CRACK PATH STABILITY IN CRACKED PANEL

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ABSTRACT

The prediction of the crack growth path plays a significant role in the estimation of the final shape of broken solids and structures. Furthermore, the study of the crack path in broken specimens renders the loading conditions just before fracture. Experiments on brittle materials, pre-cracked specimens of the same geometry, under similar loading conditions, occasionally result in different trajectories of the crack propagation. The already proposed theories for the prediction of the crack path (in) stability are based on the perturbation method in combination with analytical and finite elements methods; however, they require knowledge of the toughness equations. Therefore, they can only be applied in specimens with uncomplicated geometry and straightforward loadings. In the present paper the problem of the crack path (in) stability, is approached from a different viewpoint. Using a finite element program, the stres field is calculated, and consequently, a plotting program constructs the contours map of the strain energy density on the idealized geometry of the specimen or structure. For the determination of the predicted trajectory of the crack during unstable propagation, the minimum of the strain energy density criterion (SED) is used. The forecasted trajectory appears with the drawing of the "gorge" on the contours map of the strain energy density. Based on the estimation criterion, which claims that the degree of stability is a function of the distinctness of the gorge plot, we can predict the degree of the crack path stability. Therefore, this simple method offers good reliability in the prediction of the crack path stability for two as well as three-dimensional problems with complex geometry structures and arbitrary loadings. In order to clarify the suggested prediction method, we apply it on the central cracked panel where a rich international practical experience exists. The results that are analysed in the present work, are in good agreement with equivalent published experimental results.

Key Words: crack path stability, SED, central cracked panel.

I. INTRODUCTION

The problem of (in)stability has long been a pursuit for researchers, mainly in Mechanics, as well as other fields. The first efforts in the research of stability in mechanics were on the balance of a system and the movement of a material point. During recent years, the same problem has been emphasized in fracture mechanics. Stability in fracture mechanics has generally a different meaning from the one in classic mechanics, related to the concept of a disturbed system. While for classic mechanics, the fracture of the body is an instability phenomenon, in fracture mechanics the propagation of the crack is characterized as unstable, at times, but at other times, as stable. The search for methods for forecasting of the crack path and its type, finally became an issue of particular interest due to the increasing use of advanced materials like composites or coated materials for improvement in thermo mechanical attributes. With the increasing use of structural adhesives in construction and the aerospace and automotive industries, the need for an estimate of the locus of failure and the crack path propagation is essential to improve the durability of bonded joints. In addition, the prediction of the crack path can be beneficial in the design of safe structures and gives answers on the possible initial conditions of loading in the case of a destructive fracture.

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We will try to clarify the concepts of "stable" and "unstable" crack paths in the fracture of a solid or a structure.

Part of the path of the propagated crack can be characterized as 'stable', if, and only if, this part, resulting from repeated experiments onto bodies with the same geometry and under the same loading conditions, appears with identical geometrical characteristics. Specifically, when a stable crack path situation prevails at the spread of a fracture in a specimen and leads to the catholic destruction of the specimen, the corresponding broken pieces have the same shape.

The verification of a theory or a method for the approach of a problem is usually based on the experimental process. However, if the results are exceptionally sensitive to the initial and general conditions that exist during the experiment, then a scattering of the results is very likely. The evaluation of this scattering in phenomena of "instability" can lead to erroneous conclusions. In other words, in each experimental process we should compare the scattering of the results owing to endogenous factors of instability of the problem with the magnitude of the divergence of the conditions of the experiment, like the constrained displacements, or the geometry of the specimens. Observations on experimental results show that the propagation trajectory of an extended crack depends on the material properties, the geometry of the specimen, the rate of loading, the dynamic loading and the temperature. Furthermore, the control of the load's increase or the displacement's increase on the specimen by the loading machine also plays an important role. On the interface, the propagation trajectory also depends on the tensile strength and fracture toughness of the bonded materials. Approaching the multifactor problem of stability for a part of a crack path we could verify that all the above factors participate in the configuration of a situation that will determine the stability of the crack path, adding the history of the crack path and the magnitude of the applied forces or stresses.

The search for stability of the crack path is connected initially with the determination of the crack path propagation. We summarize the most significant among the criteria, which have been proposed to predict the initial direction of crack growth and have also being used for crack trajectory determination: the maximum tangential stress, the minimum strain energy density developed by Sih (1972-1982), the maximum energy release rate, the criterion which determinates the direction along the straight where the stress intensity factor K_{II}, vanishes for infinitesimally small crack extension. The above criteria have been used primarily for cracks in brittle homogeneous isotropic solids and yield similar results with small divergences to the observed experimentally results. The first two criteria, using the entire stress field, just before the crack begins to propagate unstably, can predict the crack path. All the above criteria can be used under the conventional step-by-step procedure to determine the crack path, when the crack propagates stably, a phenomenon that occurs in materials with elastic-plastic behaviour. We should notice that using the step-by-step method, the predicted trajectory of the propagated crack depends on the decision for the length of the incremental step. Consequently, this method can easily lead to erroneous results, as we will demonstrate below. For example in the double cantilever beam specimen, the choice of a very small growth length will always produce a straight line as the predicted crack propagation trajectory, while we well-know that curved trajectories also appear.

The interest of researchers on the determination of the crack propagation trajectory has been focused on a phenomenon already observed in experimental results: the phenomenon of the directional (in)stability and the path (in)stability of the crack propagation. Cotterel (1970) Approached this problem using the beam theory and applied it on the DCB test specimen. Cotterel and Rice (1980) showed that further information other than the stress intensity factors is necessary in order to predict the directional stability. Therefore, they showed that the T-stress plays an important role in the directional stability of the crack propagation. The T represents the uniform stress acting parallel to the axis of the crack at its tip, in the second term of William' s asymptotic stress expansion for a crack in a homogeneous material. Thus, for a small crack growth under predominate mode-I loading (K_{II}=0), the crack will deviate away from the original path and the resultant crack path will be directionally unstable if the T-stress is positive (tensile). If the T is negative (compressive) then the crack will propagate in the original direction, which is characterized as a directionally stable crack trajectory. By using higher order terms for William's expansion, the effect of the T-stress on the crack propagation manner was further investigated. Melin (1992) considered the directional stability of a static crack under biaxial remote loading σ_x (crack-parallel stress) and σ_y (crack-normal stress) and showed that the crack moved away from the original crack plane if $\sigma_x/\sigma_y>0.2146$. From the above, we conclude that the T-stress criterion for directional stability can't cover all the cases. Sumi *et al*. (1984) and Sumi (1985) developed a computer program for the numerical prediction of curved crack growth paths. The stress field is calculated by the method of superposition of analytical and finite elements solutions, and the curved crack path is obtained by the first order perturbation method. First sumi calculated the stress field ahead of the pre-existing crack tip by the finite element method and determined the parameters, which characterized the near tip field. A branched and curved predicted crack path is performed step by step by the use of the fracture criterion (iv). This approach, with virtual growth of the crack, and the use of the resultant stress field, introduces the global stress situation in the problem of the determination of the crack path and directional stability.

Several papers that studied the problem of crack path stability, do not give a distinct discrimination between the concepts that characterize the crack path: "curved and unstable". Furthermore, the majority of studies are dealing with cases where the toughness equations are known and are applied to test specimens with simple geometry shapes and loading conditions.

Interesting cases of (in)stability crack paths emerge in symmetrical specimens, under symmetrically imposed loading, where the mode I (K_{II}=0) is dominant on the preexisting crack but propagation is not always straight. Under those conditions, most researchers consider the line of the crack's axis as a stable trajectory and any deviation is taken as a sign of instability. According to the present study, there are cases, where under suitable conditions, the crack path is not a straight line and the propagation is stable despite the mode-I loading conditions.

In the present work, the problem of crackpath stability is approached from a different viewpoint. An extensive description of developed approaches exists in the paper Zacharopoulos (2004). Using a computer finite element program and carrying on with a plotting program we take the contours of strain energy density before the unstable crack propagates on the idealization of a solid plane. The grafical application of the minimum strain energy density criterion on the contours map results in the trajectory of the crack growth. Furthermore, the knowledge of the trajectory, in combination with an estimation criterion results in the degree of stability of the crack path for the fast propagation of an unstable fracture. Therefore, this simple method offers good reliability in the prediction of crackpath stability for problems with complex geometry structures and arbitrary loadings.

The above consideration offers the following new classification of the predicted crack path (in)stability for symmetric geometry and loading (K_{II}=0):

1. The path of the crack propagation is stable and has the shape of a straight line or of a curve (actually two symmetrical curves) outside its original direction.
2. The crack path is unstable, and follows many discrete curves, including the straight line.

In order to clarify the suggested prediction method, we apply it on a central cracked panel, where experimental experience exists. The results on the crack path stability presented in this work are in good agreement with experimental observations.

II. CRITERIA AND CRITICAL QUANTITIES IN STRAIN ENERGY DENSITY THEORY

Structural components failure may involve fracture and/or yielding. Loading type, structural geometry and material properties as a combination can significantly influence the failure mode. Failure its prediction is not always straight forward, when extra information is required in the failure initiation loading and location; in addition to the path of the fracture and eventual global instability. An initial crack may or may not be present.

The stored strain energy density, directly related to the potential energy in the infinitesimal element, is a global quantity. When used as a critical quantity it can control the fracture and/or yielding of a body. The Strain Energy Density theory developed by Sih (1972-1982) resolves such problems through a series of validated assumptions by making use of the energy stored in a unit volume of the solid that could simultaneously account for the proportion of distortion and dilatation.

1. Physical Meaning of Stationary Values of Energy Density

For a problem with known stresses σ_{ij} and strains ε_{ij} which are developed on a body, the strain energy per unit volume of a material can be computed from the expression

$$\frac{dW}{dV} = \int_0^{\varepsilon_{ij}} \sigma_{ij} d\varepsilon_{ij} = \frac{1}{2}\sigma_{ij}\varepsilon_{ij} \tag{1}$$

for material with linear elastic behaviour. If a point is located in the vicinity of the crack tip, then the above relation can be written $dW/dV=S/r$, where r is its distance from the crack tip. The separation of stored energy dW/dV into $(dW/dV)_d$ for distortion and $(dW/dV)_v$ for dilatation can be achieved only when the deformations are linearly elastic. These quantities will generally differ from element to element and it is not possible to know the location of yielding and/or fracture ahead of time. Assume the location where the dW/dV introduces a relative maximum, i.e. $(dW/dV)_{\max}$ the element distorted with small volume change, whereas when dW/dV attains a relative minimum, i.e. $(dW/dV)_{\min}$ then is dominated by dilatation. The proportion of energy consumed in failure by yielding and/or fracture is determined automatically by the stationary values of dW/dV. The failure usually initiates from sites of geometric defects that give rise to variation in the stored strain energy density.

Consider the local coordinate systems (x_1, y_1), (x_2, y_2), ⋯ (x_j, y_j), at each critical point P_1, P_2, ⋯ P_j ⋯, in the body and a fixed coordinate system XY. At the periphery of a small radius r_0, from each point P_j, local stationary values of strain energy density (dW/dV) may be found with respect to the angle θ_j. The radius of the core region r_0 is a scale length that separates the region of macroscopic homogeneity from those of microscopic inhomogeneity and depends on the material. The radius r_0 is the minimum value which can take the distance r, so that the geometrical representation of the crack tip and quantities of strain and stress have a physical meaning, when they depend on it. The maximum among all of the relative minima, which appear at all points P_j (j=1, 2, ⋯, n), is represented by $[(dW/dV)_{\min}^{\max}]_L$. Similarly, the maximum among all of the relative appeared maxima is denoted by $[(dW/dV)_{\max}^{\max}]_L$. The global stationary values of strain energy density which refer to coordinate system XY are denoted respectively by $[(dW/dV)_{\min}^{\max}]_G$ and $[(dW/dV)_{\max}^{\max}]_G$. Because the strain energy density is a nonnegative quantity and the relative minima chosen among those on the corresponding locations of the periphery with radius r_0, the tangential stress is tensile.

2. Basic Hypotheses

The SED theory to predict the local failure by fracture and/or yielding is based on the following hypotheses:

- Fracture initiates at the location where the $[(dW/dV)_{\min}^{\max}]_L$ appears when it reaches the critical value $(dW/dV)_c$.
- Yielding initiates at the location where $[(dW/dV)_{\max}^{\max}]_L$ appears, when it reach the critical value $(dW/dV)_d$.

and for expansion of the failure, on the hypotheses.

- In ductile material when once a crack is initiated by reaching $(dW/dV)_c$ and propagated stably and can be simulated by finite incremental steps, the amount of the crack growth lengths r_1, r_2, ⋯, r_c, is governed by the following relation:

$$\left(\frac{dW}{dV}\right)_c = \frac{S_1}{r_1} = \frac{S_2}{r_2} = \cdots = \frac{S_j}{r_j} = \cdots = \frac{S_c}{r_c}\text{, or } \frac{S_a}{r_a} \quad (3)$$

in which $S_1<S_2<\cdots<S_j<\cdots<S_c$, for $r_1<r_2<\cdots<r_j<\cdots<r_c$ correspond to increasing rate of crack growth leading to unstable fracture and $S_1>S_2>\cdots>S_j>\cdots>S_a$, for $r_1>r_2>\cdots>r_j>\cdots>r_a$ correspond to decreasing rate of crack growth resulting in crack arrest, depending on the rate of local energy release.

After the calculation of the first increase of the crack length, it is possible to compute the new stress field for the new crack shape. By reapplying the criterion, the new increase of the crack length can be also computed. Therefore, the application of this step-by-step method can determine the trajectory of the crack propagation. Under suitable loading conditions on the specimen, where unstable and fast fracture occur, the crack path can be obtained with a reasonable accuracy by applying the following hypothesis, based on the strain energy density criterion, using the stress field existing just before the onset of the crack propagation.

- For unstable and fast crack propagation, the fracture trajectory is assumed to be the path along which the maximum gradient of (dW/dV) prevails and is directed toward the location of $[(dW/dV)_{\min}^{\max}]_G$.

The critical value $(dW/dV)_c$ for fracture initiation is derived from the area under the uniaxial true stress- true strain curve of the material and is related to the ASTM of value fracture toughness K_{Ic} through the critical strain energy density factor S_c.

3. Local and Global Instability of Cracking

The traditional energetic approach to crack instability studies makes use of linear elastic fracture mechanics, where a global energy balance is employed. Fracture instability of a cracked elastic body is associated with an experimentally evaluated toughness quantity. No distinction is made between local initiation of failure and global termination of fracture.

The strain energy density theory can be used to evaluate the instability of a system from the stationary values of dW/dV and the index ℓ. The index of instability ℓ represents the length of the predicted fracture trajectory between points L and G. This trajectory is obtained from the curve which begins from point L and follows the maximum gradient of dW/dV. The hypothesis for cracking instability is "Zacharopoulos (1990)":

The quantity $S_{LS}=\ell\Delta(dW/dV)$ where $\Delta(dW/\Delta V)$= $[(dW/dV)_{\min}^{\max}]_L-[(dW/dV)_{\min}^{\max}]_G$. may be used as a measure of mechanical system instability.

The quantity S_{LG} involves a combination of material properties, loading types and structure geometry.

4. Stability or Instability of Crack Path

For any two-dimensional problem with an

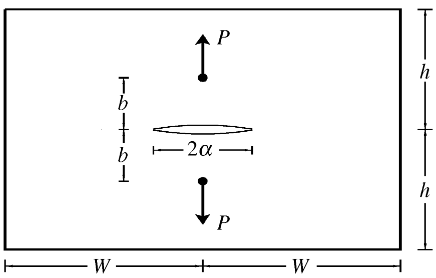

Fig. 1 Geometry and loading of the central cracked panel

arbitrary geometry of the body and arbitrary constraints, the displacement and loading can product the stress and strain fields, by the use of a computer finite elements program. Using that data we can produce the contours map of strain energy density by the use of a computer graphical program. This map has common features with a geographic map: The points where the failures will begin, according to the basic hypotheses, will be the hilltops of the topographic map respectively. They will be surrounded by closed contours that may have U shapes. The higher contours are always enclosed from lower ones. In the present work we will show how, with the elaboration of this map, we can estimate a stable or unstable crack path. When the critical locus on the plane body is the point O, which can be a crack tip, according to the above hypotheses, the beginning, the initial direction of crack growth as well as the crack path of propagation will occur.

According to the strain energy density criterion, the initiation of fracture from the O, takes place along the direction OL, $(OL)=r_c$, when the $[(dW/dV)^{\max}_{\min}]_L$ reaches the critical value $(dW/dV)_c$. The predicted crack path during the unstable propagation is the curve that starts from the point L, passes the points with the maximum gradients of (dW/dV) and ends up at point G, where the global minimum value of (dW/dV) develops.

On the map, the crack path is indicated by V shaped contours of strain energy density. If the apices of the V_s points are joined by a line, then the resulting plotting curve starts from the peak O and arrives in the vicinity of the point G. In the geographic map, the curve OG represents a gorge or a riverbed, which starts from the hilltop O. The drawing of this gorge can give additional information for the estimation of the stability of the crack path according to the following hypothesis:

- The stability of the crack path can be deduced from the degree of the sharpness with which the curve of the "gorge" is drawn.

III. RESULTS OF APPLICATION

The approach described above is applied to the test specimen, and the results are compared with the equivalent available published experimentals. Moreover, the classification map of the crack path in/stability is given. Consider the rectangular panel shown in Fig. 1, cracked in the centre, loaded with tensile force. The dimensions of the specimen which remain constant are length $2W$=30cm and thickness B=1 cm. The crack length and height h are considered variable. The point of the applied force remains constant at a distance of b=5cm from the crack axis. The values of the material properties are, the Young's modulus, E=3.0 GPa, the Poisson ratio ν=0.33, and fracture toughness K_{Ic}=1.38 MN/m$^{3/2}$.

The elastic stress field is computed by the application of a finite element program, which uses eight-node isoparametric elements. Only one quarter of the specimen needs to be solved because of symmetry on geometry and loading. Higher density of elements is undertaken around the region of the crack tip and the point where the force is applied. A computer program manipulates the results from the finite elements program and plots the contours map of the strain energy density and the other mechanical quantities on the idealized geometry of the specimen.

Specimens from three different characteristic cases are presented here, which according to the above consideration, have different behaviours regarding crack path stability. In Fig. 2 the contours of the strain energy density on the front area of the crack tip for the case a/W=0.2 and h/W=1.0 are presented. The predicted trajectory propagation of the crack is OG. The crack path is drawn with sharpness and is straight and therefore stable.

The contour map of the strain energy density for the case with ratios a/W=0.2 and h/W=0.47 is presented in Fig. 3. We observe that the trajectory (gorge) of the crack propagation cannot be drawn with sharpness because the contours are almost circular. This shows that the crack path is unstable and we expect the trajectory of the fracture to be a random curve.

The contour map of the strain energy density for the case with ratios a/W=0.2 and h/W=0.40 is presented in Fig. 4. We observe that on the contour map the "gorge" OG is clearly traced out, implying that the crack path is a stable curve.

Fig. 5 shows the classification map of the crack path (in)stability according to the above approach. The map is a diagram of the geometric characteristics of the central cracked rectangular panel: (h/w) against (a/w). The results proceded from the application of the developed estimation criterion for the crack path stability onto several cases of specimens, with different heights and crack lengths. The diagram is divided into three regions.

When the specimen is laid in the above region

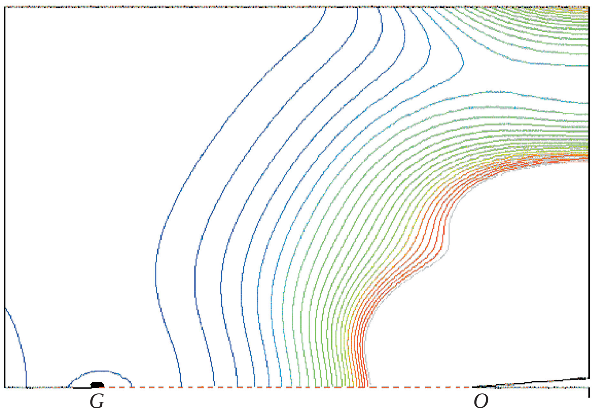

Fig. 2 Straight crack path for central cracked panel with a/W= 0.2, h/W=1.0

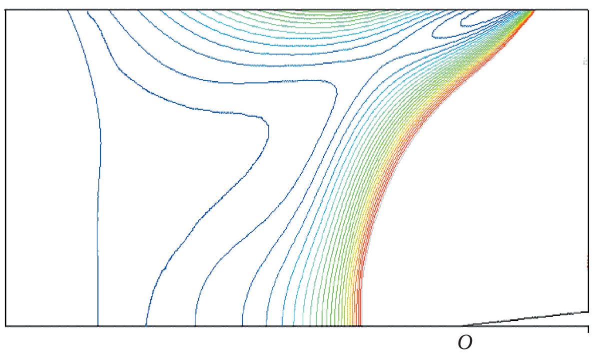

Fig. 3 Unstable crack path for central cracked panel with a/W= 0.2, h/W=0.47

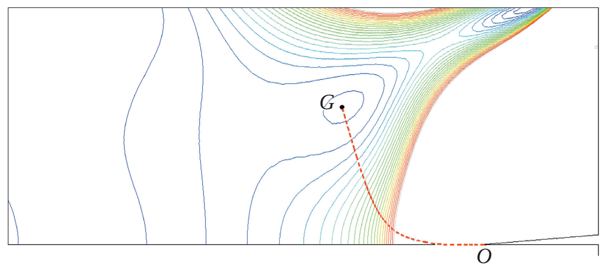

Fig. 4 Curved crack path for central cracked panel with a/W= 0.2, h/W=0.40

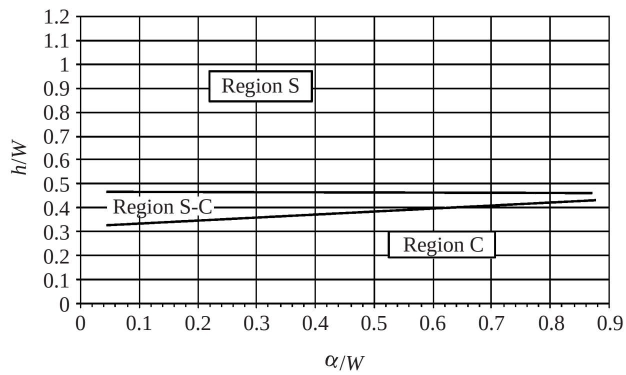

Fig. 5 The classification map of (in)stability for central cracked panel

"***S***" of the diagram (Fig. 5) the crack runs straight across its initial axis up to point G. After a temporary rest at point G it continues propagating in a straight line since it remains in the same region. When it moves towards the lower side of the diagram into the region "***S-C***" and the value of (h/w) is decreased, the crack path becomes unstable. This implies that the predicted trajectory could be straight or curved, but not unique. When the specimen is located in the lower region "***C***" of the diagram the crack path becomes a stable curve. We should point out that in those cases, symmetry favours two possible curves. In addition, the transition region where the crack path bounces from an unstable curve to a stable one, is defined from a inclined line in respect to horizontal. This means that, for example, on a specimen with ratio h/W=0.35, when the crack propagates in a straight line, then its position on the classification map will move in a horizontal line. In the next time moment it will pass from the region "***S-C***" to the region (C). Consequently, according to the description of the above map, (Fig. 5), the crack path becomes smoothly curved after some straight propagation

IV. CONCLUSIONS

In this paper, the application onto central cracked specimens of a theoretical approach to the problem of the prediction of type of propagation trajectories for pre-existing cracks is presented. A prerequisite for the validity of the theory is that the crack moves fast, unstably in parts or globally. Experimental verification for the validity of the results in this paper can be found in the book by Atkins-Mai (1985). The described experimental results in this book are in perfect agreement with those that occurred during this work.

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