第廿六屆力學會議學生論文競賽

A new meshless method for free vibration analysis of plates using radial basis function

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Outlines

1. Introduction
2. Methods of solution
3. Illustrated examples
4. Discussion
5. Conclusions

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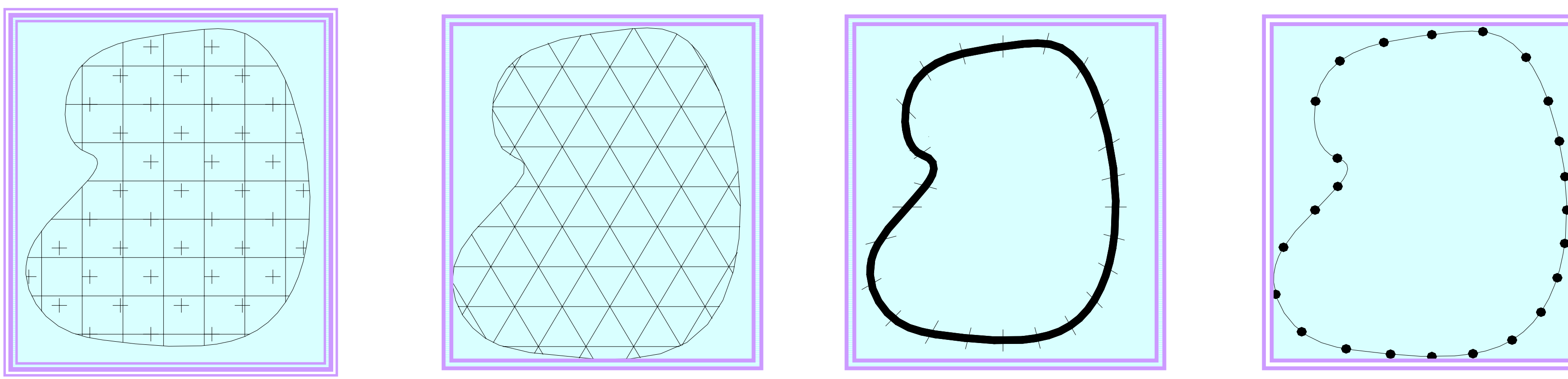
Methods of solution

Methods of solution

- Finite Difference Method
- Finite Element Method
- Boundary Element Method
- Meshless Method



Finite Difference Method	Finite Element Method	Boundary Element Method	Meshless Method
Mesh ¥	Mesh ¥	Mesh ¥ Singular integral ¥	

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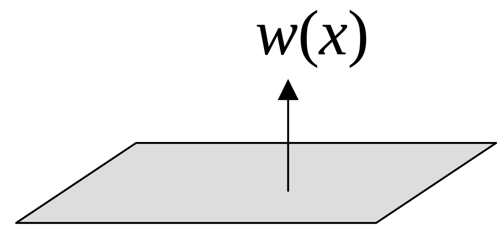
Vibration of plates

Governing Equation:



$$\nabla^4 w(x) = \lambda^4 w(x), \ x \in \Omega$$

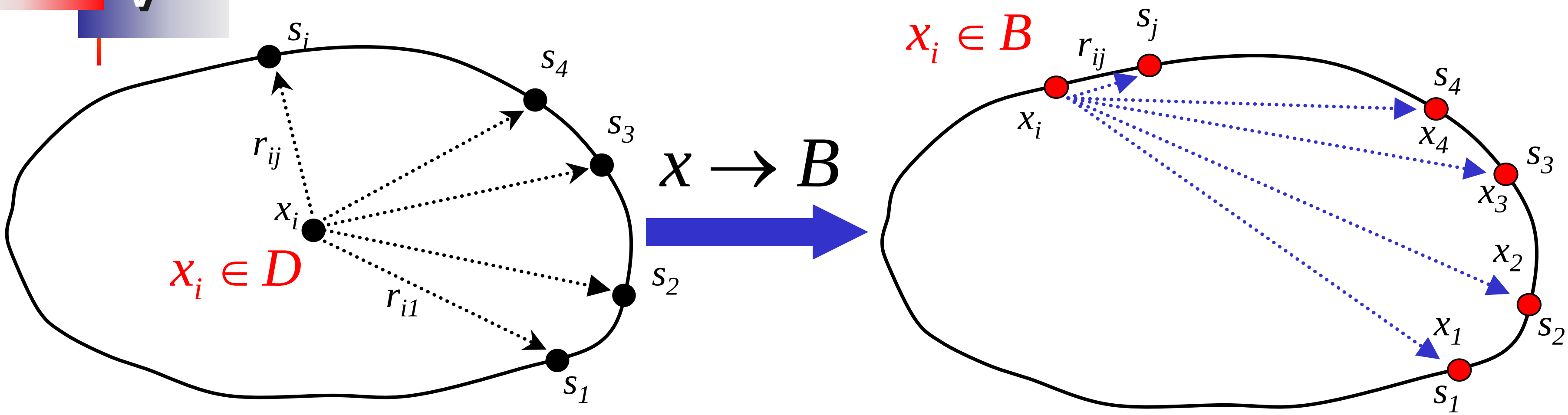
$$\lambda^4 = \frac{\omega^2 \rho h}{D}$$

$$D = \frac{E h^3}{12(1-\nu)}$$

w is the angle frequency
r is the surface density
h is the plates thickness
D is the flexural rigidity
E is the Young's modulus
n is the Poisson ratio

biharmonic operator
ateral displacement
frequency parameter
domain of the thin plates

Field representation using RBF



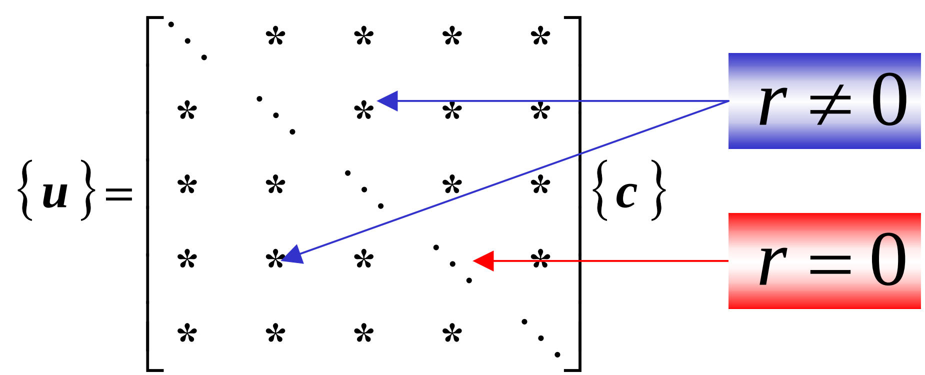
Field representation

$$u(x_i) = \sum c_j \psi(x_i, s_j)$$

$$\psi(x, s) = \psi(r)$$

$$r \equiv |s - x|$$

To match B.C.



Data bank of RBF

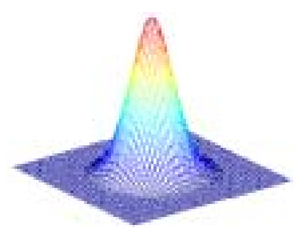
Radial basis function (RBFs)

- **Mesh method**
  - **Globally-supported RBFs**
    - **DRBEM** — Nardini, Brebbia: $\psi(r) = 1 + r$
    - **Method of particular integral** — Ahmad & Banerjee: $\psi(r) = C - r$

- **Meshless method**
  - **Compactly-supported RBFs** — C.S. Chen

$$\psi(r) = (1-r)_+^4 (4r+1)$$

$$\begin{cases} \bar{\psi}(r) = \psi(r), & 0 \le r \le 1 \\ \bar{\psi}(r) = 0, & r > 1 \end{cases}$$



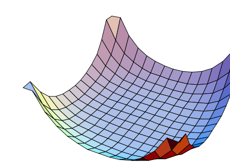
  - **Globally-supported RBFs**
    - **Volume potential** → **Method of fundamental solution**

$$\psi(r) = U_c(s,x) = U_c(|x-s|) = U_c(r)$$

(Potential theory)

    - **The present method** → **Imaginary-part fundamental solution**

$$\psi(r) = J_0(\lambda r) + I_0(\lambda r)$$



Displacement, Slope and Moment

$$w(x)=\sum_{j=1}^{2N}W(s_j,x)p(s_j)+\sum_{j=1}^{2N}\Theta(s_j,x)q(s_j)$$

Displacement

$$\theta(x)=\sum_{j=1}^{2N}W_n(s_j,x)p(s_j)+\sum_{j=1}^{2N}\Theta_n(s_j,x)q(s_j)$$

Slope

$$m(x)=\sum_{j=1}^{2N}W_m(s_j,x)p(s_j)+\sum_{j=1}^{2N}\Theta_m(s_j,x)q(s_j)$$

Moment

Imaginary-part fundamental solution

$$W(s,x) = \mathrm{Im}\{\frac{i}{8\lambda^2}(H_0^{(2)}(\lambda r) + H_0^{(1)}(i\lambda r))\}$$

$$W(s,x) = \frac{1}{8\lambda^2}(J_0(\lambda r) + I_0(\lambda r))$$

Constructing the six kernel functions

$$W(s,x) \Rightarrow \Theta(s,x) = \frac{\partial W(s,x)}{\partial n_s}$$

$\frac{\partial}{\partial n_x}$ ⇓ $\quad \frac{\partial}{\partial n_s} \quad$ $\frac{\partial}{\partial n_x}$ ⇓

$$W_n(s,x) = \frac{\partial W(s,x)}{\partial n_x} \Rightarrow \Theta_n(s,x) = \frac{\partial^2 W(s,x)}{\partial n_s \partial n_x}$$

$\frac{\partial}{\partial n_x}$ ⇓ $\quad \frac{\partial}{\partial n_s} \quad$ $\frac{\partial}{\partial n_x}$ ⇓

$$W_{nn}(s,x) = \frac{\partial^2 W(s,x)}{\partial n_x \partial n_x} \Rightarrow \Theta_{nn}(s,x) = \frac{\partial^3 W(s,x)}{\partial n_s (\partial n_x)^2}$$

$$V\Theta_m(s_j,x) = \Theta_{nn}(s_j,x) + \frac{\nu}{\rho}\Theta_n(s_j,x)$$

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Two cases

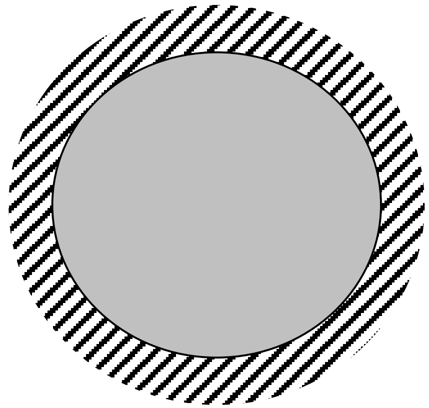
	Clamped plate	Simply-supported plate
Boundary condition	$w(x)=0$ $q(x)=0$	$w(x)=0$ $m(x)=0$
Field representation	$w(x)=[W]\{p\}+[Q]\{q\}$ $q(x)=[\mathrm{W_n}]\{p\}+[Q_n]\{q\}$	$w(x)=[W]\{p\}+[Q]\{q\}$ $m(x)=[W_m]\{p\}+[Q_m]\{q\}$
Eigenequation	$J_\ell(\lambda\rho)I_{\ell+1}(\lambda\rho)+I_\ell(\lambda\rho)J_{\ell+1}(\lambda\rho)=0$	$\dfrac{I_{\ell+1}(\lambda\rho)}{I_\ell(\lambda\rho)}+\dfrac{J_{\ell+1}(\lambda\rho)}{J_\ell(\lambda\rho)}=\dfrac{2\lambda\rho}{(1-\nu)}$

Clamped plate

$$w(x) = 0 \Rightarrow [W]\{p\} + [\Theta]\{q\} = \{0\}$$

$$\theta(x) = 0 \Rightarrow [W_n]\{p\} + [\Theta_n]\{q\} = \{0\}$$

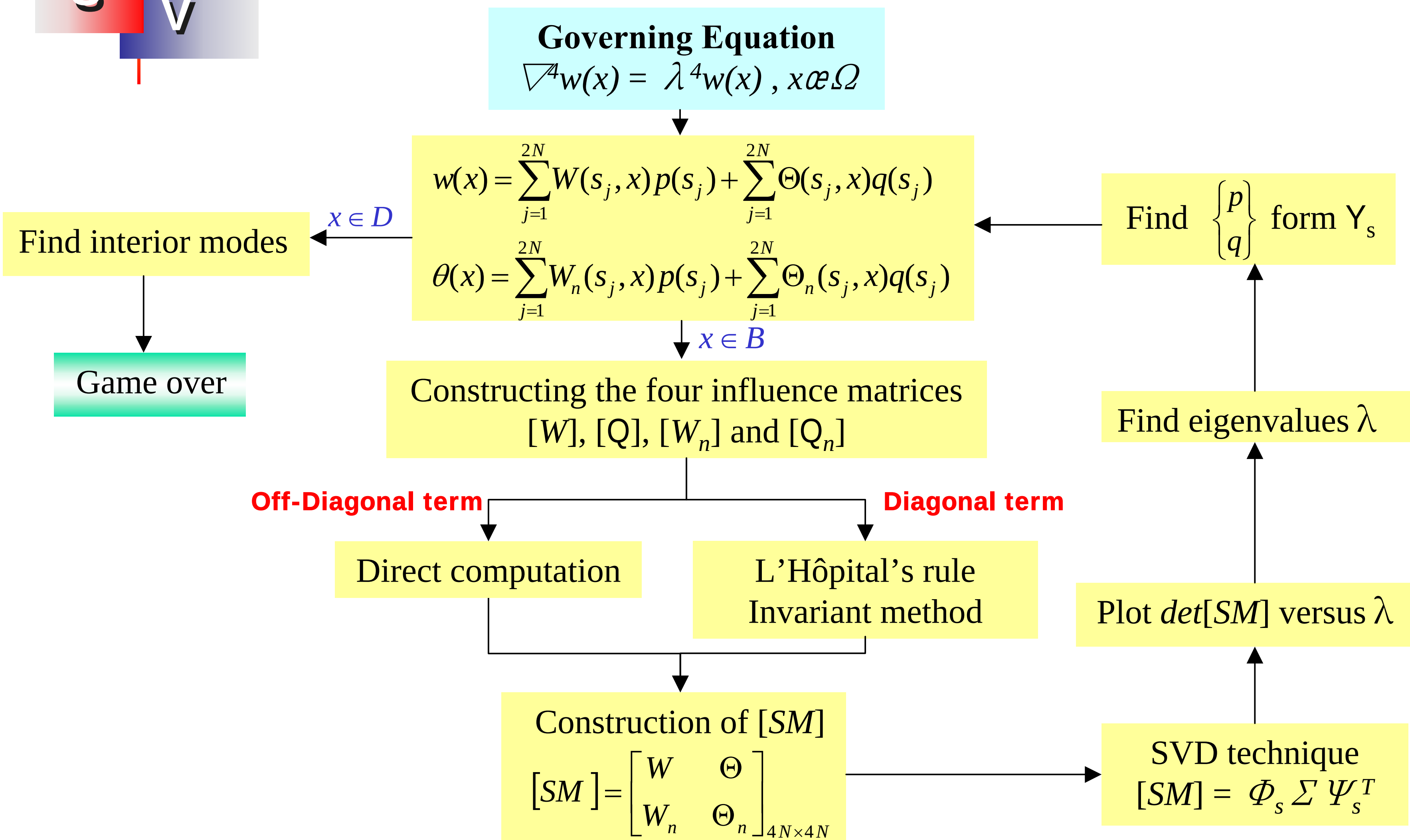
$x \in B$



Nontrivial

$$\det[SM] = 0 \Leftarrow \begin{bmatrix} W & \Theta \\ W_n & \Theta_n \end{bmatrix} \begin{Bmatrix} p \\ q \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

Flow chart of the present method for clamped case

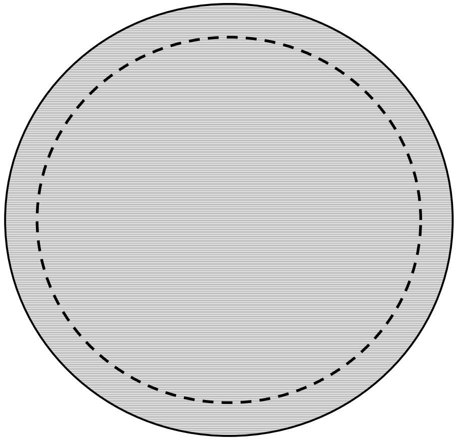


Simply-supported plate

$$w(x) = 0 \Rightarrow [W]\{p\} + [\Theta]\{q\} = \{0\}$$

$$m(x) = 0 \Rightarrow [W_m]\{p\} + [\Theta_m]\{q\} = \{0\}$$

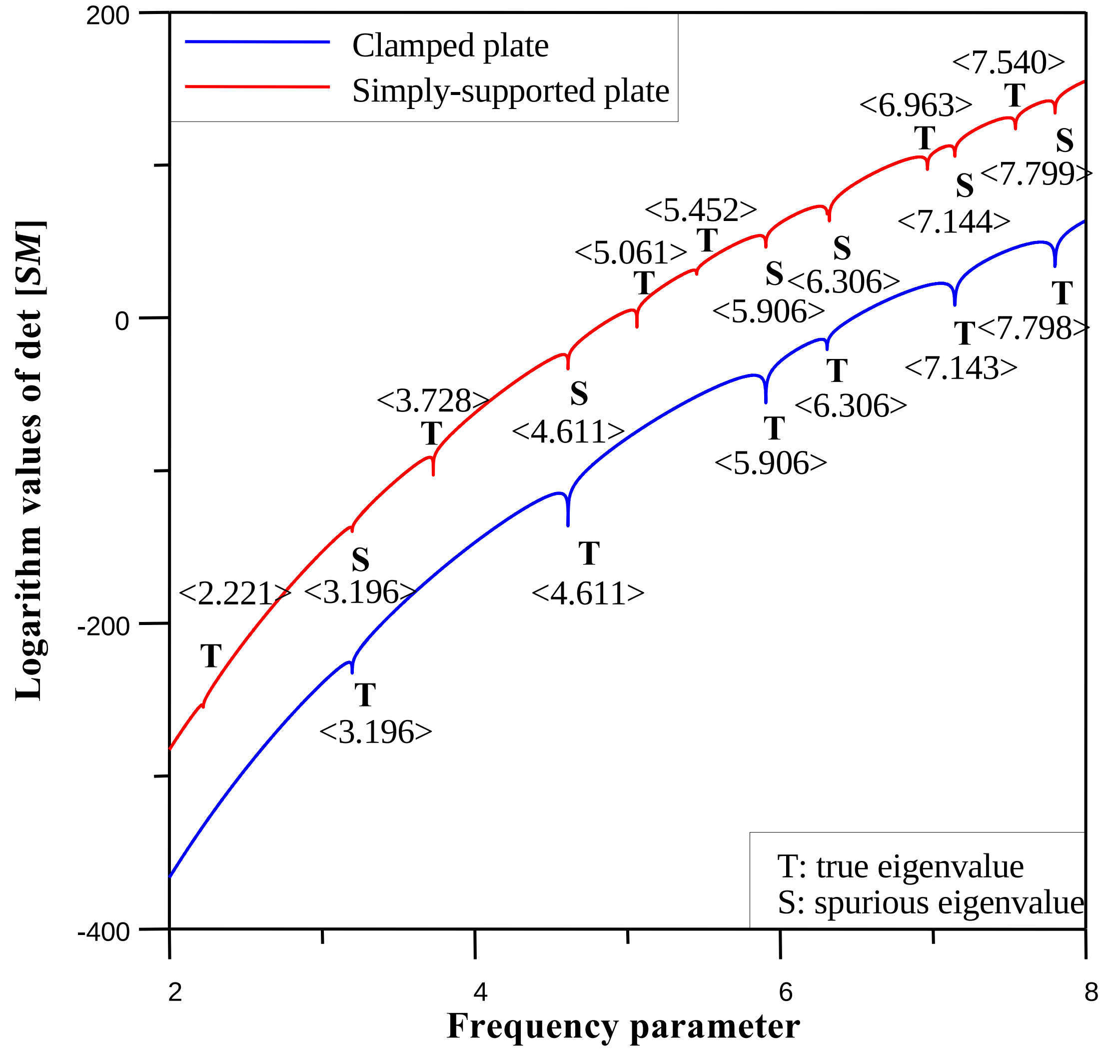
$$x \in B$$



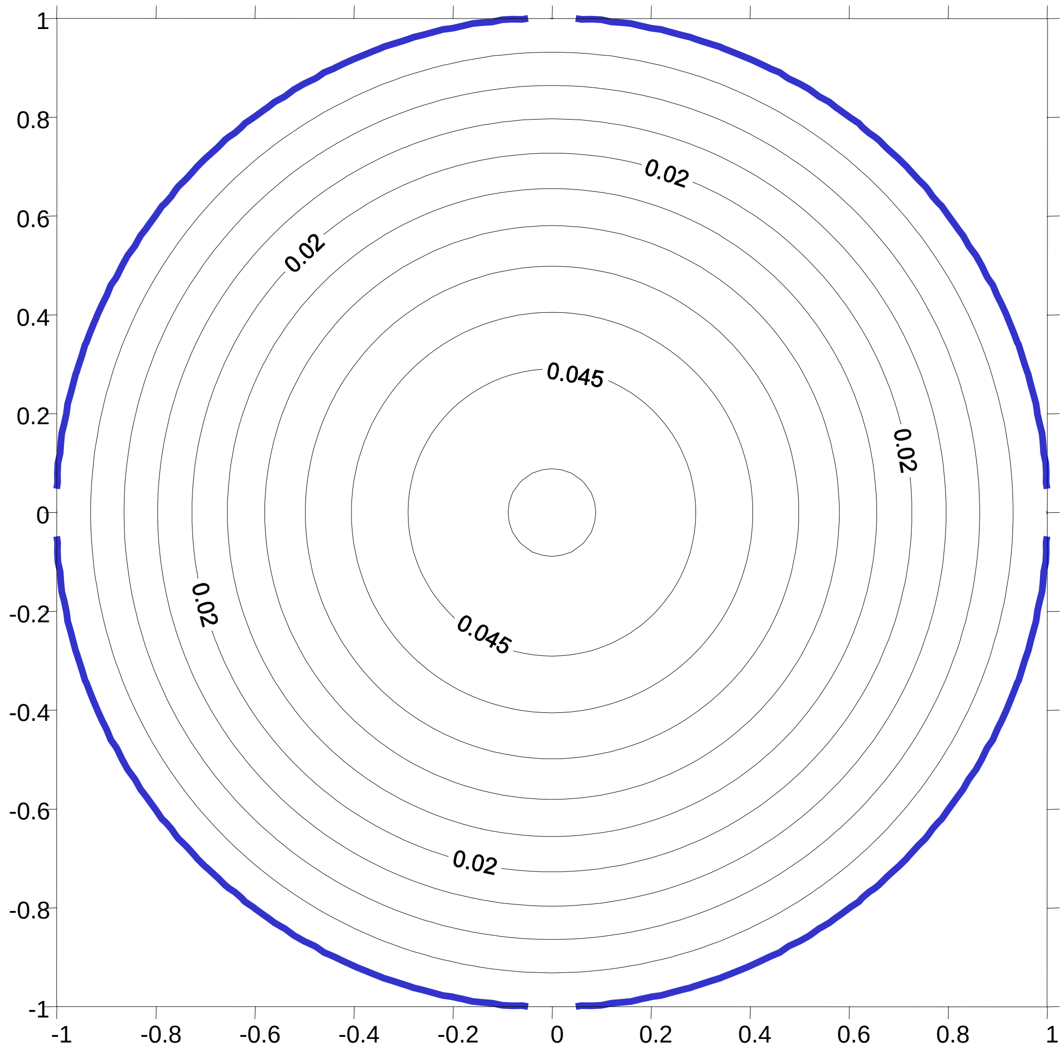
Nontrivial

$$\det[SM] = 0 \Leftarrow \begin{bmatrix} W & \Theta \\ W_m & \Theta_m \end{bmatrix} \begin{Bmatrix} p \\ q \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

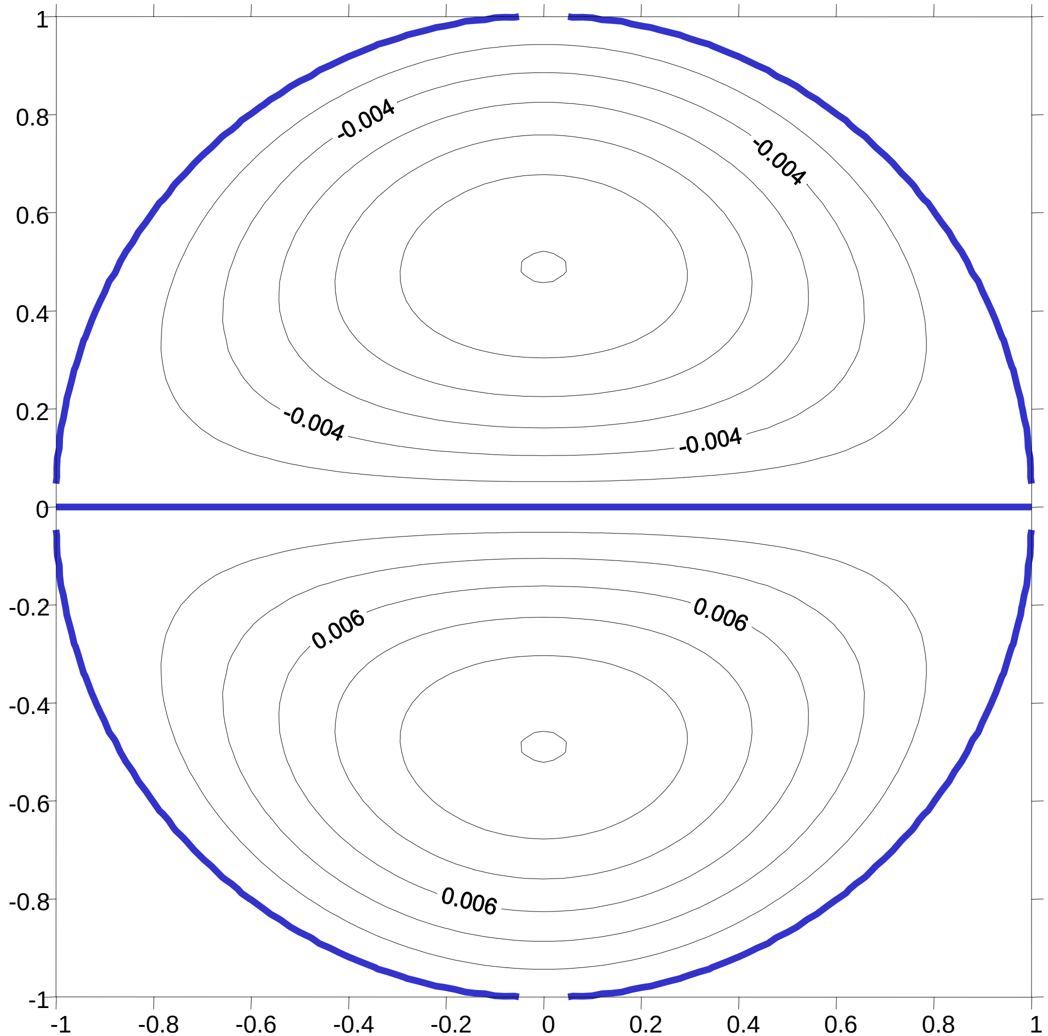
Circular clamped and simply-supported plate using the present methods



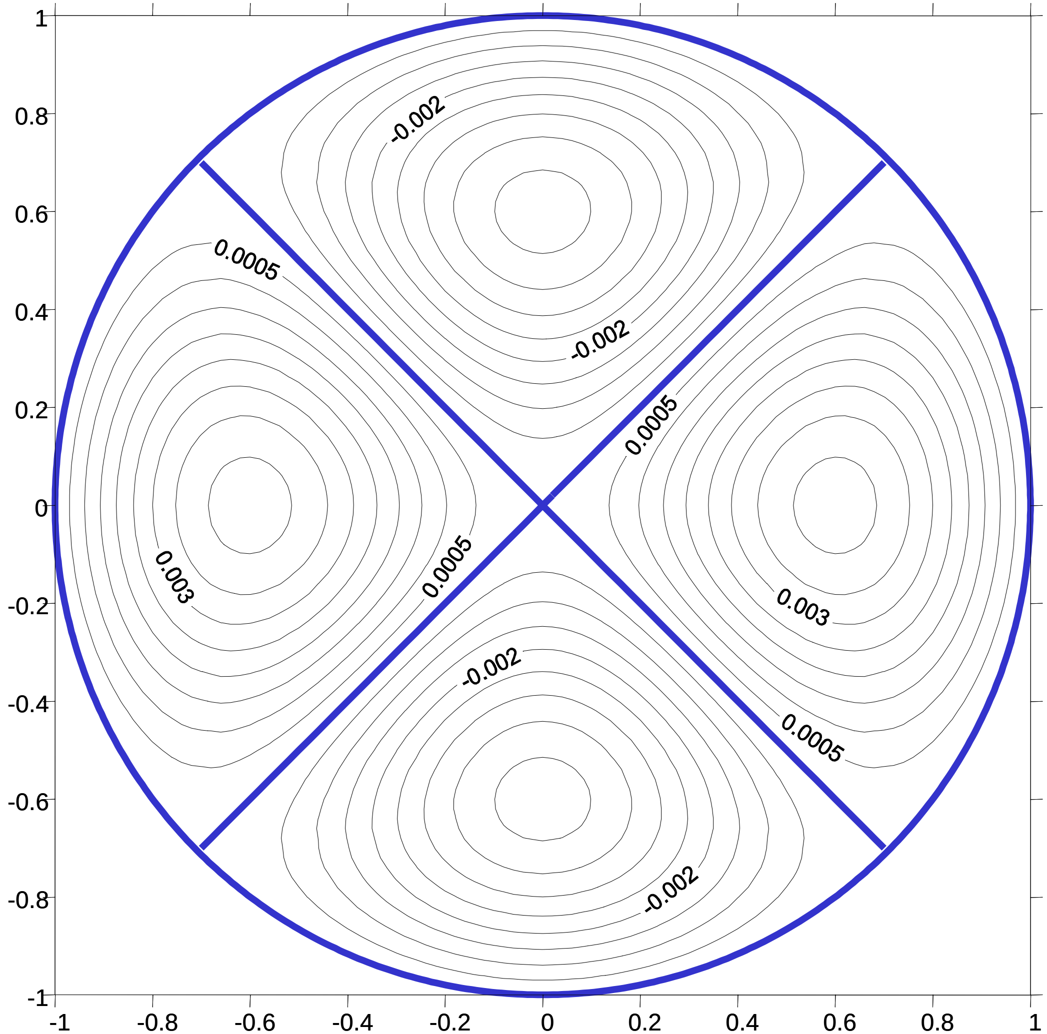
Mode 1 (λ =2.221) for simply-supported plate



Mode 2 (λ =3.728) for simply-supported plate



Mode 2 (λ =5.061) for simply-supported plate



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Circulants

Discritization into 2N nodes on the circular boundary

$$[W]=\begin{bmatrix} a_0 & a_1 & a_2 & \cdots & a_{2N-2} & a_{2N-1} \\ a_{2N-1} & a_0 & a_1 & \cdots & a_{2N-3} & a_{2N-2} \\ a_{2N-2} & a_{2N-1} & a_0 & \cdots & a_{2N-4} & a_{2N-3} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ a_2 & a_3 & a_4 & \cdots & a_0 & a_1 \\ a_1 & a_2 & a_3 & \cdots & a_{2N-1} & a_0 \end{bmatrix}$$

$$\lambda_\ell = a_0 + a_1\alpha_\ell + a_2\alpha_\ell^2 + \cdots + a_{2N-1}\alpha_\ell^{2N-1}$$

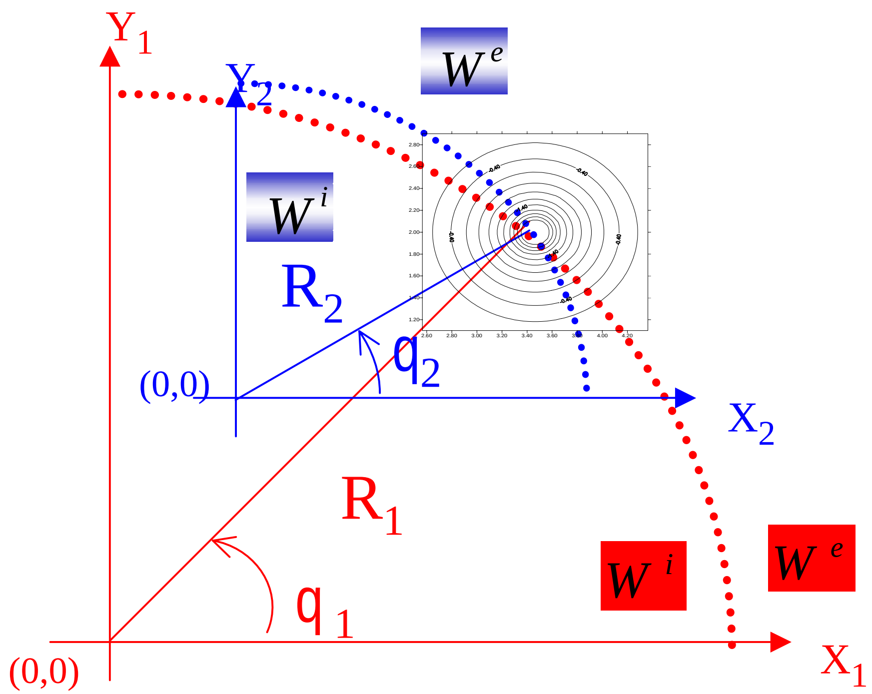
$\ell = 0, \pm 1, \pm 2, \cdots, \pm(N-1), N$ ：eigenvalue of $[W]$

Circulants

$$C_{2N} = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 0 & 1 \\ 1 & 0 & 0 & \cdots & 0 & 0 \end{bmatrix}_{2N\times 2N}$$

$$\alpha_\ell = e^{i\frac{2\pi\ell}{2N}} = \cos(\frac{2\pi\ell}{2N}) + i\sin(\frac{2\pi\ell}{2N})$$ ： eigenvalue of C_{2N}

Degenerate kernels for circular case



$$W(s,x)=\begin{cases} W^{I}(R,\theta;\ \rho,\phi)=\dfrac{1}{8\lambda^2}\displaystyle\sum_{m=-\infty}^{\infty}[J_m(\lambda R)J_m(\lambda\rho)+(-1)^m I_m(\lambda R)I_m(\lambda\rho)](\cos(m(\theta-\phi))), & R>\rho \\ W^{E}(R,\theta;\ \rho,\phi)=\dfrac{1}{8\lambda^2}\displaystyle\sum_{m=-\infty}^{\infty}[J_m(\lambda\rho)J_m(\lambda R)+(-1)^m I_m(\lambda\rho)I_m(\lambda R)](\cos(m(\theta-\phi))), & R<\rho \end{cases}$$

Eigenvalues of influence matrices

$$\lambda_\ell = \frac{N}{4\lambda^2}[J_\ell(\lambda\rho)J_\ell(\lambda\rho)+(-1)^\ell I_\ell(\lambda\rho)I_\ell(\lambda\rho)]$$

$$\mu_\ell = \frac{N}{4\lambda}[J_\ell(\lambda\rho)J'_\ell(\lambda\rho)+(-1)^\ell I_\ell(\lambda\rho)I'_\ell(\lambda\rho)]$$

$$\nu_\ell = \frac{N}{4\lambda}[J'_\ell(\lambda\rho)J_\ell(\lambda\rho)+(-1)^\ell I'_\ell(\lambda\rho)I_\ell(\lambda\rho)]$$

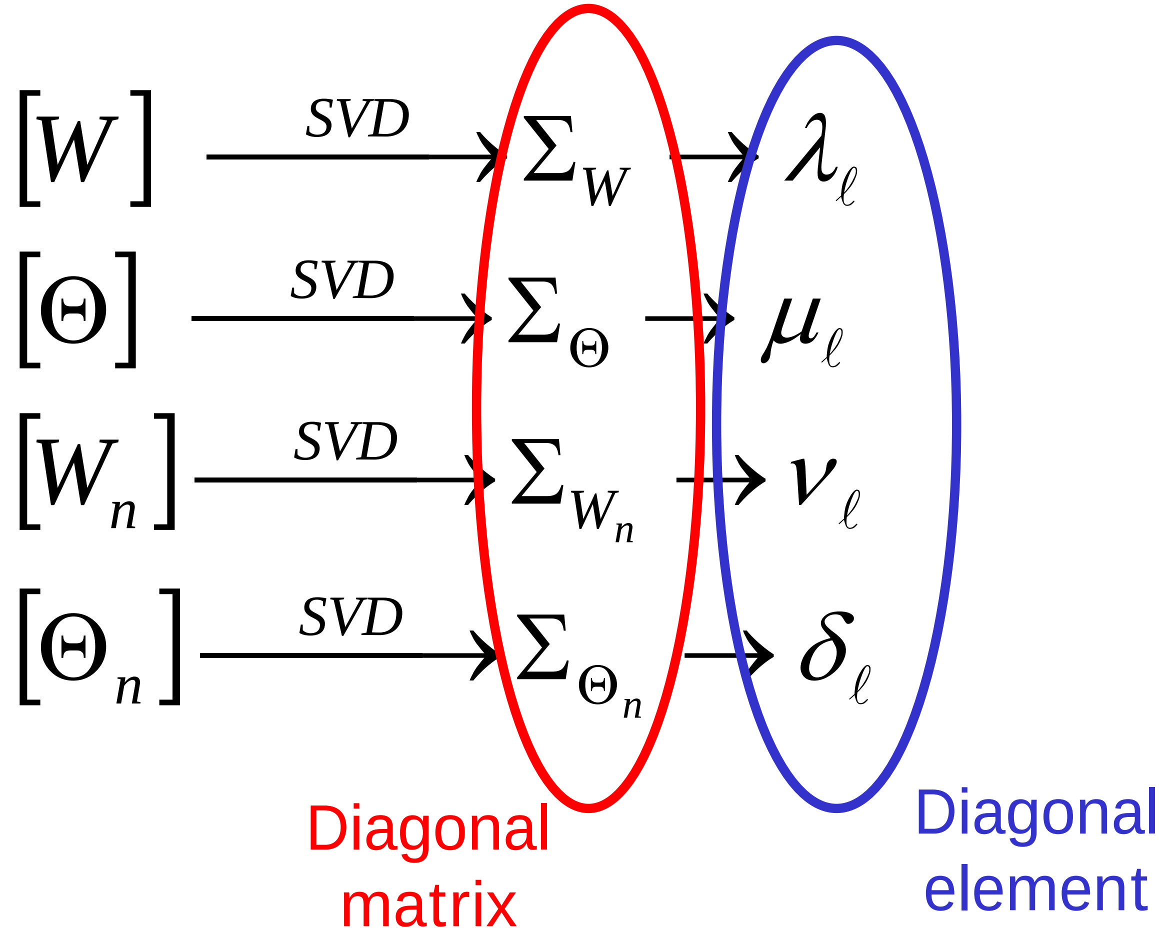
$$\delta_\ell = \frac{N}{4}[J'_\ell(\lambda\rho)J_\ell(\lambda\rho)+(-1)^\ell I'_\ell(\lambda\rho)I_\ell(\lambda\rho)]$$

The eigenvalues of matrices

$$[W] = \Phi \Sigma_W \Phi^T$$

$$= \Phi \begin{bmatrix} \lambda_0 & 0 & 0 & \cdots & 0 & 0 & 0 \\ 0 & \lambda_1 & 0 & \cdots & 0 & 0 & 0 \\ 0 & 0 & \lambda_{-1} & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & \lambda_{(N-1)} & 0 & 0 \\ 0 & 0 & 0 & \cdots & 0 & \lambda_{-(N-1)} & 0 \\ 0 & 0 & 0 & \cdots & 0 & 0 & \lambda_N \end{bmatrix}_{2N\times 2N} \Phi^T$$

Singular value of influence matrices



Determinant for clamped plate

$$[SM]=\begin{bmatrix}\Phi\Sigma_W\Phi^T & \Phi\Sigma_\Theta\Phi^T\\ \Phi\Sigma_{W_n}\Phi^T & \Phi\Sigma_{\Theta_n}\Phi^T\end{bmatrix}_{4N\times 4N}$$

$$=\begin{bmatrix}\Phi & 0\\ 0 & \Phi\end{bmatrix}\begin{bmatrix}\Sigma_W & \Sigma_\Theta\\ \Sigma_{W_n} & \Sigma_{\Theta_n}\end{bmatrix}\begin{bmatrix}\Phi & 0\\ 0 & \Phi\end{bmatrix}^T$$

$$\det[SM]=\sigma_0(\sigma_1\sigma_2\cdots\sigma_{N-1})^2\sigma_N=0$$

$$\sigma_\ell=\lambda_\ell\delta_\ell-\mu_\ell\nu_\ell$$

Eigenequation for clamped boundary

$$\sigma_\ell = \lambda_\ell \delta_\ell - \mu_\ell \nu_\ell$$

$$= \frac{N}{4} \frac{[J_\ell(\lambda\rho) I_{\ell+1}(\lambda\rho) + I_\ell(\lambda\rho) J_{\ell+1}(\lambda\rho)]^2}{J_\ell(\lambda\rho) J_\ell(\lambda\rho) - (-1)^\ell I_\ell(\lambda\rho) I_\ell(\lambda\rho)}$$

$$= 0, \quad \ell = 0, \pm 1, \pm 2, \cdots, \pm(N-1), N$$

$$J_\ell(\lambda\rho) I_{\ell+1}(\lambda\rho) + I_\ell(\lambda\rho) J_{\ell+1}(\lambda\rho) = 0$$

Exact eigensolution

Determinant for simply-supported plate

$$[SM]=\begin{bmatrix} \Phi\Sigma_w\Phi^T & \Phi\Sigma_\Theta\Phi^T \\ \Phi\Sigma_{w_m}\Phi^T & \Phi\Sigma_{\Theta_m}\Phi^T \end{bmatrix}_{4N\times 4N}$$

$$=\begin{bmatrix} \Phi & 0 \\ 0 & \Phi \end{bmatrix}\begin{bmatrix} \Sigma_w & \Sigma_\Theta \\ \Sigma_{w_m} & \Sigma_{\Theta_m} \end{bmatrix}\begin{bmatrix} \Phi & 0 \\ 0 & \Phi \end{bmatrix}^T$$

$$\det[SM]=\sigma_0(\sigma_1\sigma_2\cdots\sigma_{N-1})^2\sigma_N=0$$

Singular value of influence matrices

$$[W] \xrightarrow{SVD} \Sigma_W \rightarrow \lambda_\ell$$

$$[\Theta] \xrightarrow{SVD} \Sigma_\Theta \rightarrow \mu_\ell$$

$$[W_m] \xrightarrow{SVD} \Sigma_{W_m} \rightarrow \kappa_\ell$$

$$[\Theta_m] \xrightarrow{SVD} \Sigma_{\Theta_m} \rightarrow \eta_\ell$$

Diagonal matrix

Diagonal element

Eigenequation for simply-supported boundary

$$\sigma_\ell = \lambda_\ell \eta_\ell - \mu_\ell \kappa_\ell$$

Exact eigenequation of clamped plates

$$\Rightarrow (J_\ell(\lambda\rho) I_{\ell+1}(\lambda\rho) + I_\ell(\lambda\rho) J_{\ell+1}(\lambda\rho))$$

$$\times \left(\frac{I_{\ell+1}(\lambda\rho)}{I_\ell(\lambda\rho)} + \frac{J_{\ell+1}(\lambda\rho)}{J_\ell(\lambda\rho)} - \frac{2\lambda\rho}{(1-\nu)} \right) = 0$$

Spurious

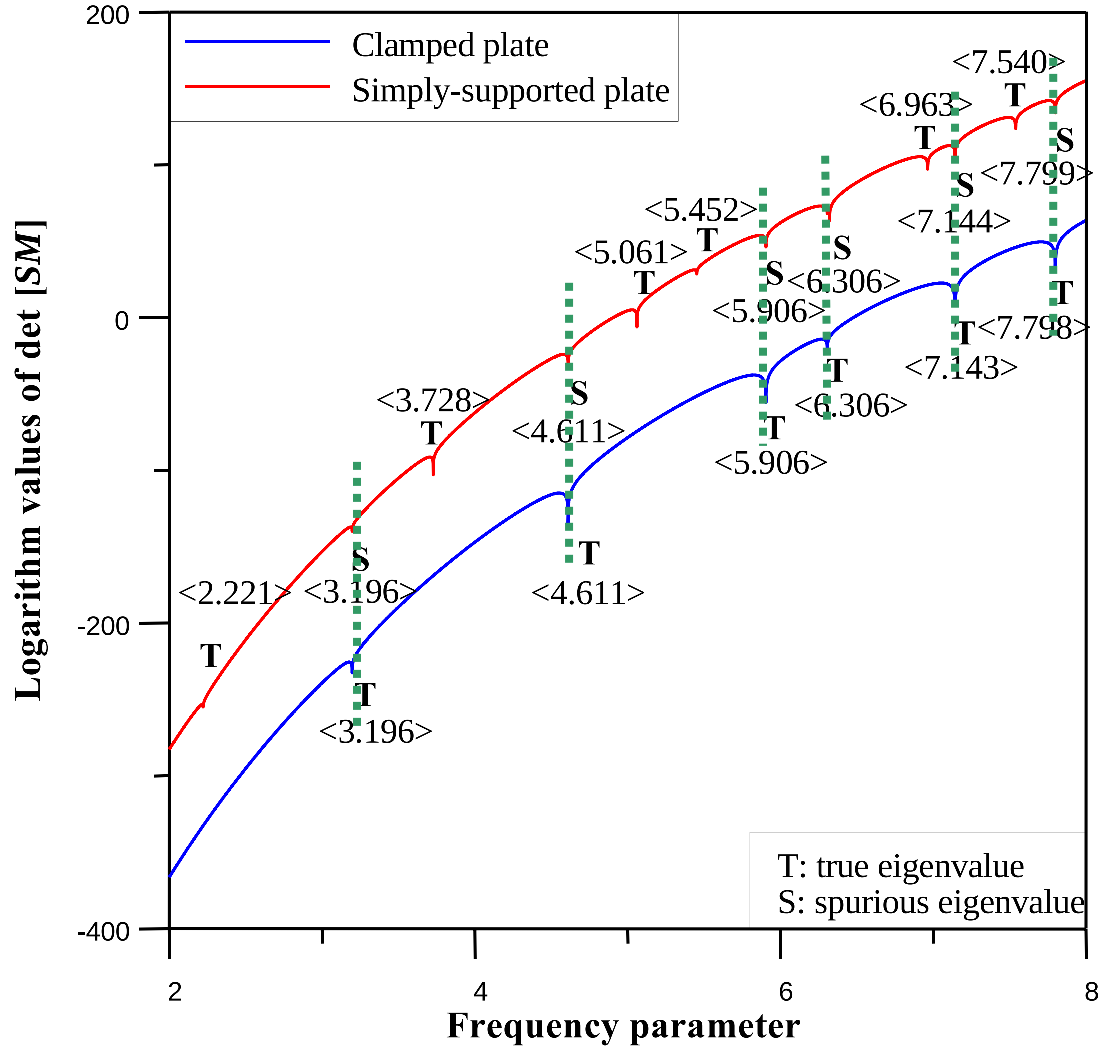
True

$$\ell = 0, \pm 1, \pm 2, \cdots, \pm(N-1), N$$

$$\frac{I_{\ell+1}(\lambda\rho)}{I_\ell(\lambda\rho)} + \frac{J_{\ell+1}(\lambda\rho)}{J_\ell(\lambda\rho)} = \frac{2\lambda\rho}{(1-\nu)}$$

Exact eigenequation of simply-supported plate

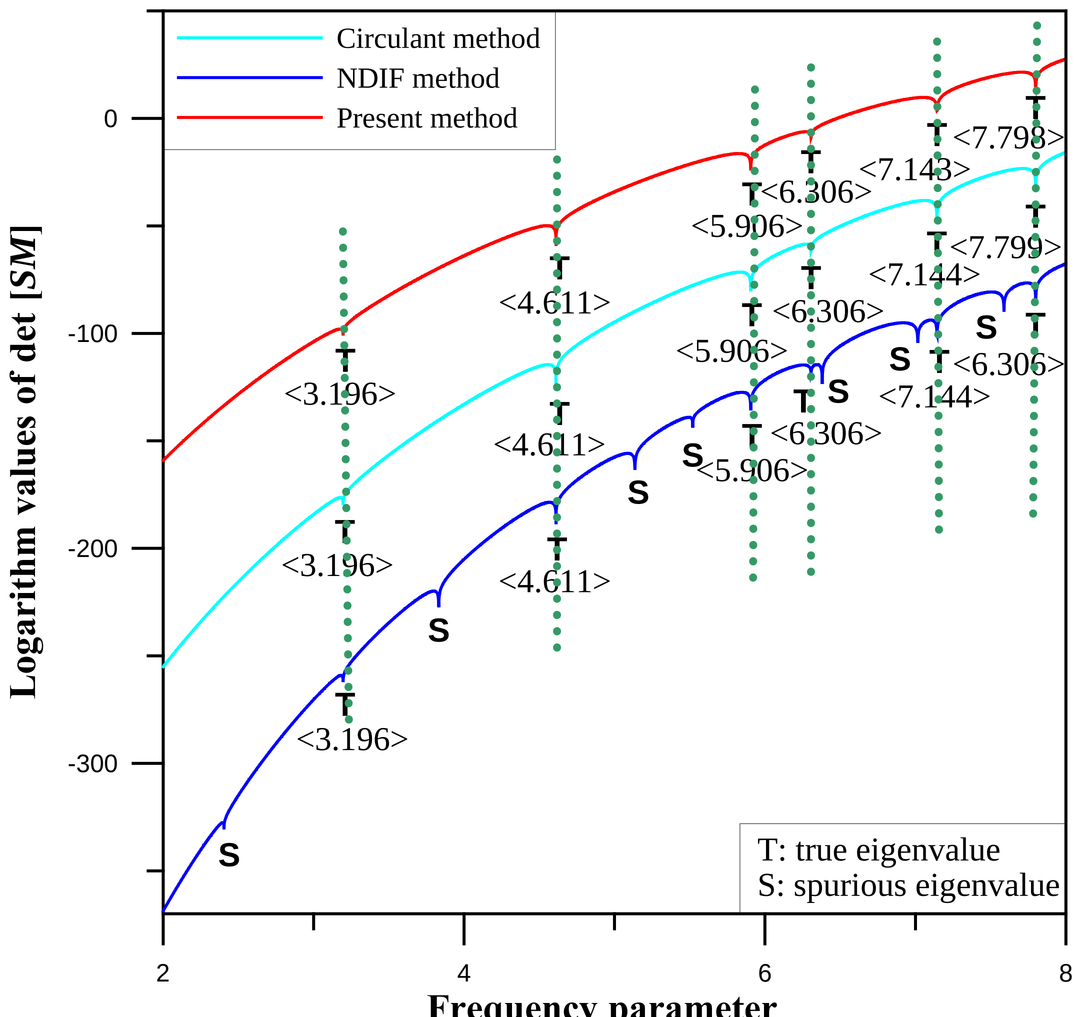
Circular clamped and simply-supported plate using the present method



Comparisons of the NDIF and present method

	Kang	Present method
RBF	$W(s,x)=J_0(\lambda r)$ $\Theta(s,x)=I_0(\lambda r)$	$W(s,x)=\frac{1}{8\lambda^2}(J_0(\lambda r)+I_0(\lambda r))$ $\Theta(s,x)=\frac{\partial W(s,x)}{\partial n_s}$
Clamped plate	$J_\ell(\lambda r)I_{\ell+1}(\lambda r)+J_{\ell+1}(\lambda r)I_\ell(\lambda r)=0$	$J_\ell(\lambda r)I_{\ell+1}(\lambda r)+J_{\ell+1}(\lambda r)I_\ell(\lambda r)=0$
	$J_\ell(\lambda r)=0$	No
Simply-supported plate	$\frac{I_{\ell+1}(\lambda r)}{I_\ell(\lambda r)}+\frac{J_{\ell+1}(\lambda r)}{J_\ell(\lambda r)}=\frac{2\lambda r}{(1-\nu)}$	$\frac{I_{\ell+1}(\lambda r)}{I_\ell(\lambda r)}+\frac{J_{\ell+1}(\lambda r)}{J_\ell(\lambda r)}=\frac{2\lambda r}{(1-\nu)}$
	$J_\ell(\lambda r)=0$	$J_\ell(\lambda r)I_{\ell+1}(\lambda r)+J_{\ell+1}(\lambda r)I_\ell(\lambda r)=0$
Treatment	Net approach	CHEEF method Dual formulation with SVD updating

Circular clamped plate using different methods



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Conclusions

1. Field solution can be superimposed by using the two-point function (RBF).
2. Diagonal terms in the influence matrices can be derived by L'Hôpital's rule or invariant method.
3. The eigensolutions for clamped plate can be easily determined using our approach.
4. The eigensolution of simply-supported problem is contaminated by the true eigensolution of clamped problem.

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The End

Thanks for your kind attention