol)	2.04E+4
Gas constant R (cal/mol-K)	1.987
Yield stress σ_y, MPa	6.0

Table 4 95.5Sn-3.9Ag-0.6Cu solder material parameters (temperature dependent)

Temperature (°K)	298	373
A_4 (GPa)	8.4	4.2
A_5 (1/MPa)	84	262.8
A_6 (1/MPa-sec)	0	0

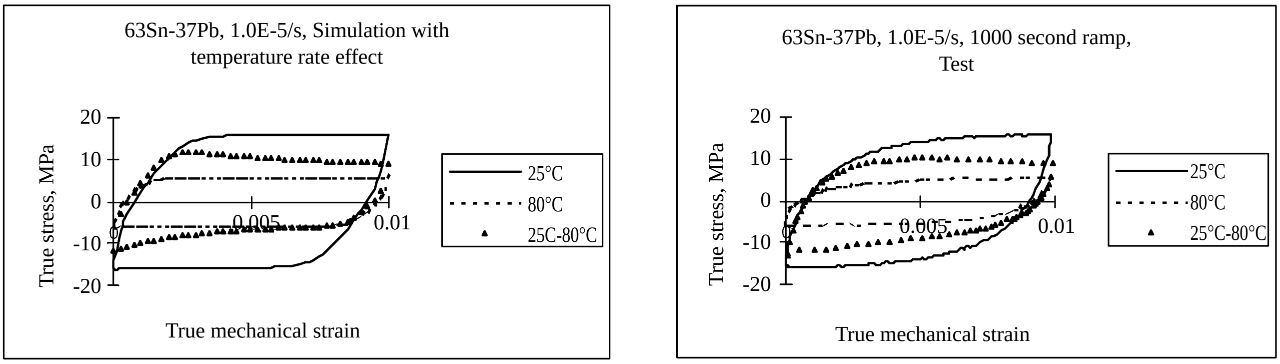


(a) Simulation



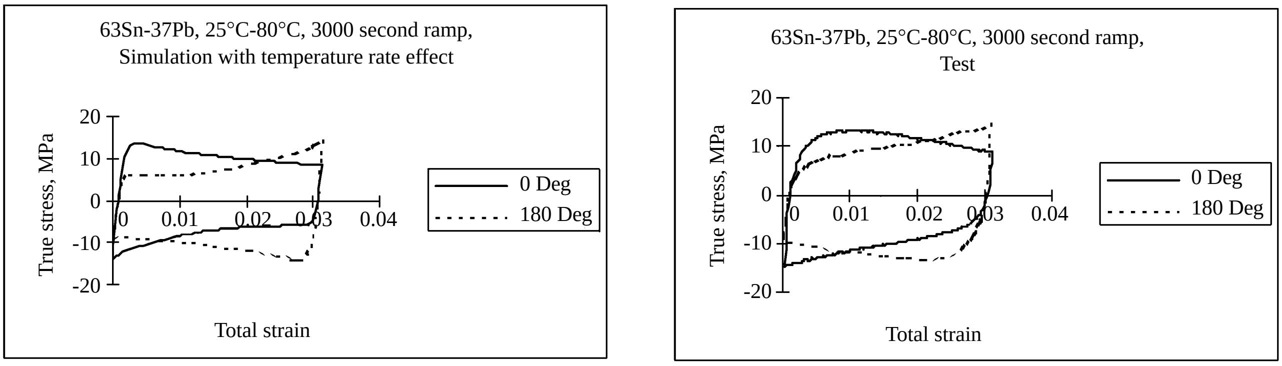
(b) Test

Fig. 1 Isothermal hysteresis loops for 63Sn-37Pb solder under different temperatures at strain rate of 10^{-5}/s



(a) Simulation (b) Test

Fig. 2 Isothermal and TMF hysteresis loops for 63Sn-37Pb solder at strain rate of 10^{-5}/s



(a) Simulation (b) Test

Fig. 3 In-phase and Out-of-phase TMF hysteresis loops for 63Sn-37Pb solder at strain rate of 10^{-5}/s

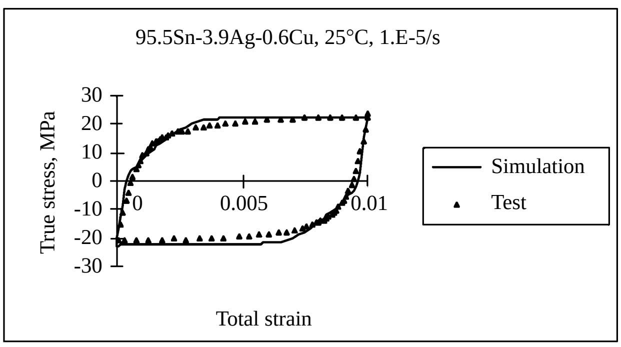


Fig. 4 Isothermal hysteresis loops for 95.5Sn-3.9Ag-0.6Cu solder under 25°C at strain rate of 10^{-5}/s

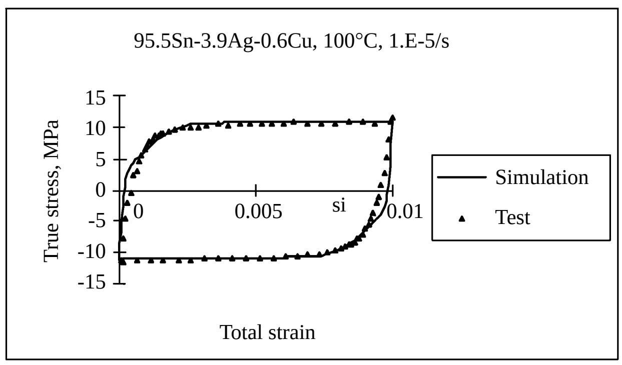


Fig. 5 Isothermal hysteresis loops for 95.5Sn-3.9Ag-0.6Cu solder under 100°C at strain rate of 10^{-5}/s

strain range from 0 to 3.14% with 3000-second ramp (about 10^{-5}/s strain rate) from 25°C to 80°C. The results are shown in Fig. 3.

A lead-free solder of 95.5Sn-3.9Ag-0.6Cu was chosen for the validation of the TMF material model. The applied cyclic strain range from 0 to 1% at the strain rate of 10^{-5}/s under two different test temperatures, 25°C and 100°C were applied. The calculated hysteresis loops for the first few cycles are compared with the measured data as shown in Figs. 4 and 5. In order to validate the proposed TMF model for the lead-free solder, an in-phase thermomechanical cyclic test was conducted with the test temperatures varying from 25°C to 100°C under a strain range from 0 to 1% at a strain rate of 10^{-5}/s. The calculated thermomechanical response was compared with the test result in Fig. 6. The predicted thermomechanical profiles are found similar in shape, compared with the measurement.

This TMF model was also applied to predict fatigue life of 63Sn-37Pb solder with 120-second ramp time according to Guo's testing condition (Guo, *et*

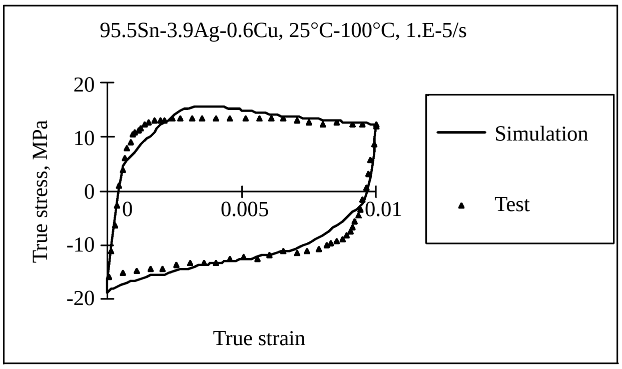


Fig. 6 In-phase TMF hysteresis loops for 95.5Sn-3.9Ag-0.6Cu solder under the temperature range from 25°C to 100°C at strain rate of 10^{-5}/s

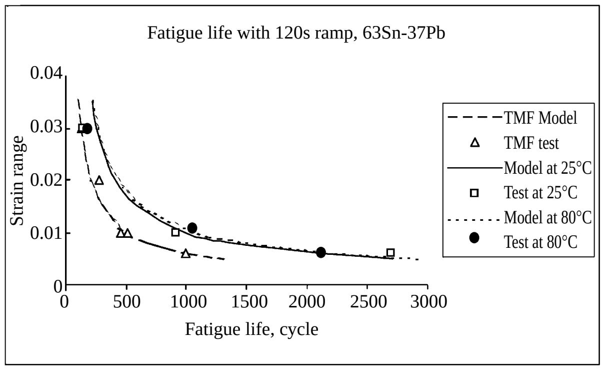


Fig. 7 Isothermal and TMF fatigue life for 63Sn-37Pb solder

al., 1992). The tests were performed for two isothermal fatigue loads (at 25°C and 80°C) and one in-phase TMF load (from 25°C to 80°C) The fatigue failure life is determined from the accelerated stress-range drop. The calculated load-drop curve was employed to predict fatigue life under the strain-controlled fatigue loading. The predicted curves of the fatigue life are shown to agree well with all three sets of testing data in Fig. 7.

IV. CONCLUSIONS

Two solder materials were chosen for the validation of a damage-coupled viscoplstic TMF material model. Several conclusions are listed as following:

1. A damage-coupled TMF model is established. Several variables, such as strain rate, temperature, temperature rate, microstructure change and damage accumulation, are taken into account.
2. The model can be used to calculate mechanical response of solder materials under both isothermal and TMF loads.
3. The model is capable of simulating load-drop curve under strain-controlled cyclic loading. Accordingly, isothermal and TMF fatigue life can be predicted.
4. More TMF data to determine state variables and material constants are required to establish accurate and robust damage evolution

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