CRACK GROWTH AT LOW ΔK'S AND THE FROST-DUGDALE LAW

R. Jones*, S. Barter, L. Molent, and S. Pitt

ABSTRACT

This paper examines the fatigue crack growth histories, at low ΔK's, of a range of test specimens and service loaded components and concludes that, as a first approximation, there is a linear relationship between the log of the crack length or depth and the service history (number of load cycles). We also show that, for the cases studied, that the log linear method can give a better prediction of experimental data than a conventional crack growth model.

Key Words: Frost-Dugdale, crack growth, low ΔK

I. INTRODUCTION

The growth of cracks with low ΔK's plays a central role when designing structural components, assessing through life performance, determining maintenance programs, and determining the effectiveness of life extension techniques. Paris *et al.* (1961) were the first to relate the stress intensity factor range, ΔK to the crack growth rate, da/dN. However, the work of Frost and Dugdale (1958), that predated Paris *et al.*, reported that crack growth under constant amplitude loading could be described via a simple log linear relationship. A comprehensive review of the current status of crack growth modelling is presented by Schijve (2003). It should be noted that the log-linear relationship is also consistent with the energy density criterion developed by Sih (1973-1981) for mixed mode fracture. In this approach the direction crack growth is assumed to coincide with the minimum of the $(dW/dV)_{\min}$. This corresponds to the location where volume change dominates. That is the location where energy dissipation via shape change is relatively small and where the change in the volume strain energy with respect to the change in the surface area (dW/dA) is maximum. In this theory failure by cracking is assumed to occur when $(dW/dV)_{\min}$ reaches a critical value $(dW/dV)_c$, $=S_c/r_c$, that is a characteristic of the material and where r_c is the critical ligament size ahead of the crack that triggers unstable fracture. Key features of this approach are that stable crack growth increments r_1, r_2, etc., are assumed to coincide with:

$$\left(\frac{dW}{dV}\right)_c = \frac{S_1}{r_1} = \frac{S_2}{r_2} = \cdots = \frac{S_j}{r_j} = \cdots = \frac{S_c}{r_c} \tag{1}$$

for Mode I 2D brittle crack growth we can write $dW/dV=S(\theta)/r=a_{11}K_I^2/r$, where a_{11} is a material constant, and S is the strain energy density factor. Integrating this law yields a log linear relationship.

The present paper examines a range of published aircraft fatigue test results as well as fatigue tests on cracked structures repaired with externally bonded composite repairs (patches) to reveal the applicability of the Frost and Dugdale law for crack growth at low ΔK's. In some cases the prediction based on Frost-Dugdale law and a conventional LEFM model (FASTRAN II Newman (1992) containing Elber's (1971) crack closure formulation (Elber, 1971) were used to predict published experimental data. It should be stressed that this work is in the development stage and that the formulation of a single law that transitions between the low to medium ΔK regime is still under consideration.

II. CRACK GROWTH AT LOW ΔK'S

As a structure ages we frequently encounter the

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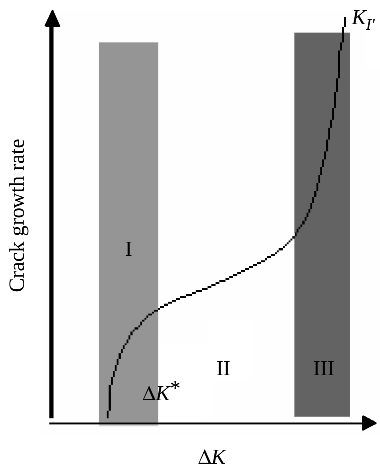

Fig. 1 A schematic diagram of the crack growth rate versus ΔK relationship

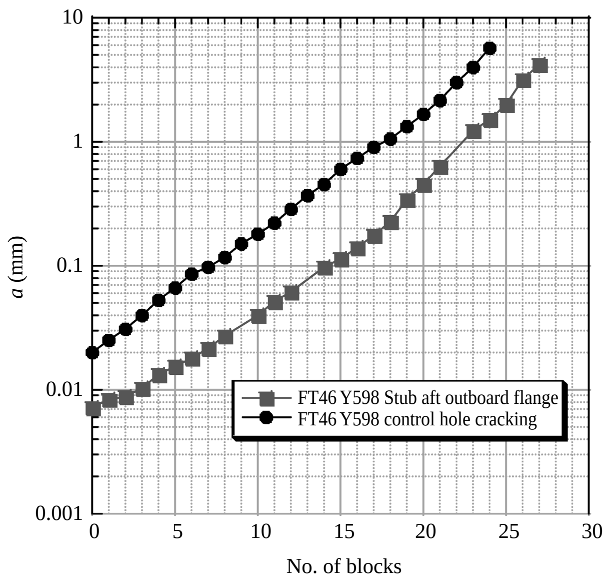

Fig. 2 Crack growth in F/A-18 FT46 full-scale fatigue test, from summary paper (Barter)

phenomenon of crack/damage growth. In general this process is commonly described as shown in Fig. 1 where there are three distinct regions. Region III is associated with rapid crack growth and as such this region is thought to account for a small fraction of the total life. Region II has received the greatest attention as it is in this region where the "Paris" crack growth law (Paris *et al*., 1961), is applied, viz:

$$da/dN=C\Delta K^m \quad (2)$$

where C and m are experimentally obtained constants, and where a range of associated variants (Schijve, 2003; Elber, 1971; Newman, 1992) have been developed. In contrast, Region I is associated with the growth of cracks with low ΔK's, and it is this region that usually accounts for a significant proportion of the fatigue life of a structure.

The Frost and Dugdale law, which predates the approach above, showed that crack growth could be described via a simple log linear relationship, viz:

$$ln(a)=\beta N+ln(a_0) \quad (3)$$

where N is the "fatigue life," β is a parameter that is geometry and load dependent, i.e. $\beta=f(\sigma)$, and a_0 is the initial flaw-like size. For constant amplitude loading Frost and Dugdale (1958) found that β could be expressed as:

$$\beta=\lambda(\Delta\sigma)^{\alpha} \quad (4)$$

where λ is problem dependent, and α=3. Researchers at the Australian Defence Science and Technology Organisation (DSTO) have subsequently presented a wealth of experimental data, that is summarised in the work of Barter *et al*. (2004), that supports this hypothesis when K is essentially a linear function of $\sqrt{a}$. Cracks considered included those grown from; semi- and quarter elliptical surface cuts, holes, pits and inherent material discontinuities in test specimens, full-scale aircraft fatigue tests, fuselage lap joints, welded butt joints, and complex tubular jointed specimens and include cracks grown under uniaxial and biaxial loading for both constant amplitude and complex in-service spectra.

The apparent exponential rate of crack growth at small crack lengths has also been mentioned in references (Clark *et al*., 1997; Wang, 1982; Zhang, 2000; Berens *et al*., 1991) The work of Clark *et al*. (1997) is particularly important in recognizing the usefulness of this relationship. As a result of its success in describing cracking in the RAAF Macchi fleet Clark *et al*. stated that this law "will be used more widely and more confidentially in any similar applications in future."

The following presents several examples to illustrate the Frost-Dugdale log-linear behavior for low ΔK's.

F/A-18 fatigue test: Fig. 2 presents two results from the DSTO F/A-18 simultaneous buffet and manoeuvre full-scale fatigue test, taken from the summary presented in (Barter *et al*., 2004). These cracks were grown in 7050-T7451 components with a complex manoeuvre plus dynamic buffet spectrum.

Cracking in F-16 fatigue tests: Fig. 3 presents the crack growth results from D6ac ultra-high strength steel centre hole specimens with a simple 'blocked' 400Hr F-16 spectrum, Speaker (1982). Here cracks from several different loads cases are shown. All specimens were fitted with fasteners and the cracks

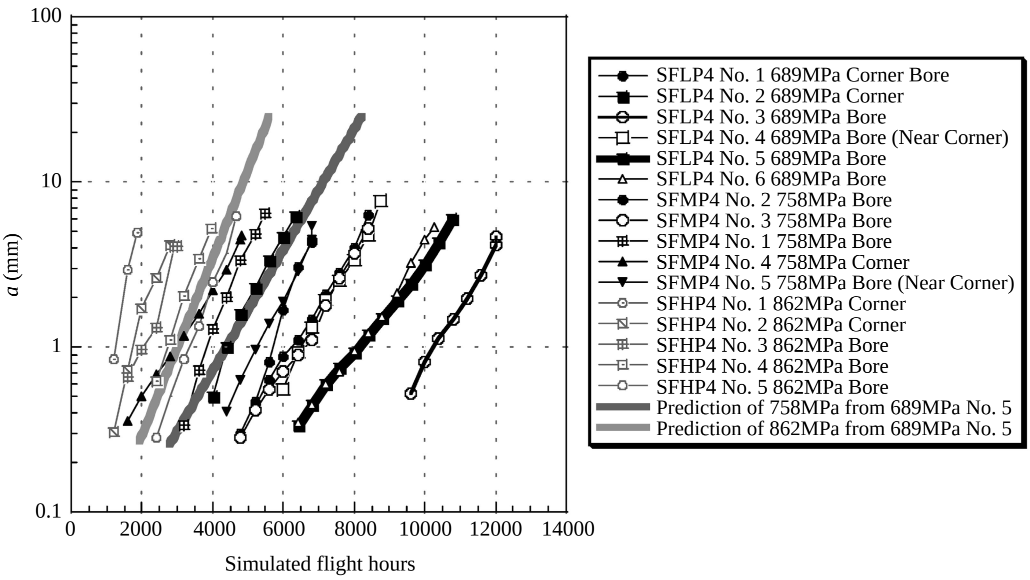

Fig. 3 Crack growth in F-16 coupon tests of D6ac steel (Speaker *et al*., 1982). One of the sets of 689MPa results was used to predict the other two stress levels using an a_0 of 0.025mm

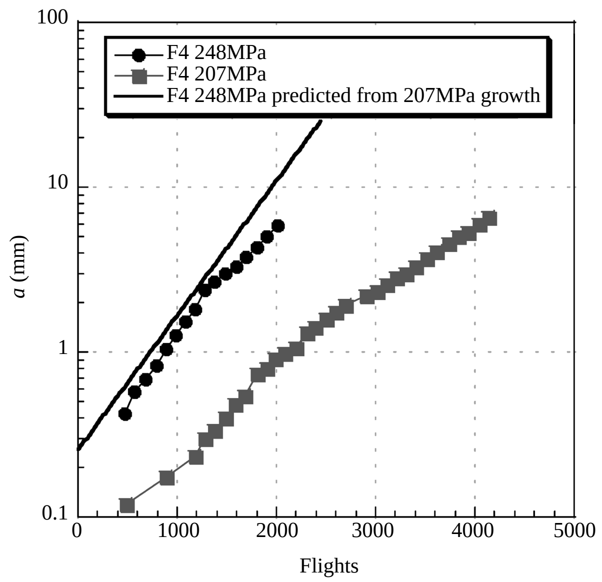

Fig. 4 Crack growth data for F4 wing. (USAF Damage Tolerance Design Handbook, 1998) and summary paper (Barter)

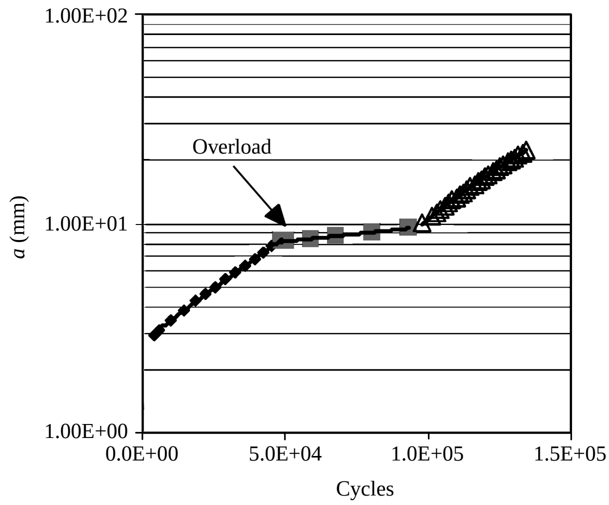

Fig. 5 Effect of overloads on a D16Cz aluminium alloy test specimen (Schijve *et al*., 2004)

started from the holes. To further illustrate the applicability of the Frost-Dugdale law the test results for specimen SFMP4 No. 5 was used together with Eqs. (3) and (4) and assuming an initial crack (a_o) size of 0.001inch (0.0254 mm) to predict the slope of another two stress levels for a similar type of crack. These predictions are also included in Fig. 3.

Cracking in F-4 fatigue tests: Fig. 4 presents crack growth data obtained in an F4 wing test program, presented in the USAF Damage Tolerance Design Handbook (1998), at both 207 and 248 MPa. Also presented is the predicted crack growth at 248 using the 207 MPa data together with Eqs. (3) and (4). Note that both crack growth curves are essentially log-linear.

1. Example of Crack Growth with Overloads

The Frost-Dugdale law implies that for tests with periodic overloads we should see a piecewise log-linear relationship with the change in the slope following an overload reflecting the retardation stresses induced in front of the crack. This is indeed illustrated in Fig. 5 which presents the results obtained by Schijve *et al*. (2004) for the aluminium alloy D16Cz

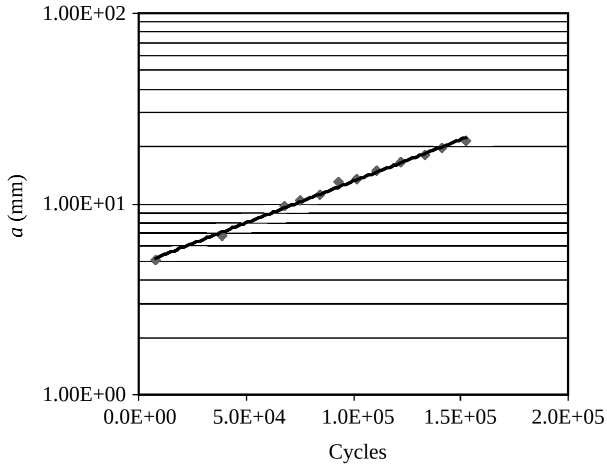

Fig. 6 Crack length verus number of cycles (Baker, 1998)

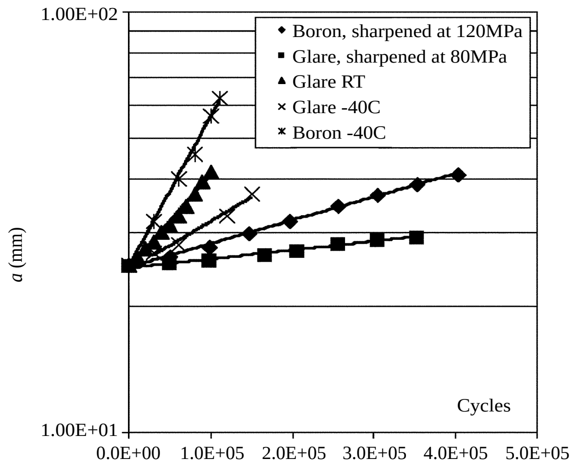

Fig. 7 Total crack length 2a verus number of cycles (Bater, 2004)

under constant amplitude testing with periodic overloads. These crack growth fatigue tests were carried out on centre-cracked tension specimens containing a starter notch to initiate a fatigue crack. The maximum cyclic fatigue stress was 64 MPa, R=0.0, with overloads at 128 MPa. The crack growth data are shown in Fig. 5, where we see a piecewise linear relationship. Here only the regime immediately prior to and following the first overload is shown.

III. CRACK GROWTH UNDER COMPOSITE REPAIRS

In the previous section we considered the low ΔK's behaviour to be associated with the growth of relatively small cracks. However, low stress intensity factors can also be obtained by bonding a composite patch over a cracked member. In this class of problems the crack lengths are frequently relatively large. Indeed, externally bonded composite patches have proven to be an effective method (see Baker and Jones, 1988, and Baker *et al*., 2002) of extending the fatigue life of cracked, or damaged, structural components. Jones *et al*. (2004) have recently shown that the Frost-Dugdale law also applies for this class of problems. To illustrate this we will consider several cases taken from (Jones *et al*., 2004).

Case 1: Baker (1998) presented the results for a 5 mm crack in 160 mm wide and 3.14 mm thick 2024-T3 aluminium alloy specimen patched with a seven ply (0.889mm thick) semi-circular uni-directional composite patch with a radius of 80 mm. The specimen was subjected to constant amplitude fatigue testing with σ_{max}=138 MPa and R=0.1, see Baker (1998) Figure 6.23. The resultant crack growth data, from Baker (1998), is shown in Fig. 6. This figure supports the hypothesis of a log-linear relationship. The initial flaw size used in this test was 5 mm, and the size predicted from the linear fit was 4.8 mm.

Case 2: In Chapter 14 Fredell et al (2002) compared the efficiency of Glare and Boron epoxy patches, and the results are shown in Fig. 7. In this case tests were performed on centre-cracked panels and only one side of the panel was patched. Tests were performed at room temperature and at –40°C, and patches were applied after growing cracks to a nominal 25 mm under stress amplitudes of either 80 or 120 MPa. In this study we again see that the log crack length versus cycles relationship was essentially linear and that in each case we predict a similar initial flaw size of approximately 24.5 mm as against a nominal initial crack length of 25 mm.

Case 3: Fatigue tests with R=0.1 and various σ_{max} levels were also reported in (Baker *et al*., 2002) for the seven ply boron epoxy repair as detailed in Case 1. Fig. 8 presents the crack growth data for the cases when there were no inbuilt disbonds tested at σ_{max}=160 and 80 MPa, together with the crack growth data predicted for the case σ_{max}=160 MPa using equations (3) and (4) and the experimental data for the case σ_{max}=80 MPa. From this Figure we see that the predicted and measured data are in good agreement and that the data are essentially log-linear.

IV. FROST AND DUGDALE VERSUS CONVENTIONAL LEFM

The previous examples have established the ability of the Frost-Dugdale law (1958) to represent crack growth, at low ΔK's at crack lengths ranging 0.1mm to 10's of mm's. It has been recognised that in some cases LEFM crack growth models tend to often underestimate the crack growth rates at low ΔK's.

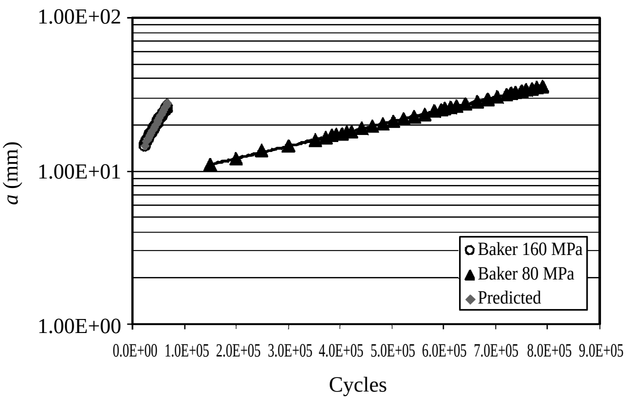

Fig. 8 Crack length a versus number of cycles for an edge cracked 2024 T3 aluminium plate (Baker *et al*., 2002)

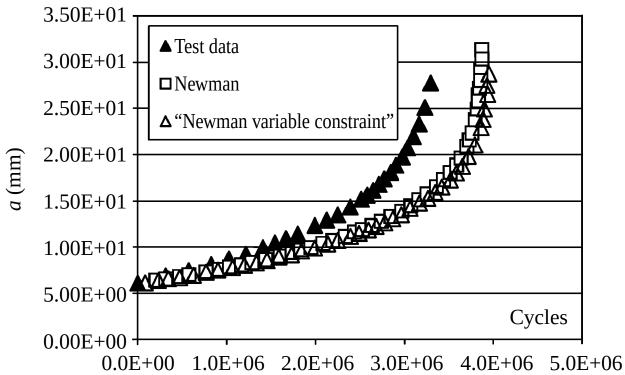

Fig. 9 Depth versus cycles growth data for a surface flaw in 7075 aluminium alloy (Newman *et al*., 1994)

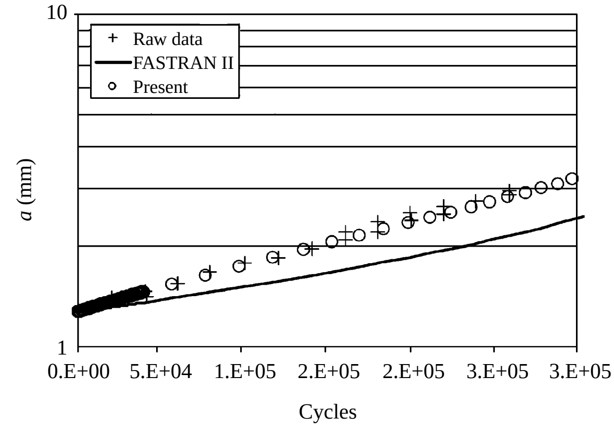

Fig. 10 Depth a versus cycles, for R=0.7 (McDonald and Daniewicz, 2001)

Indeed, in Section 5.3 the USAF Damage Tolerance Design Handbook states that: "the plasticity induced closure based predictions by FASTRAN do not correlate with the small crack growth data under periodic over loads .... The predictions of small crack life under spectrum loads were even farther off."

This is illustrated in Fig. 9, where we present the results of a NASA-China study into crack growth modelling, (Newman *et al*., 1994). Here two variants of FASTRAN were used to predict the experimental results. In the following section, several random examples are used to show that if FASTRAN was modified to produce log-linear growth, then better predictions can be obtained. The modification used was of the form:

$$da/dN=Ca^{(1-w/2)}\ (\Delta K)^{w} \tag{5}$$

that resembles the Paris growth law, and where setting w=3 yields the Frost-Dugdale law.

The Frost-Dugdale law reveals that, as commented on in the USAF Damage Tolerance Design Handbook, in the low ΔK regime the relationship between crack growth (da/dN) and ΔK is not unique, i.e. it also depends on crack length.

1. Simple Constant Amplitude Block Loading - Surface Flaw

McDonald and Daniewicz (2001) studied the growth of centrally located surface flaws in a rectangular block subject to a remote cyclic load applied to a rectangular aluminium specimen. The specimens were made from 7075-T651 aluminium alloy, with a yield strength of 524 MPa, and an ultimate tensile strength of 586 MPa. The specimens were 12.7 mm thick and had a total width of 25.4 mm. The fatigue tests had R=0.7 with the load spectrum containing two blocks, see (McDonald and Daniewicz, 2001) for more details. For this case the load spectra contained two blocks. The first block consisted of 20,000 cycles with an σ_{max}=152 MPa and σ_{min}=106.2 MPa. The second block was the marker block used to create visible bands that could be viewed on the crack face at the conclusion of the test. This block consisted of a load spectra cycling at an R ratio of 0.9 with σ_{max}=152 MPa and σ_{min}=137 MPa. The marker block contained 40,000 cycles, to ensure visibility of the bands.

The results of this test, for low ΔK's, are shown in Fig. 10, which include the predictions made using both FASTRAN II (Newman, 1992) and the log-linear modification for low ΔK's. In this study we used the Paris crack growth parameters of C=3.7313×10^{-10} and m=3.325 values given in (McDonald and Daniewicz, 2001). This example shows that, in this case, adopting the Frost-Dugdale law improves the agreement between the measured and predicted crack growth.

2. Constant Amplitude Complex Tubular 'XX' Joint

Let us next consider the fatigue performance of

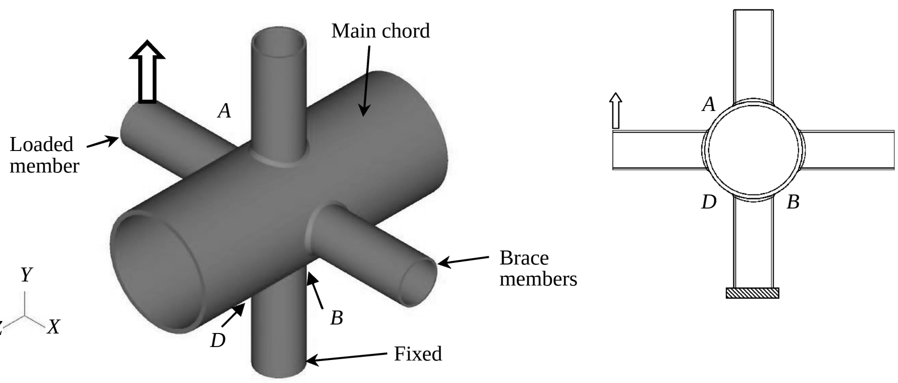

Fig. 11 Schematic representation of the tubular 'XX' joint geometry

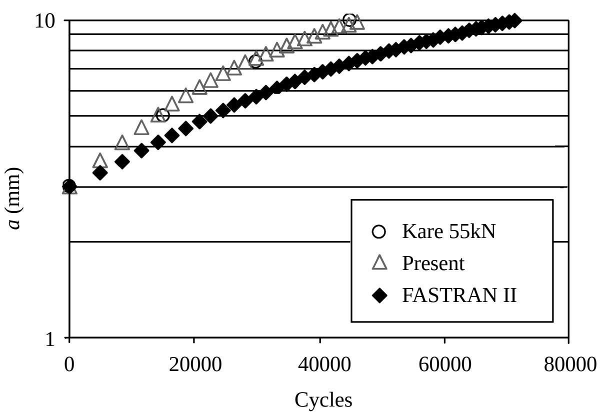

Fig. 12 Depth a versus cycles for the XX joint at point A (Kare, 1989)

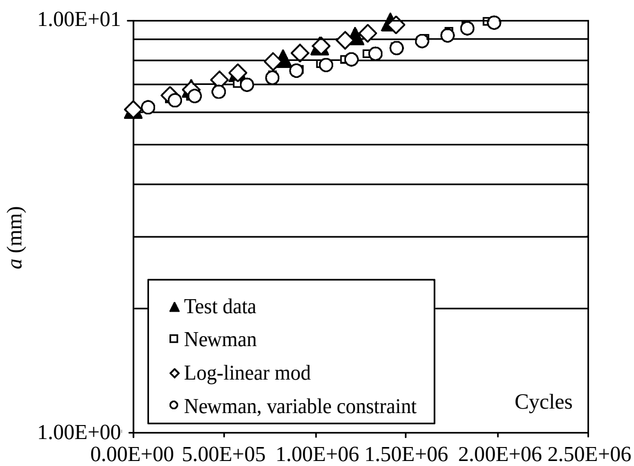

Fig. 13 Depth versus cycles growth data for a surface flaw in 7075 aluminium alloy (Newman *et al*., 1994)

a BS4360 50D seamless tubular steel double 'XX' tubular joint fixed by the end of one brace and subjected to an out of plane bending moment to a brace perpendicular to the fixed brace, see Fig. 11. The dimensions of the main chord length, thickness and diameter were 2000, 32, and 800 mm respectively whilst the brace length, thickness and diameter was 750, 24, and 324 mm respectively. A cyclic load of 55 kN, with an R ratio of –1, was applied to the structure at the end of one of the lacings producing and out of plane bending moment. Due to the constraints applied to the fixed lacing member, the stress field in the structure was non-symmetrical. The crack growth at point "A, see Fig. 11, is shown in Fig. 12. The crack growth parameters used by Kare (1989) were, viz: C=2.92×10^{-12} and m=3.0. Here we again see that the modified formulation provides an improved result.

3. Newman's Medium Crack Test

Newman *et al*. presented the crack growth results for a 7075-T6 aluminium alloy specimen containing a surface flaw, tested under spectrum loading. The crack growth predictions from (Newman *et al*., 1994) and those obtained using the modified formulae are presented in Fig. 13. The crack growth law used in (Newman *et al*., 1994) used tabular input for the da/dN versus ΔK relationship. This example again reveals that an improvement is obtained by modifying the closure model to reflect the Frost-Dugdale crack growth law (Frost and Dugdale, 1958).

V. CONCLUSION

This paper has shown that at low ΔK's crack growth is consistent with the Frost-Dugdale law, namely that a linear relation exists between the log of the crack size and the number of cycles applied. For the cases considered, modifying a conventional crack growth model to allow for this observation improved the agreement with the experimental data. It should be stressed that this work is in the development stage and that the formulation of a single law that transitions from such a modified growth law to the standard growth laws that are widely used for the medium ΔK regime is still under consideration. The

Frost-Dugdale law reveals that in the low ΔK regime the relationship between crack growth (da/dN) and ΔK is not unique but also depends on crack length. To this end a more extensive theoretical study of any other potential extension of the existing conventional growth laws is required.

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