Journal of the Chinese Institute of Engineers, Vol. 23, No. 3, pp. 289-297 (2000)

STOCHASTIC INVERSE BOUNDARY ELEMENT METHOD FOR DEFECT AND STRUCTURAL RELIABILITY PARAMETER PREDICTION

Yong Liu*, Gaoshun Jia, Jian Yang and L.H.Liang
Zhejiang University of Technology
Hangzhou, 310014, China

Key Words: boundary element method (BEM), structural reliability, stochastic inverse Analysis.

ABSTRACT

A stochastic inverse boundary element method is developed. The stochastic boundary integral equations are combined with the improved Kalman filtering algorithm to formulate the inverse iterative scheme and minimize the difference between the initial value and the objective value. Several numerial examples which show the effectiveness of this paper, including unknown defect and unknown parameter distribution of random boundary traction, are presented and discussed.

I. INTRODUCTION

Predicting structural reliability is a key problem for large scale and complex structures. An efficient method for predicting structural reliability parameters is attractive to many engineers. Inverse analysis is one of the more efficient and powerful numerical methods. Unlike the direct analysis method, it can be used to estimate and predict some unknown information, such as the inverse finite element method (Maniatty *et al.*, 1989; Maniatty and Zabaras, 1994) and the inverse boundary element method (IBEM) (Zabaras and Morellas, 1989). Up to now, the inverse analysis method has been widely used for the identification of structural parameters and unknown defects, structure design and structural optimization, etc. In engineering numerical analysis, the finite element method has efficiently solved many large-scale and complex structural problems. However, the inverse finite element method requires re-meshing in iterations for some problems, for instance, for predicting an interior defect, which will cost more CPU time. For the linear boundary element method, its elements are divided only on the boundary, therefore, no such interior re-meshing is required. So if the stochastic inverse boundary element method (SIBEM) is developed for structural reliability analysis, it might become an efficient and powerful method for engineering reliability prediction. This is the objective of the paper.

In recent years, many BEM researchers have focused on the IBEM and its application. Kobayashi (Nishimura and Kobayashi, 1991) and his co-worker have developed IBEM for inspection of flaws or defects in structural components. Saigal and his co-workers (Zeng and Saigal, 1992; Bezerra and Saigal, 1993) presented the boundary integral formulation for detection of flaws in planar structural members. Ulrich and Moslehy (1996) have presented a boundary element method based on the Hooke-Jeeves pattern search solution for the determination of internal cavities. Kobayashi and Nishimura (1994), Tosaka and Utani (1995) presented unknown defect identification in an elastic field by boundary element method with filtering procedure, which shows the efficiency of filtering. However, all of the above inverse BEMs

*Correspondence addressee

do not deal with stochastic problems.

On the other hand, the stochastic boundary element method (SBEM) is more and more being applied to structural reliability analysis. For example, Ettouney (1989) presented a stochastic boundary element with perturbation for stress analysis. Wen (Weidong, 1994) presented a 3D stochastic boundary element method for structural strength reliability. Liu (1992) presented a probabilistic boundary element method for structural system safety analysis. Liu and Yang (1997) developed a stochastic boundary element method for reliability analysis of general static and dynamic structural problems.

However, the stochastic inverse boundary element method (SIBEM) has not yet been completely investigated, especially for structural reliability prediction. Therefore, in this paper, the SIBEM is presented for structural reliability prediction based on the stochastic displacement estimation given at some boundary locations and applied random loading. With random loading, the investigation starts by estimating the stochastic distribution of random properties for assuming the objective value and any distributions as the initial values. It is known that such inverse problems formulated by a finite element require a lot of re-meshing work, which makes the iteration procedure computationally expensive and cumbersome. However, the boundary element method, which has the property of dimensional deduction, may effectively solve the problem. Thus, this paper will further investigate SBEM and combine it with the Kalman filtering algorithm to formulate the iterative procedure and eliminate the difference between the initial guess and the objective value. Three numerical examples, including unknown defects and unknown parameters of boundary traction (load), are presented to demonstrate the effectiveness of the reliability prediction method.

II. SBEM FOR RELIABILITY ANALYSIS

Assuming the fundamental stochastic variables X_i (i=1, 2, ..., n) are independent of each other, $x_i \in set\{X\}$. Let objective stochastic variables $Y_l(i=1, 2, ..., m) \in set\{Y\}$, $Y_l=y_l(x_1, x_2, ..., x_n)$, e.g., stress objective stochastic variable $\sigma_{ij}=\sigma_{ij}(x_1, x_2, ..., x_n)$, strain objective stochastic variable $\varepsilon_{ij}=\varepsilon_{ij}(x_1, x_2, ..., x_n)$. Y_l can be expanded into a Taylor series at a mean space point $(X_1^*, X_2^*, \cdots, X_n^*)$.

$$Y_l(X_1, X_2, \cdots, X_n) \cong Y_l(X_1^*, X_2^*, \cdots, X_n^*)$$
$$+\sum_{l=1}^{n}(X_i - X_i^*)(\frac{\partial Y_l}{\partial X_i})^* \quad (1)$$

In the above the second and higher oeder terms are considered to be small, and therefore are omitted. If Y_l and X_n are stochastic distributions, the mean value and the variance of Y_l can be written as:

$$\begin{cases} E[Y_l] = Y_l^* + \sum_{k=1}^{n}(\mu_{X_k} - X_k^*)(\frac{\partial Y_l}{\partial X_k})^* & \in mean\ space \\ Var[Y_l] = \sum_{k=1}^{n}(\frac{\partial Y_l}{\partial X_k})^{*2}\mathrm{var}[X_k] & \in variance\ space \end{cases} \quad (2)$$

where, μ_{X_k} is the mean value of X_k, var$[X_k]$ is the variance of X_k. If X_i^* is given, Y_1^* and $(\partial Y_1/\partial X_i)^*$ may be solved by SBEM, therefore the stochastic variable Y_l can be determined by Eq. (2).

The standard boundary integral equation can be written as

$$C_{ij}u_j + \int_\Gamma P_{ij}u_j d\Gamma = \int_\Gamma U_{ij}p_j d\Gamma \quad (3)$$

where P_{ij} and U_{ij} denote fundamental solutions of traction and displacement (Brebbia and Telles, 1984; Liu and Antes, 1999).

The above boundary integral equation can be solved by the usual boundary element method by using the prescribed boundary conditions. The final form of Eq. (3) can be expressed in the following discrete matrix form

$$\boldsymbol{A\delta=F} \quad (4)$$

where $\boldsymbol{A}$ is the known coefficient matrix, $\boldsymbol{F}$ is the prescribed vector, and $\boldsymbol{\delta}$ stands for the unknown traction and displacements at the boundary.

Due to the stochastic variable X_i, Eq. (4) can be written as

$$\boldsymbol{A} = \overline{\boldsymbol{A}}\big|_{X^*} + \Delta\boldsymbol{A}, \quad \Delta\boldsymbol{A} = \frac{\partial \boldsymbol{A}}{\partial X_i}\Big|_{X^*}\Delta X_i$$

$$\boldsymbol{\delta} = \overline{\boldsymbol{\delta}}\big|_{X^*} + \Delta\boldsymbol{\delta}, \quad \Delta\boldsymbol{\delta} = \frac{\partial \boldsymbol{\delta}}{\partial X_i}\Big|_{X^*}\Delta X_i$$

$$\boldsymbol{F} = \overline{\boldsymbol{F}}\big|_{X^*} + \Delta\boldsymbol{F}, \quad \Delta\boldsymbol{F} = \frac{\partial \boldsymbol{F}}{\partial X_i}\Big|_{X^*}\Delta X_i \quad (5)$$

where $\overline{\boldsymbol{A}}$, $\overline{\boldsymbol{\delta}}$ and $\overline{\boldsymbol{F}}$ are mean space matrix and vector, while $\Delta\boldsymbol{A}$, $\Delta\boldsymbol{\delta}$ and $\Delta\boldsymbol{F}$ are deviation space matrix and vector. Eq. (5) actually is the first order Taylor expansion. Inserting Eq. (5) into (4), and separating the variables yields:

$$\overline{\boldsymbol{A}}\cdot\overline{\boldsymbol{\delta}}=\overline{\boldsymbol{F}} \quad (6a)$$

$$\boldsymbol{A}\cdot\Delta\boldsymbol{\delta}=\Delta\boldsymbol{F}-\Delta\boldsymbol{A}\cdot\overline{\boldsymbol{\delta}} \quad (6b)$$

Equation (6a), solves $\overline{\boldsymbol{\delta}}$, i.e. the Y_l^* ; Eq. (6b) solves $\Delta\boldsymbol{\delta}$. Note the independent relation of Eq. (5): $\{\frac{\partial \delta}{\partial X_i}\}^* \Delta X_i = \Delta\delta_i$, from which the coefficient $\{\frac{\partial \delta}{\partial X_i}\}^*$ can be determined. For interior displacement and stress, their expressions are taken as:

$$\begin{cases} u_i^I = \int_\Gamma U_{ij} p_j d\Gamma - \int_\Gamma P_{ij} u_j d\Gamma \\ \sigma_{ij}^I = \int_\Gamma D_{ijm} p_m d\Gamma - \int_\Gamma S_{ijm} u_m d\Gamma \end{cases} \quad \forall \text{ interior points} \tag{7}$$

where D_{ijm}, S_{ijm} are fundamental Kelvin solutions (Brabbia and Telles, 1984; Liu and Antes, 1999).

Eq. [8] can be expanded into mean and deviation equations respectively.

$$\overline{\boldsymbol{u}}^I = \overline{\boldsymbol{H}}\,\overline{\boldsymbol{p}} - \overline{\boldsymbol{G}}\,\overline{\boldsymbol{u}}$$

$$\Delta \boldsymbol{u}^I = \overline{\boldsymbol{H}}\,\Delta\boldsymbol{p} + \Delta\boldsymbol{H}\,\overline{\boldsymbol{p}} - \overline{\boldsymbol{G}}\,\Delta\boldsymbol{u} - \Delta\boldsymbol{G}\,\overline{\boldsymbol{u}} \tag{8}$$

$$\overline{\boldsymbol{\sigma}}^I = \overline{\boldsymbol{D}}\,\overline{\boldsymbol{p}} - \overline{\boldsymbol{S}}\,\overline{\boldsymbol{u}}$$

$$\Delta \boldsymbol{\sigma}^I = \overline{\boldsymbol{D}}\,\Delta\boldsymbol{p} + \Delta\boldsymbol{D}\,\overline{\boldsymbol{p}} - \overline{\boldsymbol{S}}\,\Delta\boldsymbol{u} - \Delta\boldsymbol{S}\,\overline{\boldsymbol{u}} \tag{9}$$

where $\overline{\boldsymbol{u}}^I$, $\overline{\boldsymbol{\sigma}}^I$ are interior mean displacement and stress; $\Delta\boldsymbol{u}^I$, $\Delta\boldsymbol{\sigma}^I$ are interior deviations of displacement and stress; $\overline{\boldsymbol{H}}$, $\overline{\boldsymbol{G}}$ and $\overline{\boldsymbol{D}}$ are the mean fundamental Kelvin solution matrices $\Delta\boldsymbol{H}$; $\Delta\boldsymbol{G}$, and $\Delta\boldsymbol{D}$ are the deviation matrices of Kelvin solutions.

III. STRUCTURAL STRENGTH RELIABILITY

Considering failure criteria to be failures of static strength, the critical state function of structural strength can be expressed as

$$Z = g(X_i) = R(X_i) - S(X_i) \quad X_i \in (X_1, X_2, \ldots, X_n) \tag{10}$$

where S is the effective stress; R is the strength of material (i.e. the yield function); The structural reliability parameter (or index) β is defined as

$$\beta = E[g]/\sqrt{\mathrm{var}[g]} \tag{11}$$

Based on Eq. (11), the structural reliability can be determined by

$$R_R = (\text{or } P_R) = \Phi(\beta) \tag{12}$$

where Φ is a probabilistic function of stochastic distribution, from Eq. (12) it is known that the structural reliability parameter β plays a key role in reliability analysis, once it is determined, the structural reliability can be obtained. In this paper, a standard first-order second-moment method is used to evaluate β

$$\beta = \frac{E[R] - E[S]}{\sqrt{var[R] + var[S]}} \tag{13}$$

$E[R]$ and $var[R]$ can be obtained from a probabilistic material manual. $E[S]$ and $var[S]$ are determined from Eq. (2) and SBEM. (Note: The failure criterion function (10) is suitable for structures made of brittle materials, but not for ductile materials. Therefore, a design safety factor for Eq. (10) should be introduced so that it can be suitable for more materials in engineering analysis).

IV. KALMAN FILTERING ALGORITHM

1. Kalman Filtering (Tosaka and Utani, 1995)

The state equation is:

$$\boldsymbol{x}_{k+1} = \Phi_k \boldsymbol{x}_k + \omega_k \tag{14}$$

where $\boldsymbol{x}_k$ is the state vector (which actually is an initial guess for the original vector (e.g. signal, image, defect)), Φ_k is the state transition matrix of the system, ω_k is the zero-mean system noise vector.

The observation equation can be written as:

$$\boldsymbol{y}_k = H^* \boldsymbol{x}_k + \boldsymbol{v}_k \tag{15}$$

in which, $\boldsymbol{y}_k$ is the observed vector (which is determined by BEM in this paper), H^* is the observation matrix (degradation matrix), $\boldsymbol{v}_k$ is the observation noise vector. Both ω_k and $\boldsymbol{v}_k$ are assumed to possess the following stochastic characteristics:
Zero – mean noise

$$\begin{cases} E(\omega_k) = 0, \ E(\omega_k \omega^T{}_l) = Q_k \delta_{kl}, \ E(\omega_k \boldsymbol{x}^T{}_l) = 0 \\ E(v_k) = 0, \ E(v_k v^T{}_l) = R_k \delta_{kl}, \ E(v_k \boldsymbol{x}^T{}_l) = 0 \end{cases} \tag{16}$$

where Q_k is the covariance matrices of system noise, R_k is the observation noise covariance matrices.

The Kalman filter algorithm is developed based on the *Wiener Filter*, in which the filter gain G^* (restoration matrix) is taken as the form

$$G_k^* = \hat{P}_{k-1} H_k^{*T} (H_k^* \hat{P}_{k-1} H_k^{*T} + R_k)^{-1} \tag{17}$$

where $\hat{P}_{k-1}$ is the covariant matrix of estimation error.

The Kalman filter scheme can be constructed as:

$$\begin{cases} \hat{x}_k = \hat{x}_{k-1} + G_k^*(y_k - H_k^*\hat{x}_{k-1}) \\ \hat{x}_{k+1} = \Phi_k \hat{x}_k \end{cases} \tag{18}$$

The estimated error covariant matrices $\hat{P}_k$ and $\hat{P}_{k+1}$ are obtained from Eqs. (14) and (18):

$$\begin{cases} \hat{P}_k = \hat{P}_{k-1} - G_k^* H_k^* \hat{P}_{k-1} = (I - G_k^* H_k^*)\hat{P}_{k-1} \\ \hat{P}_{k+1} = \Phi_k \hat{P}_k \Phi_k^T + Q_k \end{cases} \tag{19}$$

For stationary static conditions, the state equation in this case can be expressed in the following form in which the unknown parameters to be identified should be kept constant in each time step:

$$x_{k+1} = I x_k + 0 \tag{20}$$

Hence, the state transition matrix Φ_k reduces to the unit matrix I, and the system noise ω_k may be ignored. Consequently, the suffix k in this situation does not indicate the time step, but the iteration number. Note in this case:

(1) $E(\omega_k \omega^T{}_l) = Q_k \delta_{kl} \equiv 0$ which leads to $Q_k = [0]$.

(2) $\hat{P}_{k+1} = I\hat{P}_k I^T + 0$, this means the second equation of Eq. (19) does not make sense, only the first is effective.

If the Kalman gain is taken as a *Project Filter*

$$G_k^* = [H_k^{*T} R_k^{-1} H_k^*]^{-1} H_k^{*T} R_k^{-1} \tag{21}$$

which does not depend on the covariant matrix P^*_{k-1}, it leads to fast convergence. This is the so-called improved Kalman filter.

The covariance matrix of observation noise R_k, which relates to the distribution of observation noise v_k and x_k, can be chosen by experience. Based on our calculation results, the value of R_k only affects the convergence speed without significantly affecting the accuracy of solution. For example, in this paper, R_k is taken to be

$$R_k = \begin{bmatrix} 0.25 & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & 0.25 \end{bmatrix} \tag{22}$$

2. Inverse SBEM with Kalman Filtering

It is known that in order to predict an unknown defect (or a unknown boundary traction) of a structure, it is necessary to measure some values at the boundary (e.g. displacement) so that we can construct the state equation. However, we do not use the experimental method to measure the displacements at some boundary points. We, instead utilize the boundary displacements calculated numerically from forward SBEM, and then use them to form the objective vector (a vector that will be used to compare the results from the inverse method).

Assume that the matrix function $\boldsymbol{m}$ which stands for using the forward SBEM to evaluate the boundary displacements at the measuring points, can be written as

$$\overline{\boldsymbol{u}}_m = \boldsymbol{m}(\overline{\boldsymbol{x}}_{exact})\,;\ \Delta\boldsymbol{u}_m = \Delta\boldsymbol{m}(\Delta\boldsymbol{x}_{exact}) \tag{23}$$

where $\overline{\boldsymbol{x}}_{exact}$ is the mean state vector of exact defect, $\Delta\boldsymbol{x}_{exact}$ is the deviation state vector of exact defect, $\overline{\boldsymbol{u}}_m$ is the mean displacement at some measure points, $\Delta\boldsymbol{u}_m$ is the deviation displacement at some measured points. Eq. (23) is the objective vector of boundary displacements at measured points.

The observation equation can be derived from boundary integral Eq. (6). Separating the unknown displacements and traction reveals the relation concerning the nodal displacement vector and the defect state vector $\overline{\boldsymbol{x}}_k$ and $\Delta\boldsymbol{x}_k$

$$\begin{cases} \overline{\boldsymbol{u}}_m = [\overline{A}_m]^{-1} \bullet \overline{\boldsymbol{F}}_m, \\ \Delta\boldsymbol{u}_m = [\overline{A}_m]^{-1} \bullet \Delta\boldsymbol{F}_m - [\Delta A_m]^{-1} \bullet \overline{\boldsymbol{u}}_m \end{cases} \tag{24}$$

which is obtained at a defect state vector x_k and Δx_k (i.e. a given estimated defect). Eq. (24) can be further written in the form

$$\overline{\boldsymbol{u}}_{m,k} \triangleq \boldsymbol{m}_k(\boldsymbol{x}_k, \Delta\boldsymbol{x}_k),\ \ \Delta\boldsymbol{u}_{m,k} \triangleq \Delta\boldsymbol{m}_k(\boldsymbol{x}_k, \Delta\boldsymbol{x}_k) \tag{25}$$

where $\boldsymbol{m}_k = [\overline{A}_m]^{-1} \bullet \overline{\boldsymbol{F}}_m$,

$$\Delta\boldsymbol{m}_k = [\overline{A}_m]^{-1} \bullet \Delta\boldsymbol{F}_m - [\Delta A_m]^{-1} \bullet \overline{\boldsymbol{u}}_m$$

The observation equation can then be expressed by a linear approach

$$\boldsymbol{y}_k = H_k^* \boldsymbol{x}_k + \overline{v}_k\ ,\ \ \Delta\boldsymbol{y}_k = H_k^* \Delta\boldsymbol{x}_k + \Delta v_k \tag{26}$$

where the observation matrix H_k^* is

$$H_k^* = [\frac{\partial \boldsymbol{m}_k}{\partial \boldsymbol{x}_k}, \frac{\partial \Delta\boldsymbol{m}_k}{\partial \boldsymbol{x}_k}, \frac{\partial \boldsymbol{m}_k}{\partial \Delta\boldsymbol{x}_k}, \frac{\partial \Delta\boldsymbol{m}_k}{\partial \Delta\boldsymbol{x}_k}]^T \tag{27}$$

It is very hard to find the exact solution of (27). Thus we use the finite difference method (FDM) to evaluate its components.

Based on the Kalman filter algorithm, the

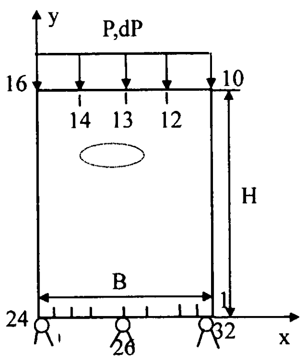

Fig. 1 The model of structure

scheme of Inverse SBEM is presented as follows:

STEP 1. Input the data concerning the stochastic geometry, elastic constants and boundary conditions.

STEP 2. Set up the initial condition for the first guess of the unknown defect and/or the unknown boundary traction, (i.e. the initial state variable)

$$\hat{x}_0 = \overline{x}_0, \ \Delta\hat{x}_0 = \Delta x_0 \tag{28}$$

STEP 3. Computing the stochastic displacements at measurement points using forward SBEM from an estimated state vector $\hat{x}_k$ and $\Delta\hat{x}_k$ (Eq. (18))

STEP 4. Set up the observation vectors y_k, Δy_k by Eq. (26)

STEP 5. Update the mean estimated state vector $\hat{x}_k$ and deviation estimated state vector $\Delta\hat{x}_k$ by

$$\begin{cases} \hat{x}_k = \hat{x}_{k-1} + G_k^*(y_k - H_k^*\hat{x}_{k-1}) \\ \Delta\hat{x}_k = \Delta\hat{x}_{k-1} + G_k^*(\Delta y_k - H_k^*\Delta\hat{x}_{k-1}) \end{cases}$$

STEP 6. Check the convergence:

$$\left\| \overline{u}_{m,k} - \overline{u}_m \right\| < \varepsilon, \ \left\| \Delta\overline{u}_{m,k} - \Delta\overline{u}_m \right\| < \Delta\varepsilon, \tag{29}$$

where ε and $\Delta\varepsilon$ are the given convergence tolerances. If Eq. (29) is not satisfied, go to step 3, repeat the iteration procedure. If Eq. (29) is satisfied, stop the iterations, output the mean and deviation estimated state vectors $\hat{x}_k$ and $\Delta\hat{x}_k$, recover the unknown stochastic distribution of estimated defect and boundary traction (load) using Eq. (2), and do the structural reliability parameter calculation and analysis.

V. NUMERICAL RESULTS AND DISCUSSION

Consider a 2D structure with an inner defect.

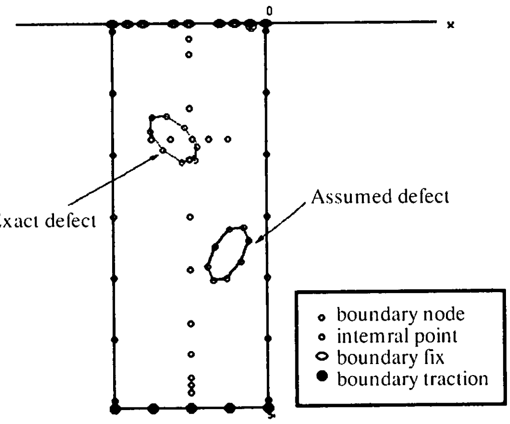

Fig. 2 The model with defect and mesh

Assume a plane stress problem where the defect can be expressed as $\frac{(x-x_0)^2}{a^2} + \frac{(y-y_0)^2}{b^2} = 1$. The predicted parameters include the location (x_0, y_0), the length of the long-axis (a), the length of the short-axis (b) and the rotation angle (alpha). The material constants are assumed to be: Young's modulus E=2.4×10^4 MPa and Poisson's ratio ν=0.3. Height H=5.0m, width B= 2.0m. The structure model is shown in Figure 1, which includes 24 linear elements on the boundary, 8 linear elements on the inner boundary of the unknown defect. The load is applied at the top side. As we mentioned before, we use the boundary displacements given by forward SBEM at some measured points to form the objective vector (a vector that will be used to compare the results from the inverse method). The measured points are chosen to be the points (10,12,13,14 and16) of the top boundary (see Fig.1). The convergence tolerance ε and $\Delta\varepsilon$ are taken as 1.e-6 and 1.e-8.

1. Structure with Unknown Ellipse Defect

The model with BEM grid is given in Fig. 2. The computing results are shown in Figs. 3-6. Fig. 3 and Fig. 4 show the convergence procedure of IBEM and SIBEM, respectively. Fig. 5 and Fig. 6 illustrate the convergence property with respect to the defect shape, respectively.

From the above results we can see that both IBEM and SIBEM are effective for determining the defects, however SIBEM can be used for the prediction of reliability parameters, whereas IBEM can't.

2. Reliability Parameter Prediction of Structure with Unknown Ellipse Defect and Random Loads

Using SIBEM, it is possible to predict the structural reliability parameter. Here, we assume the

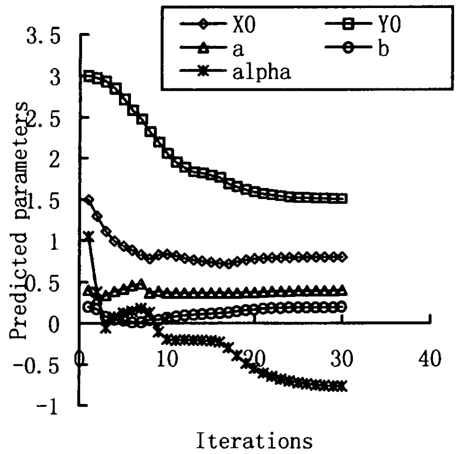

Fig. 3 Identified result by IBEM

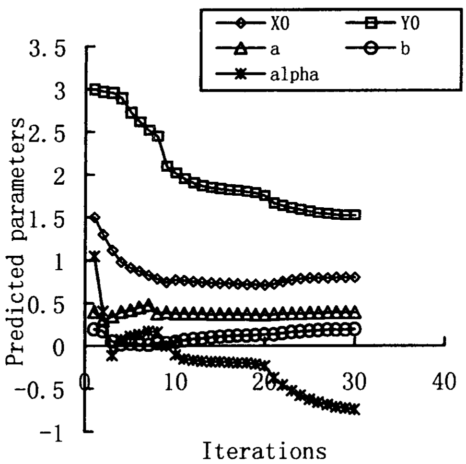

Fig. 4 Identified result by SIBEM

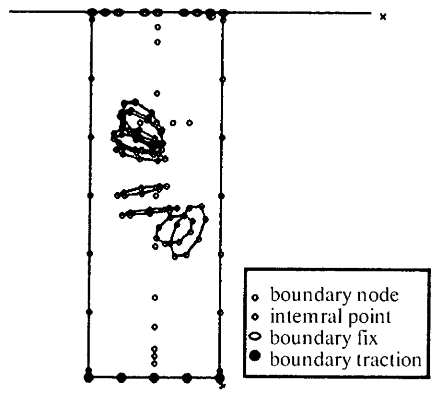

Fig. 5 Process of IBEM prediction

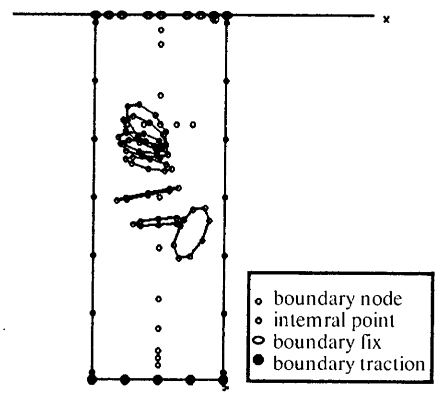

Fig. 6 Process of SIBEM prediction

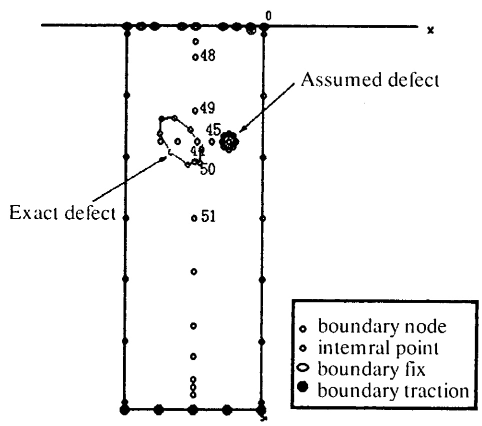

Fig. 7 The mesh model

model (shown in Fig. 7) with unknown ellipse defect and random loads. The parameters include the location (x_0, y_0), the length of the long- axis (a), the length of the short-axis (b), the rotating angle (alpha), and the random boundary loads (including Px, dPx, Py, dPy) which are applied at the top side of the model. There are 9 parameters to be predicted. Fig. 8 shows the convergence property with respect to the defect shape. Fig. 9 shows the identified results of the parameters. Fig. 10 presents the structural reliability parameter at some internal points (including No.44, 45, 48, 49, 50, 51) and their prediction procedure.

The objective vector of the location (x_0, y_0), the length of the long-axis (a), the length of the short-axis (b), the rotating angle (alpha), and the random boundary loads (including Px, dPx, Py, dPy) is (0.8, 1.5, 0.2, 0.4, $5\pi/4$, 0.0, 0.0, -0.2, -0.02). Assumed initial vector is (1.5, 1.5, 0.1, 0.1, 0.0, 0.5, 0.6, -0.1, -0.05). After 50 iterations steps, the prediction vector is (0.80011, 1.50003, 0.19998, 0.4, -2.35619, 0, 0, -0.2, -0.02) where -2.35619 is $5\pi/4$. The assumed objective structural reliability parameter β at some internal points (No. 44, 45, 48, 49, 50, 51) are (2.557345, 5.062733, 5.313857, 5.263923, --, 5.298350), respectively. The structural reliability parameter, β, on these points are (2.582521, 5.062647, 5.313863, 5.264038, -9.003421, 5.298374), respectively. Furthermore, according to Fig. 7 and Fig. 10, point No. 44 is near the boundary of the

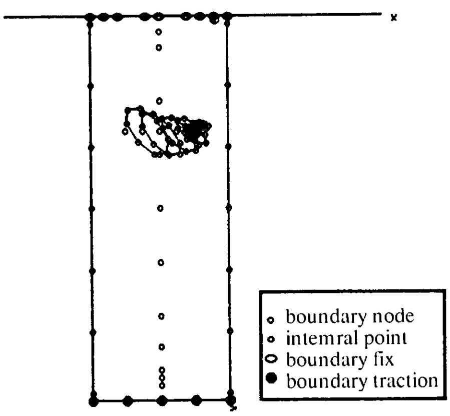

Fig. 8 Process of SIBEM prediction

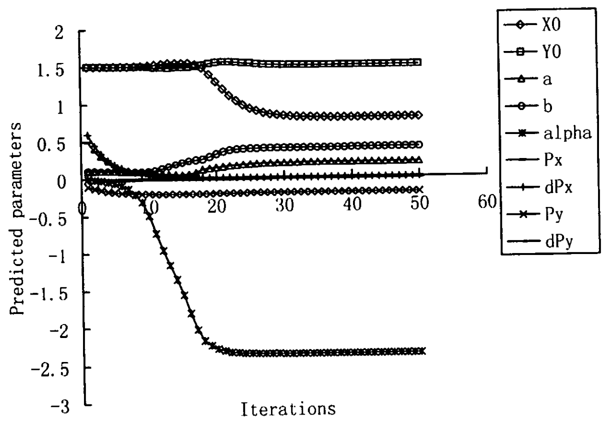

Fig. 9 Identified result by SIBEM

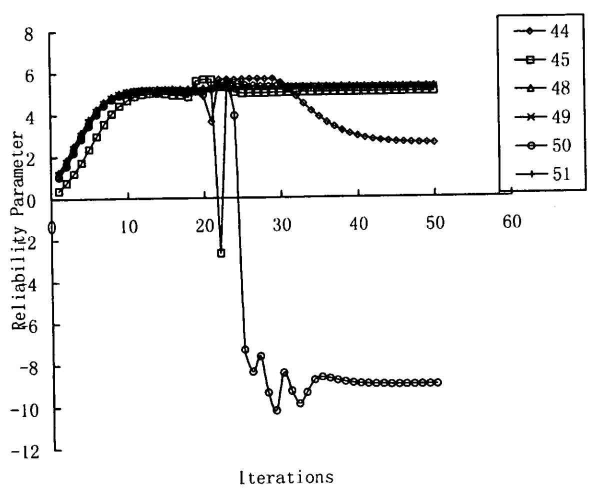

Fig. 10 Reliability parameter β at some points by SIBEM identification

defect. Its reliability parameter is lower than that of other points. Point No. 50 is located inside the defect. Since it is not located in the body of the structure component, its reliability parameter is negative

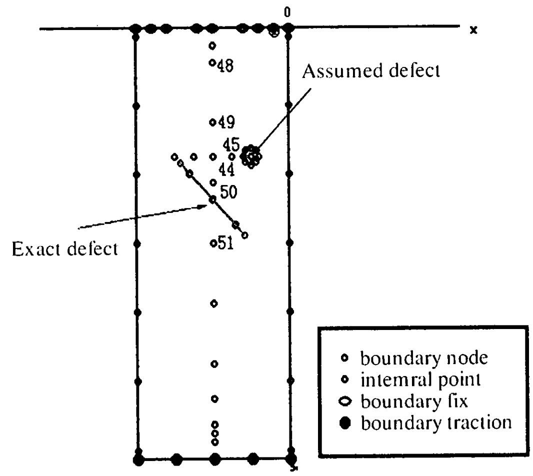

Fig. 11 The crack model with BEM mesh

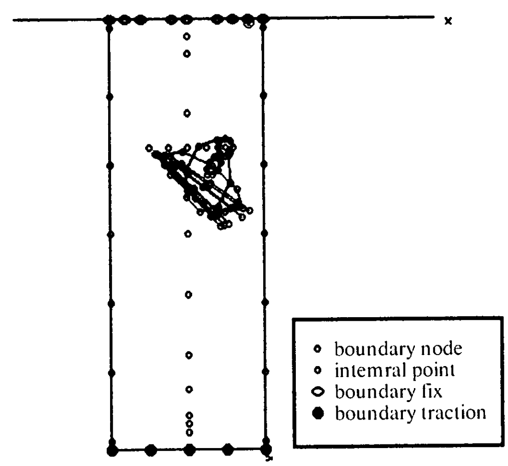

Fig. 12 Process of SIBEM prediction

(-9.003421), which does not make sense.

3. Reliability Parameter Prediction of Structure with Unknown Crack Defect and Random Loads

Here, we consider a crack, i.e. a super ellipse defect with long long-axis (a) and short short-axis (b). The model is shown in Fig. 11, where the ratio of a:b is 60. Fig. 12 shows the convergence property with respect to the defect shape. Fig. 13 shows the identified results of the parameters. Fig. 14 presents the structural reliability parameter β at some internal points (see last example) and their prediction process. The objective vector of the location (x_0, y_0), the length of the long-axis (a), the length of the short-axis (b), rotating angle (alpha), and the random boundary load (including Px, dPx, Py, dPy) is (1.0, 2.0, 0.01, 0.6, $3\pi/4$, 0.0, 0.0, -0.2, -0.02). Assumed initial vector is (1.5, 1.5, 0.1, 0.1, 0.0, 0.0, 0.0, -0.1, -0.05). After 50 iteration steps, the prediction vector is (1.00000,

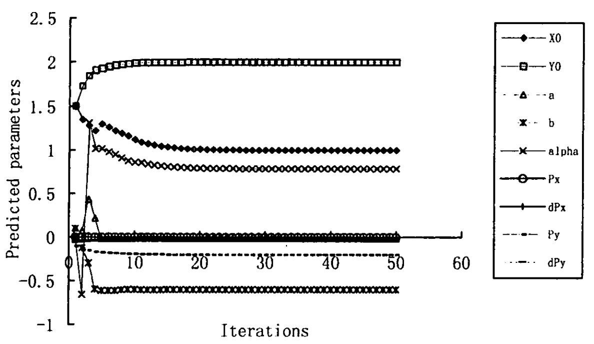

Fig. 13 Identified result by SIBEM

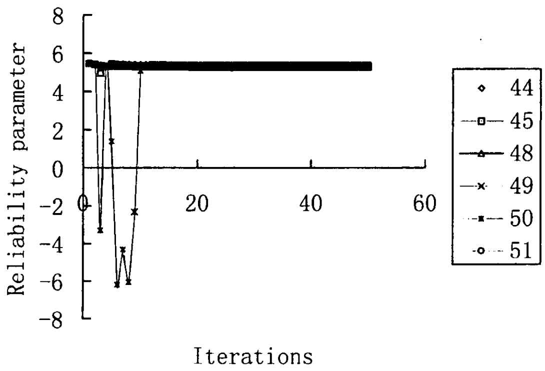

Fig. 14 Reliability parameter β at some points by SIBEM identification

2.00000, 0.01000, 0.60001, $3\pi/4$, 0.00000, 0.00000, -0.20000, -0.02000). The assumed objective structural reliability parameters β at some internal points (No. 44, 45, 48, 49, 50, 51) are (5.341835, 5.268374, 5.262212, 5.308814, 5.397956, 5.342162) respectively. The predication results of structural reliability parameter β on these points are (5.341827, 5.268373, 5.262214, 5.308814, 5.397943, 5.342154). Note: since point No.50 is located in the defect (not really body) of the structure component, its reliability parameter appears negative. This state makes no sense. However, the final result becomes good. In addition, it is found that for unknown flaw prediction, if the ratio of a/b is very large, the covariance matrix R_k should be adjusted. Otherwise, the convergence speed will be very slow.

VI. CONCLUSION

In this paper, a stochastic inverse boundary element method is developed by combining the stochastic BEM with the Kalman filtering algorithm. The contribution of this paper is that it presents a stochastic Kalman filtering scheme and uses it to predict the structural reliability parameters and the stochastic boundary traction (load), which is different from existing research. The method was applied to solve planar structural problems with both unknown defect and unknown random boundary traction. The unknown defect identification can be carried out simultaneously with the prediction of unknown random boundary traction and the structural reliability parameter. Numerical examples support the effectiveness of the method. As is known, the inverse problem for seeking a defect is not a well-posed problem, and the stability of solution is therefore usually not guaranteed. In addition, how the boundary conditions affect the accuracy of the defect determination when the defect is far small enough and/or is far away enough from measured points. This paper does not include these problems, which are further research the authors will investigate.

ACKNOWLEDGEMENTS

This research was supported by the Zhejiang Province Natural Science Foundation Special Funds (RC9601) and Chinese National Science Foundation Project (No. 19972062).

REFERENCES

1. Bezerra, L.M., and Saigal, S., 1993, "A boundary Element Formulation for the Inverse Elastostatics Problem (IESP) of Flaw Detection." *Int. J. Numer. Methods Eng*, Vol. 36, pp. 2189-2202.
2. Brebbia C.A., and Telles, J.C.F., 1984, Boundary Element Techniques: Theory and Application in Engineering. Berlin, Heidelberg, New York, Tokyo: Springer-Verlag.
3. Ettouney, M., 1989, "Boundary Element Methods in Probabilistic Structural analysis (PBEM)," *Appl. Math. Modeling*, Vol. 13, pp. 430-440.
4. Kobayashi, S., and Nishimura, N., 1994, "Determination of Cracks having Arbitrary Shapes with the Boundary Integral Equation Method," *Engineering Analysis with Boundary Elements*, Vol. 15, pp. 189-195.
5. Liu, X., 1992, "Probabilistic Boundary Element Method.Reliability," *Engineering and System Safety*. Vol. 37, pp. 123-127.
6. Liu, Y., and Yang, J., 1997, "Research on Stochastic Boundary Element Method for Strength Reliability of Mechanical Structure," *J. of Mechanical Strength*, Vol. 19 (in Chinese), pp.13-22.
7. Liu, Y., and Antes, H., 1999, "An improved Implicit Algorithm for the Elastic Viscoplastic Boundary Element Method," *ZAMM, Z. Angew. Math. Mech.*, Vol. 79, pp. 317-334.
8. Maniatty, A, Zabaras, N., and Stelson. K., 1989,

"Finite Element Analysis of Some Inverse Elasticity Problems." *J. Eng. Mech.*,Vol. 115, pp. 1303-1317.

9. Maniatty, A.M., and Zabaras, N., 1994, "Investigation of Regularization Parameters and Error Estimating in Inverse Elasticity Problems." *International Journal for Numerical Methods in Engineering*, Vol. 37, pp. 1039-1052.
10. Nishimura, N., and Kobayashi, S., 1991, "A Boundary Integral Equation Method for an Inverse Problem Related to Crack Detection." *Int. J. Numer. Methods Eng.* Vol. 32, pp. 1371-1387.
11. Tosaka, N, and Utani, A., 1995, "Unknown Defect Identification in Elastic Field by Boundary Element Method with Filtering Procedure." *Engineering Analysis with Boundary Elements*, Vol. 15, pp. 207-215.
12. Ulrich, T.W., and Moslehy, F.A., 1996, "BEM Based Pattern Search Solution for a Class of Inverse Elastostatic Problems." *International Journal of Solids and Structures*, Vol. 33, pp. 2123-2131.
13. Weidong, W., 1994, "Stochastic Boundary Element Methods Applied to 3D Elastic Problems and Reliability of Structural Strength." *Communication in Numerical Methods in Engineering*. Vol. 10, pp. 583-589.
14. Zabaras, N, and Morellas, V., 1989, "Spatially Regularized Solution of Inverse Elasticity Problems using BEM," *Comm. Appl. Numer. Meth.*, Vol. 5, pp. 547-553.
15. Zeng, X., and Saigal, S., 1992, "Inverse Formulation with Boundary Elements," *J. of Applied Mechanics, Trans .ASME*, Vol. 59, pp. 835-840.

Discussions of this paper may appear in the discussion section of a future issue. All discussions should be submitted to the Editor-in-Chief.

Manuscript Received: Nov. 18, 1999
Revision Received: Jan. 05, 2000
and Accepted: Feb. 15, 2000

隨機逆邊界元素法於瑕疵及結構可靠度之參數預測

劉 勇 賈高順 楊 健 梁利華

中國杭州浙江工業大學

摘 要

本文發展隨機逆邊界元素法，隨機邊界積分方程式與改良式卡門濾波法同時被採用來推導反算疊代規劃。因此可縮短初始猜值與最佳值的差距。包含未知瑕疵及未知參數分布的隨機邊界曳引力的數值算例均驗證本文之有效性。

關鍵詞：邊界元素法，結構可靠度，隨機逆分析。