CRACK GROWTH IN AN ORTHOTROPIC MEDIUM

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ABSTRACT

The elastostatic problem of an orthotropic body having a central inclined crack and subjected at infinity to a uniform biaxial load is considered. It is assumed that the crack line does not coincide with an axis of elastic symmetry of the body. The problem must be considered as one of general orthotropy, due in particular to the fact that the elastic coefficients of the material change with rotation of the reference system. The stress fields at the crack tip are reported and the presence of the non-singular terms underlined. The Strain Energy Density Theory is extended to orthotropic materials. It is assumed that the Critical Strain Energy Density Factor has a polar variation. The crack initiation is determined via minimization of the ratio of the strain energy density over the material critical strain energy density, pointing out the effects of orthotropy and load biaxiality. The effects of the non-singular terms on crack growth for different orthotropic materials is also studied, underling the relation between orthotropy and non-singular terms.

Key Words: orthotropy, fracture criterion, fracture loci, biaxial load.

I. INTRODUCTION

Composite materials, like fiber reinforced polymers and laminates, are often orthotropic and find many applications in technology. The complex mechanical behaviour of composite materials can be explained with sufficient accuracy by using the theory of homogeneous orthotropic materials. Due to the wide application of composites, there has been a great interest in understanding their fracture response and many theoretical results have been provided by applying elastic fracture mechanics analysis.

The elastostatic problem of an orthotropic body having a central inclined crack and subjected at infinity to a uniform biaxial load has been studied by Nobile *et al*. (2003). It is assumed that the crack line does not coincide with an axis of elastic symmetry of the body.

One of the main purposes of this paper is to predict the crack growth. The Strain Energy Density Theory (Sih, 1973, 1974, 1991) has been used for isotropic solids and applied by the Authors (Carloni and Nobile, 2002) to orthotropic solids with a crack aligned with an axis of elastic symmetry. It is assumed that the critical strain energy density function has a polar variation (Buczek and Herakovich, 1985; Saouma *et al*., 1987; Ayari and Zhiming, 1995; Carloni *et al*. 2003). In this work, the Strain Energy Density Theory is extended to the general case with the crack not aligned to one of the directions of elastic symmetry. The crack initiation is determined via minimization of the ratio of the strain energy density over the material critical strain energy density, pointing out the effects of orthotropy and load biaxiality on the near tip elastic fields and on the angle of incipient crack propagation.

The effect of the non-singular terms on crack growth is also studied. For isotropic materials, some Authors (Eftis *et al*., 1977; Liebowitz *et al*., 1978; Eftis and Subramonian, 1978) have noted that retaining the second term of the series expansion, can be extremely important to study the effect of biaxial load.

For orthotropic materials it will be seen that the effect of the non-singular terms depends on the value of the biaxial load parameter and on the mechanical behavior of the material. Making use of the Strain Energy Density Theory the Fracture Loci are derived and their dependence on the orthotropy is studied.

II. THE PROBLEM OF THE INCLINED CRACK

Consider a homogeneous, orthotropic and

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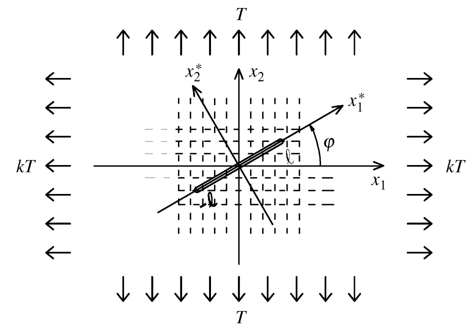

Fig. 1 The inclined crack geometry

infinite medium, having a central crack, of length 2ℓ, inclined at an angle, φ, with respect to the x_1-axis of the Cartesian co-ordinates system O (x_1, x_2) (Fig. 1). Suppose that the crack is not aligned with one of the orthogonal axes of elastic symmetry of the body, coincident with the co-ordinates system O (x_1, x_2). The crack line is defined by the direction of the x_1^*- axis, inclined at an angle, φ, respect to x_1.

Moreover, admit that the orthotropic body is subjected at infinity to a uniform biaxial load, applied along x_1 and x_2-directions.

The uniform load at infinity $\sigma_{11}^{*(\infty)} \equiv T_1$, $\sigma_{22}^{*(\infty)} \equiv T_2$, $\sigma_{12}^{*(\infty)} \equiv T_3$ referred to x_1^*, x_2^*, is a function of the crack inclination angle:

$$(\sigma_{11}^*)^{(\infty)} \equiv T_1 = \frac{T}{2}[(1+k)-(1-k)\cos 2\varphi] \quad (1)$$

$$(\sigma_{22}^*)^{(\infty)} \equiv T_2 = \frac{T}{2}[(1+k)-(1-k)\cos 2\varphi] \quad (2)$$

$$(\sigma_{12}^*)^{(\infty)} \equiv T_3 = \frac{T}{2}(1-k)\sin 2\varphi \quad (3)$$

k is the biaxial load parameter.

The fracture response of an orthotropic plate having a crack not aligned with one of the axes of elastic symmetry has been carried out by Tsukrov and Kachanov (2000), Prabhu and Lambros (2002) and Nobile et al. (2003), among others. In terms of polar coordinates (r, ϑ) centred at the crack tip, the stress field near the crack tip is expressed by (Nobile and Carloni, 2003; Carloni, 2004):

$$\sigma_{11}^* = \frac{K_I}{\sqrt{\pi\ell}}[\lambda\tilde{\sigma}_{11}^0 + h_{11}^1] + \frac{K_{II}}{\sqrt{\pi\ell}}h_{11}^2$$
$$+\frac{K_I}{\sqrt{2\pi r}}[\frac{1}{\sqrt{g_1(\vartheta)}}(\bar{a}_1\cos\frac{\vartheta_1}{2}-\bar{a}_2\sin\frac{\vartheta_1}{2})$$
$$-\frac{1}{\sqrt{g_2(\vartheta)}}(\bar{a}_3\cos\frac{\vartheta_2}{2}-\bar{a}_4\sin\frac{\vartheta_2}{2})]$$
$$+\frac{K_{II}}{\sqrt{2\pi r}}[\frac{1}{\sqrt{g_1(\vartheta)}}(\bar{\bar{a}}_1\cos\frac{\vartheta_1}{2}-\bar{\bar{a}}_2\sin\frac{\vartheta_1}{2})$$
$$-\frac{1}{\sqrt{g_2(\vartheta)}}(\bar{\bar{a}}_3\cos\frac{\vartheta_2}{2}-\bar{\bar{a}}_4\sin\frac{\vartheta_2}{2})] \quad (4)$$

$$\sigma_{22}^* = \frac{K_I}{\sqrt{\pi\ell}}[\lambda\tilde{\sigma}_{22}^0 + h_{22}^1] + \frac{K_{II}}{\sqrt{\pi\ell}}h_{22}^2$$
$$+\frac{K_I}{\sqrt{2\pi r}}[\frac{1}{\sqrt{g_1(\vartheta)}}(\bar{b}_1\cos\frac{\vartheta_1}{2}-\bar{b}_2\sin\frac{\vartheta_1}{2})$$
$$-\frac{1}{\sqrt{g_2(\vartheta)}}(\bar{b}_3\cos\frac{\vartheta_2}{2}-\bar{b}_4\sin\frac{\vartheta_2}{2})]$$
$$+\frac{K_{II}}{\sqrt{2\pi r}}[\frac{1}{\sqrt{g_1(\vartheta)}}(\bar{\bar{b}}_1\cos\frac{\vartheta_1}{2}-\bar{\bar{b}}_2\sin\frac{\vartheta_1}{2})$$
$$-\frac{1}{\sqrt{g_2(\vartheta)}}(\bar{\bar{b}}_3\cos\frac{\vartheta_2}{2}-\bar{\bar{b}}_4\sin\frac{\vartheta_2}{2})] \quad (5)$$

$$\sigma_{12}^* = \frac{K_I}{\sqrt{\pi\ell}}[\lambda\tilde{\sigma}_{12}^0 + h_{12}^1] + \frac{K_{II}}{\sqrt{\pi\ell}}h_{12}^2$$
$$+\frac{K_I}{\sqrt{2\pi r}}[\frac{1}{\sqrt{g_1(\vartheta)}}(\bar{c}_1\cos\frac{\vartheta_1}{2}-\bar{c}_2\sin\frac{\vartheta_1}{2})$$
$$-\frac{1}{\sqrt{g_2(\vartheta)}}(\bar{c}_3\cos\frac{\vartheta_2}{2}-\bar{c}_4\sin\frac{\vartheta_2}{2})]$$
$$+\frac{K_{II}}{\sqrt{2\pi r}}[\frac{1}{\sqrt{g_1(\vartheta)}}(\bar{\bar{c}}_1\cos\frac{\vartheta_1}{2}-\bar{\bar{c}}_2\sin\frac{\vartheta_1}{2})$$
$$-\frac{1}{\sqrt{g_2(\vartheta)}}(\bar{\bar{c}}_3\cos\frac{\vartheta_2}{2}-\bar{\bar{c}}_4\sin\frac{\vartheta_2}{2})] \quad (6)$$

where $\lambda = T_1/T_2$, and $K_I = T_2\sqrt{\pi\ell}$, $K_{II} = T_2\sqrt{\pi\ell}$ are the stress intensity factor for Mode-I and Mode-II respectively. The coefficients appearing in Eqs. (4)-(6) are reported in Nobile and Carloni (2003). Note that the stress intensity factors correspond to the ones of the isotropic case. This means that the expressions of the stress intensity factors do not depend on the inclination of the crack with respect to the directions of elastic symmetry of the material. The stress intensity factors depend only on the geometry. Note that also the non-singular terms are included. They affect all the stress components σ_{ij}^*, despite of the isotropic case.

III. STRAIN ENERGY DENSITY THEORY

The strain energy density criterion can be applied to predict the crack propagation in isotropic materials. In this work this criterion is extended to an orthotropic medium with an inclined crack not aligned with one of the axes of elastic symmetry of the body.

Making use of the stress and strain components σ_{ij} and ε_{ij}, the strain energy density can be written as

$$\frac{dW}{dV} = \int_0^{\varepsilon_{ij}} \sigma_{ij} d\varepsilon_{ij} \quad (7)$$

Supposing that we have a linear elastic behaviour, the strain energy density will be:

$$\frac{dW}{dV} = \frac{1}{2}\sigma_{ij}\varepsilon_{ij} \tag{8}$$

The strain energy density can be written as (Sih, 1974):

$$\frac{dW}{dV} = \frac{S}{r} \tag{9}$$

according to the case of isotropic materials. The strain energy density factor S is defined as:

$$S = a_{11}K_I^2 + 2a_{12}K_IK_{II} + a_{22}K_{II}^2 \tag{10}$$

where:

$$a_{11} = \frac{1}{2}[S_{11}^*\alpha_1^2 + S_{22}^*\beta_1^2 + S_{66}^*\gamma_1^2 + 2S_{12}^*\alpha_1\beta_1 + 2S_{16}^*\alpha_1\gamma_1 + 2S_{26}^*\beta_1\gamma_1] \tag{11}$$

$$a_{12} = \frac{1}{2}[S_{11}^*\alpha_1\alpha_2 + S_{22}^*\beta_1\beta_2 + S_{66}^*\gamma_1\gamma_2 + S_{12}^*(\alpha_1\beta_2 + \alpha_2\beta_1) + S_{26}^*(\beta_2\gamma_1 + \beta_1\gamma_2) + S_{16}^*(\alpha_2\gamma_1 + \alpha_1\gamma_2)] \tag{12}$$

$$a_{22} = \frac{1}{2}[S_{11}^*\alpha_2^2 + S_{22}^*\beta_2^2 + S_{66}^*\gamma_2^2 + 2S_{12}^*\alpha_2\beta_2 + 2S_{16}^*\alpha_2\gamma_2 + 2S_{26}^*\beta_2\gamma_2] \tag{13}$$

and

$$\alpha_1 = (\sqrt{\frac{2r}{\ell}}(\lambda\tilde{\sigma}_{11}^0 + h_{11}^1) + \overline{a}), \quad \alpha_2 = (\sqrt{\frac{2r}{\ell}}h_{11}^2 + \overline{\overline{a}}) \tag{14a,b}$$

$$\beta_1 = (\sqrt{\frac{2r}{\ell}}(\lambda\tilde{\sigma}_{22}^0 + h_{22}^1) + \overline{b}), \quad \beta_2 = (\sqrt{\frac{2r}{\ell}}h_{22}^2 + \overline{\overline{b}}) \tag{15a,b}$$

$$\gamma_1 = (\sqrt{\frac{2r}{\ell}}(\lambda\tilde{\sigma}_{12}^0 + h_{12}^1) + \overline{c}), \quad \gamma_2 = (\sqrt{\frac{2r}{\ell}}h_{12}^2 + \overline{\overline{c}}) \tag{16a,b}$$

$$\overline{a} = [\frac{1}{\sqrt{g_1(\vartheta)}}(\overline{a}_1\cos\frac{\vartheta_1}{2} - \overline{a}_2\sin\frac{\vartheta_1}{2}) - \frac{1}{\sqrt{g_2(\vartheta)}}(\overline{a}_3\cos\frac{\vartheta_2}{2} - \overline{a}_4\sin\frac{\vartheta_2}{2})] \tag{17a,b}$$

$$\overline{b} = [\frac{1}{\sqrt{g_1(\vartheta)}}(\overline{b}_1\cos\frac{\vartheta_1}{2} - \overline{b}_2\sin\frac{\vartheta_1}{2}) - \frac{1}{\sqrt{g_2(\vartheta)}}(\overline{b}_3\cos\frac{\vartheta_2}{2} - \overline{b}_4\sin\frac{\vartheta_2}{2})] \tag{18a,b}$$

$$\overline{c} = [\frac{1}{\sqrt{g_1(\vartheta)}}(\overline{c}_1\cos\frac{\vartheta_1}{2} - \overline{c}_2\sin\frac{\vartheta_1}{2}) - \frac{1}{\sqrt{g_2(\vartheta)}}(\overline{c}_3\cos\frac{\vartheta_2}{2} - \overline{c}_4\sin\frac{\vartheta_2}{2})] \tag{19a,b}$$

$$\overline{\overline{a}} = [\frac{1}{\sqrt{g_1(\vartheta)}}(\overline{\overline{a}}_1\cos\frac{\vartheta_1}{2} - \overline{\overline{a}}_2\sin\frac{\vartheta_1}{2}) - \frac{1}{\sqrt{g_2(\vartheta)}}(\overline{\overline{a}}_3\cos\frac{\vartheta_2}{2} - \overline{\overline{a}}_4\sin\frac{\vartheta_2}{2})] \tag{20a,b}$$

$$\overline{\overline{b}} = [\frac{1}{\sqrt{g_1(\vartheta)}}(\overline{\overline{b}}_1\cos\frac{\vartheta_1}{2} - \overline{\overline{b}}_2\sin\frac{\vartheta_1}{2}) - \frac{1}{\sqrt{g_2(\vartheta)}}(\overline{\overline{b}}_3\cos\frac{\vartheta_2}{2} - \overline{\overline{b}}_4\sin\frac{\vartheta_2}{2})] \tag{21a,b}$$

$$\overline{\overline{c}} = [\frac{1}{\sqrt{g_1(\vartheta)}}(\overline{\overline{c}}_1\cos\frac{\vartheta_1}{2} - \overline{\overline{c}}_2\sin\frac{\vartheta_1}{2}) - \frac{1}{\sqrt{g_2(\vartheta)}}(\overline{\overline{c}}_3\cos\frac{\vartheta_2}{2} - \overline{\overline{c}}_4\sin\frac{\vartheta_2}{2})] \tag{22a,b}$$

IV. CRACK GROWTH

For isotropic material the relative minimum of dW/dV is assumed to be associated with the direction of crack growth, that occurs when dW/dV reaches a critical value $(dW/dV)_c$ linked to the characteristics of the material.

For orthotropic material, finding the relative minimum of dW/dV is not enough to predict the crack extension angle. As the mechanical behaviour of the orthotropic material varies with direction, the critical value $(dW/dV)_c$ is not uniquely defined and it varies with the direction too.

The critical strain energy density $dW/dV)_c$ is assumed to have a polar variation (as well as K_{IC}^{ϑ} the critical stress intensity factor for Mode-I). A simple expression of the variation with the polar angle has been proposed (Carloni and Nobile, 2002):

$$(\frac{dW}{dV})_c^{\vartheta} = (\frac{dW}{dV})_c^{x_1}\cos^2(\vartheta + \varphi) + (\frac{dW}{dV})_c^{x_2}\sin^2(\vartheta + \varphi) \tag{23}$$

where $(\frac{dW}{dV})_c^{x_1}$ and $(\frac{dW}{dV})_c^{x_2}$ are the critical strain energy densities in x_1- and x_2- directions, respectively.

The crack initiation angle, ϑ_0, for orthotropic materials, can be obtained by minimizing the ratio:

$$R_W^* = \frac{(\frac{dW}{dV})^*}{(\frac{dW}{dV})_c^{\vartheta}} \tag{20}$$

Relation (20) can be normalized for numerical analysis as follows

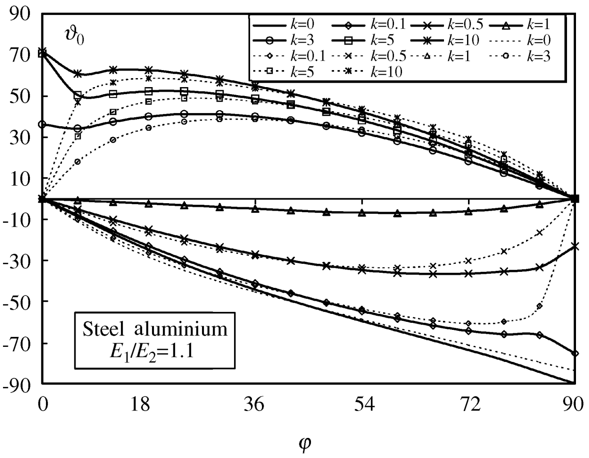

Fig. 2 Steel Aluminium: crack initiation angle ϑ_0 vs. crack inclination angle ω for different values of the biaxial load parameter k

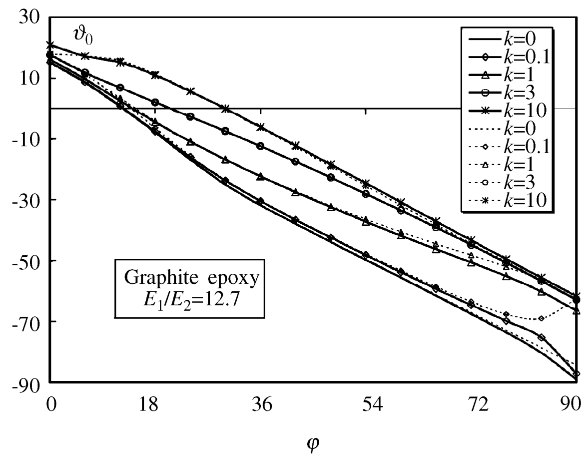

Fig. 4 Graphite Epoxy: crack initiation angle ϑ_0 vs. crack inclination angle ω for different values of the biaxial load parameter k

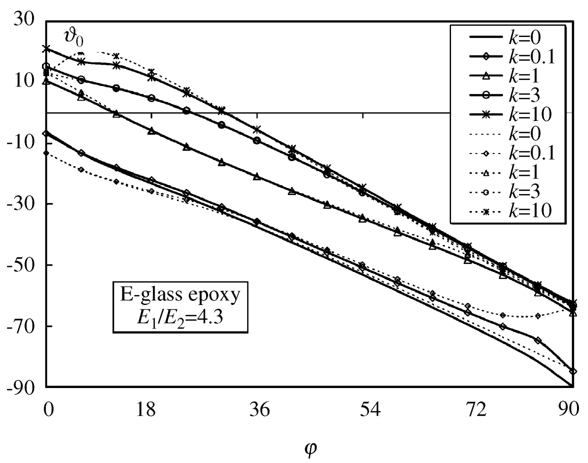

Fig. 3 E-Glass Epoxy: crack initiation angle ϑ_0 vs. crack inclination angle ω for different values of the biaxial load parameter k

$$\overline{R}_W^* = (\overline{\frac{dW}{dV}})^* \Big/ (\frac{dW}{dV})_c^*$$

$$= \frac{\dfrac{C_{66}}{T^2}\,\dfrac{2r}{\ell}(\dfrac{dW}{dV})^*}{[\dfrac{(dW/dV)_c^{x_1}}{(dW/dV)_c^{x_2}}\cos^2(\vartheta+\varphi)+\sin^2(\vartheta+\varphi)]} \quad (21)$$

Referring to the three different orthotropic materials (Arcisz and Sih, 1984), Steel Aluminium (Fig. 2), E-Glass Epoxy (Fig. 3), and Graphite Epoxy (Fig. 4), with a given $(dW/dV)_c^{x_1}/(dW/dV)_c^{x_2}$ ratio assumed to be equal to the square of the elastic moduli ratio along the axes of elastic symmetry, the crack initiation angle is represented as a function of the crack inclination angle, for different values of the biaxial load parameter k (continuous lines).

Figures 2-4 point out the effect of the non-singular terms on the crack initiation angle. The angle of incipient growth is also represented neglecting the non-singular terms inside the stress components (dot lines).

Note that the crack initiation angle depends on the elastic properties of the orthotropic material. In particular, when the elastic moduli ratio E_1/E_2 becomes close to one, the crack extension angles, predicted by the strain energy density theory, approximate the values obtained in the isotropic case. This means that the value of the crack extension angle depends not only on the biaxial load parameter but also on the orthotropic behaviour of the material. For a fixed value of the crack inclination angle, the crack initiation angle is an increasing function of k. Note also that for k>1 the crack initiation angle can be negative.

Making use of the Strain Energy Density Criterion, the effect of the non singular terms is evident even if the mechanical behavior of the orthotropic material is not close to the isotropic one, although for Steel Aluminium the importance of the non-singular terms is more relevant. Note that for Glass Epoxy and Graphite Epoxy, when ω=0°, the crack initiation angle is not zero even if the non-singular terms are neglected. The value of the crack initiation angle for ω=0° becomes independent of k.

The effect of the non-singular terms depends on the value of k. Referring to Steel Aluminium, when k>1, the effect of the non-singular terms increases when k increases. For k<1, the effect of the non-singular terms increases when k decreases. For Glass Epoxy and Graphite Epoxy this seems not to be true.

The crack initiation angle is also represented as a function of the biaxial load parameter for different

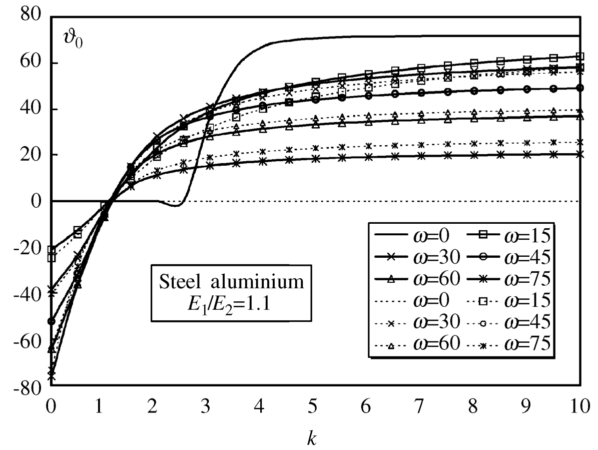

Fig. 5 Steel Aluminium: crack initiation angle ϑ_0 vs. biaxial load parameter k for different values of the crack inclination angle ω

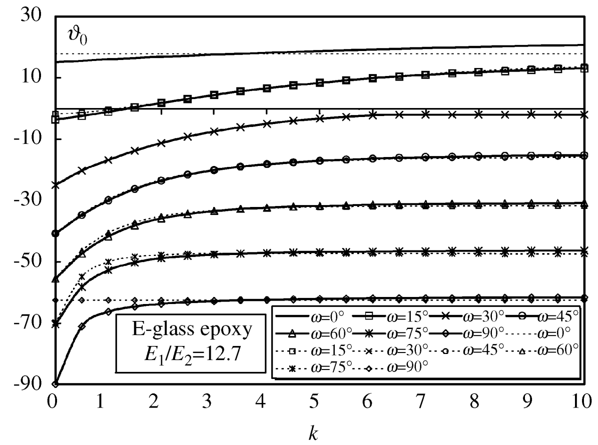

Fig. 7 Graphite Epoxy: crack initiation angle ϑ_0 vs. biaxial load parameter k for different values of the crack inclination angle ω

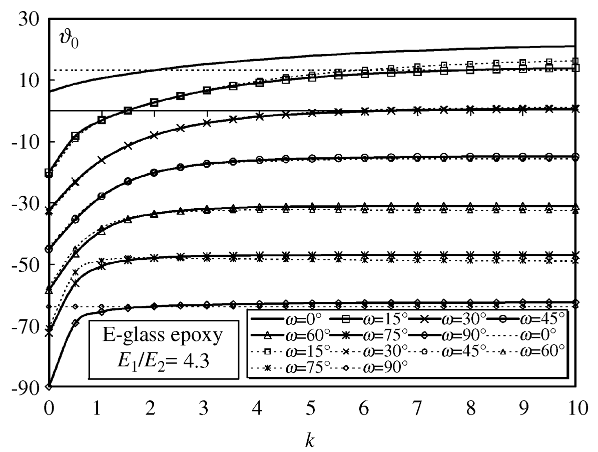

Fig. 6 E-Glass Epoxy: crack initiation angle ϑ_0 vs. biaxial load parameter k for different values of the crack inclination angle ω

values of φ, referring again to the three orthotropic materials (Figs. 5-7). For Steel Aluminium (Fig. 5) and ω=0°, the value of the biaxial load corresponding to $\vartheta_0 \neq 0°$ can be inferred. The effect of the non singular terms is also studied for the representation of the crack extension angle as a function of the biaxial load parameter.

Fixing the value of ω and k>1, the direction of crack extension seems to become asymptotic when k increases.

V. FRACTURE LOCI

The Strain Energy Density Theory for orthotropic materials states that crack extension angle ϑ_0 satisfies the following fracture conditions:

$$\frac{\partial}{\partial\theta}\left(\frac{dW}{dV}\Big/\left(\frac{dW}{dV}\right)_c^{\theta}\right)\Bigg|_{\theta=\theta_0} = 0 \tag{22}$$

$$\frac{dW}{dV}\Bigg|_{\theta=\theta_0} = \left(\frac{dW}{dV}\right)_c^{\vartheta} \tag{23}$$

Denoting by $K_{IC}^{x_1}$ and $K_{IC}^{x_2}$ the critical stress intensity factors for Mode-I along x_1- and x_2-directions, and introducing the dimensionless stress intensity factors:

$$K_I^* = \frac{K_I}{K_{IC}^{x_2}} \tag{24}$$

$$K_{II}^* = \frac{K_{II}}{K_{IC}^{x_2}} \tag{25}$$

from Eqs. (22) and (23) the parametric expression of the fracture loci, depending on λ, can be found. Remember that λ is the collinear stress parameter, defined by $\lambda = T_1/T_2$.

Figures 8-10 shows the fracture loci for the three different orthotropic materials for different values of λ. Once again, the influence of the orthotropic behaviour of the material can be underlined. Note that the value of K_{II}^* corresponding to K_I^*=0 seems not to depend on the material behavior.

It can be noted that when the elastic moduli ratio increases the effect of λ is further emphasised, underlining the fact that the orthotropic material behaves differently from the isotropic one. The pure Mode-I behaviour is affected by the presence of the collinear stress, and this is emphasized when E_1/E_2 is close to one. For example taking λ=5, looking to all the three materials, when K_{II}^*=0, K_I^* is different from zero, although it is close to one when E_1/E_2 become higher.

VI. CONCLUSIONS

The elastostatic problem of an orthotropic body

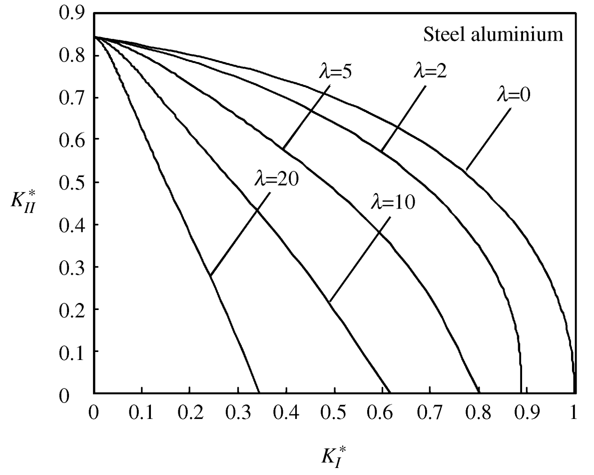

Fig. 8 Steel Aluminium: dependence of the Fracture Loci (K_I^*, K_{II}^*) on the parameter λ

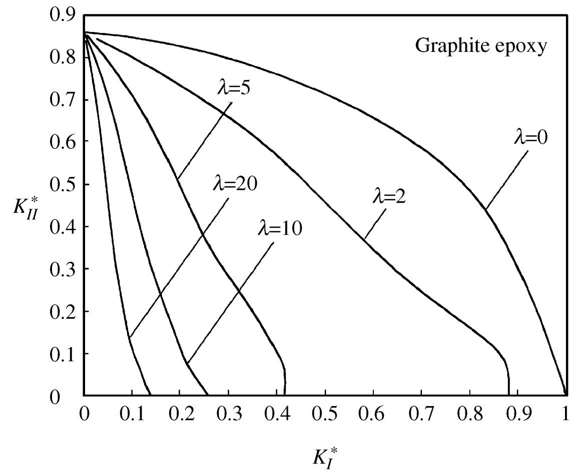

Fig. 10 Graphite Epoxy: dependence of the Fracture Loci (K_I^*, K_{II}^*) on the parameter λ

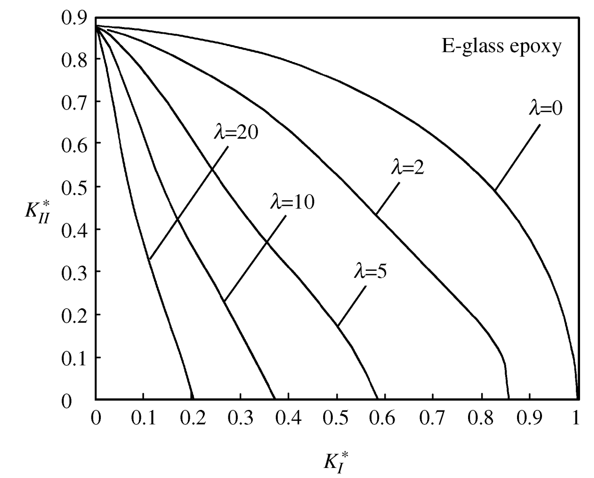

Fig. 9 E-Glass Epoxy: dependence of the Fracture Loci (K_I^*, K_{II}^*) on the parameter λ

having a central inclined crack and subjected at infinity to a uniform biaxial load has been reported. The crack line does not coincide with an axis of elastic symmetry of the body. The Strain Energy Density Theory has been extended to orthotropic materials in order to predict the crack initiation angle, defining a polar variation for the Critical Strain Energy Density. The crack initiation angle has been represented as a function of the crack inclination angle, for different values of the biaxial load parameter k. The dependence of crack initiation angle on the elastic properties of the orthotropic material and on the biaxial load parameter are underlined, noticing that when the elastic moduli ratio becomes close to one, the crack extension angles approximate the values of the isotropic case. The effect of the non-singular terms has been also studied.

Fracture Loci are reported for a wide range of orthotropic materials, underlining once again the different behavior from the isotropic case and the dependence on the collinear stress parameter.

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NOMENCLATURE

$(\frac{dW}{dV})_c^{\vartheta}$	polar variation of the critical strain energy density
$(\frac{dW}{dV})_c^{x_1}$, $(\frac{dW}{dV})_c^{x_2}$	critical strain energy density along the directions of elastic symmetry
E_1, E_2	Young's modulus along the directions of elastic symmetry (x_1, x_2)
k	biaxial load parameter
K_I, K_{II}	Stress intensity factor for Mode-I and Mode-II respectively
$K_{IC}^{x_1}$, $K_{IC}^{x_2}$	critical stress intensity factors for Mode-I along the directions of elastic symmetry
ℓ	half crack length
r, ϑ	polar coordinates with the origin at the crack tip
O (x_1, x_2)	Cartesian Coordinate system coincident with the directions of elastic symmetry
O (x_1^*, x_2^*)	Cartesian Coordinate system referred to the crack
ϑ_0	crack initiation angle
λ	collinear stress parameter
σ_{ij}^*	stress components referred to the crack system
φ	crack inclination angle respect to x_1 direction

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