True and Spurious Eigensolutions of Two-dimensional Acoustic Cavities with the Mixed-type Boundary Conditions using BEMs

邊界元素法於混合型邊界條件下之二維聲場真假根探討

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Outlines

- Motivation
- Theoretical analysis
- Numerical examples and results
- Conclusions

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Motivation

There are no spurious eigenvalues using the complex-valued BEM.

There are spurious eigenvalues using the real-part BEM, the imaginary-part BEM and MRM.

Motivation

The eigenproblems with either the Dirichlet or the Neumann B.C. have been discussed for many years.

There are only a few papers focused on the the problem with the mixed-type B.C..

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Governing equation

$$\left(\nabla^2 + k^2\right)u(\mathrm{x}) = 0, \qquad \mathrm{x} \in D$$

where

D : the domain of interest,

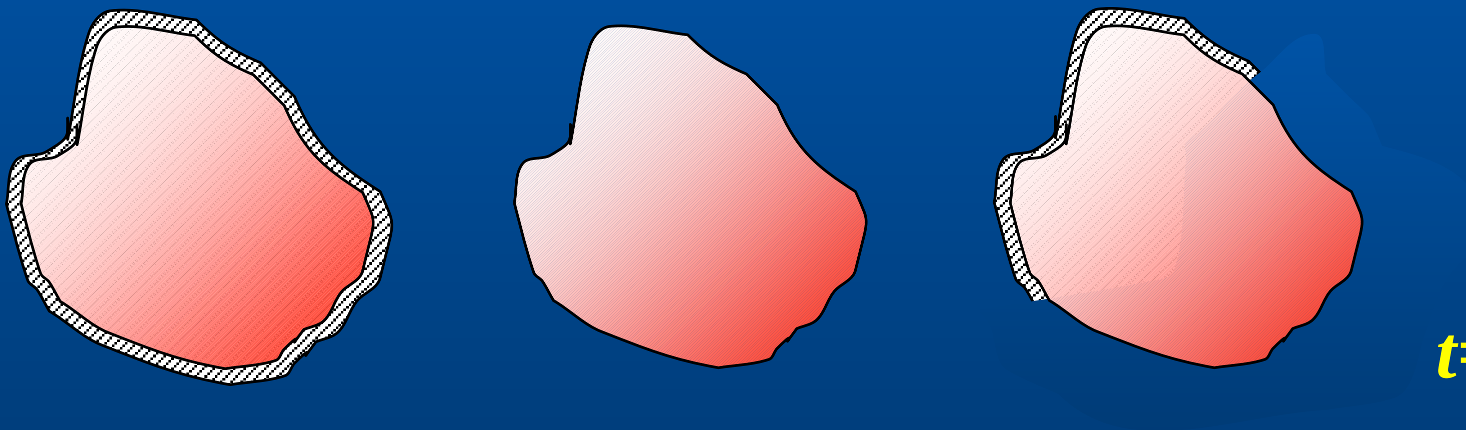
k : the wave number,

x : the domain point,

u(x): the acoustic potential.

Types of boundary conditions

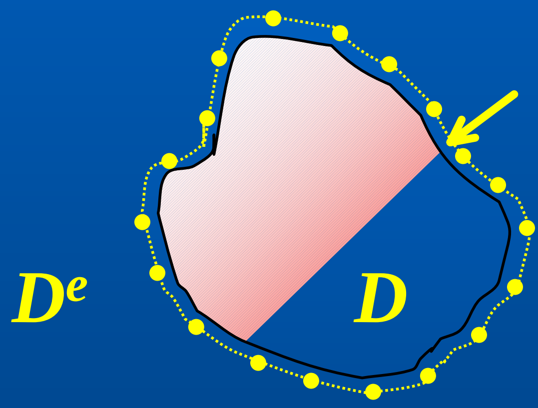
$u=0$ $\qquad$ $t=\dfrac{\partial u}{\partial n}=0$ $\qquad$ $u=0$



$t=0$

Dirichlet $\qquad$ Neumann $\qquad$ Mixed-type

The null field integral formulations



$$0=\int_B T(s,x)\,u(s)dB(s)-\int_B U(s,x)\,t(s)dB(s),\qquad x\in D^e$$

$$0=\int_B M(s,x)\,u(s)dB(s)-\int_B L(s,x)\,t(s)dB(s),\qquad x\in D^e$$

The null field integral formulations

D^e: the complementary domain of *D*,

x,s: the field and source point,

u(s): the potential on the boundary

t(s): the normal derivative of potential on the boundary

U (s,x): the kernel function,

$$T(\mathbf{s},\mathbf{x}) \equiv \frac{\partial U(\mathbf{s},\mathbf{x})}{\partial \boldsymbol{n}_{\mathbf{s}}} \qquad M(\mathbf{s},\mathbf{x}) \equiv \frac{\partial U(\mathbf{s},\mathbf{x})}{\partial \boldsymbol{n}_{\mathbf{x}} \boldsymbol{n}_{\mathbf{s}}}$$

$$L(\mathbf{s},\mathbf{x}) \equiv \frac{\partial U(\mathbf{s},\mathbf{x})}{\partial \boldsymbol{n}_{\mathbf{x}}}$$

$U_{(s,x)}$ kernel in different methods

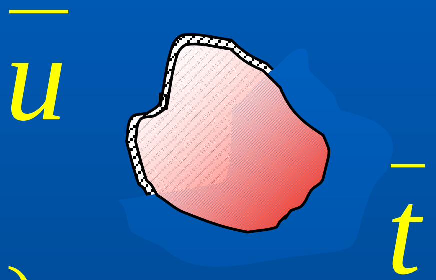
Complex-valued BEM: $$U(s,x)=\frac{-i\pi H_0^{(1)}(kr)}{2}$$

Real-part BEM: $$U(s,x)=\frac{\pi Y_0(kr)}{2}$$

Imaginary-part BEM: $$U(s,x)=\frac{\pi J_0(kr)}{2}$$

MRM: $$U(x,s)=\frac{\pi}{2}Y_0(kr)-[\ln\frac{k}{2}+\gamma]J_0(kr)$$

Discretization (singular formulation)



$$[U]_{N\times N}\{t\}_{N\times 1} = [T]_{N\times N}\{u\}_{N\times 1}$$

$$[U_L \vdots U_R]_{N\times N}\begin{Bmatrix} t \\ \bar{t} \end{Bmatrix}_{N\times 1} = [T_L \vdots T_R]_{N\times N}\begin{Bmatrix} \bar{u} \\ u \end{Bmatrix}_{N\times 1}$$

Rearrangement of the influences matrices

$$\left[U_L \vdots -T_R\right]_{N\times N}\begin{Bmatrix} t \\ u \end{Bmatrix}_{N\times 1} = \left[T_L \vdots -U_R\right]_{N\times N}\begin{Bmatrix} \bar{u} \\ \bar{t} \end{Bmatrix}_{N\times 1}$$

$$[A]_{N\times N}\{p\}_{N\times 1} = [B]_{N\times N}\{q\}_{N\times 1}$$

Derivation of boundary modes

$$[A]_{N\times N}\{p\}_{N\times 1}=0,\qquad \{p\}=\begin{Bmatrix} t \\ u \end{Bmatrix},$$

$$\{u\}=\begin{Bmatrix} 0 \\ u \end{Bmatrix}_{N\times 1},\qquad \{t\}=\begin{Bmatrix} t \\ 0 \end{Bmatrix}_{N\times 1}$$

Discretization (hypersingular formulation)

$$[L]_{N\times N}\{t\}_{N\times 1}=[M]_{N\times N}\{u\}_{N\times 1}$$

$$[C]_{N\times N}\{p\}_{N\times 1}=[D]_{N\times N}\{q\}_{N\times 1}$$

$$[C]_{N\times N}\{p\}_{N\times 1}=0$$

Extraction of the true eigensolutions by using SVD updating terms

$$[A(k)]\{p\} = 0$$

$$[C(k)]\{p\} = 0$$

$$\begin{bmatrix} [A(k_t)] \\ [C(k_t)] \end{bmatrix} \{\varphi\} = 0$$

$\boldsymbol{k_t}$: the true eigenvalues

Detection of the spurious eigensolutions by using SVD updating documents

$$\left[A(k)\right]^T\{\phi\}=0$$

$$\left[B(k)\right]^T\{\phi\}=0$$

by using the Fredhohm's alternative theorem

$$\begin{bmatrix}\left[A(k_s)\right]^T\\\left[B(k_s)\right]^T\end{bmatrix}\{\phi\}=0$$

T: transpose

ks: the spurious eigenvalues

Derivation of the eigensolutions

Singular formulation:

$$2\pi\, u(\mathrm{x}) = \int_B T(\mathrm{s},\mathrm{x})\, u(\mathrm{s})\, dB(\mathrm{s}) - \int_B U(\mathrm{s},\mathrm{x})\, t(\mathrm{s})\, dB(\mathrm{s}), \quad \mathrm{x} \in D$$

Hypersingular formulation:

$$2\pi\, t(\mathrm{x}) = \int_B M(\mathrm{s},\mathrm{x})\, u(\mathrm{s})\, dB(\mathrm{s}) - \int_B L(\mathrm{s},\mathrm{x})\, t(\mathrm{s})\, dB(\mathrm{s}), \quad \mathrm{x} \in D$$

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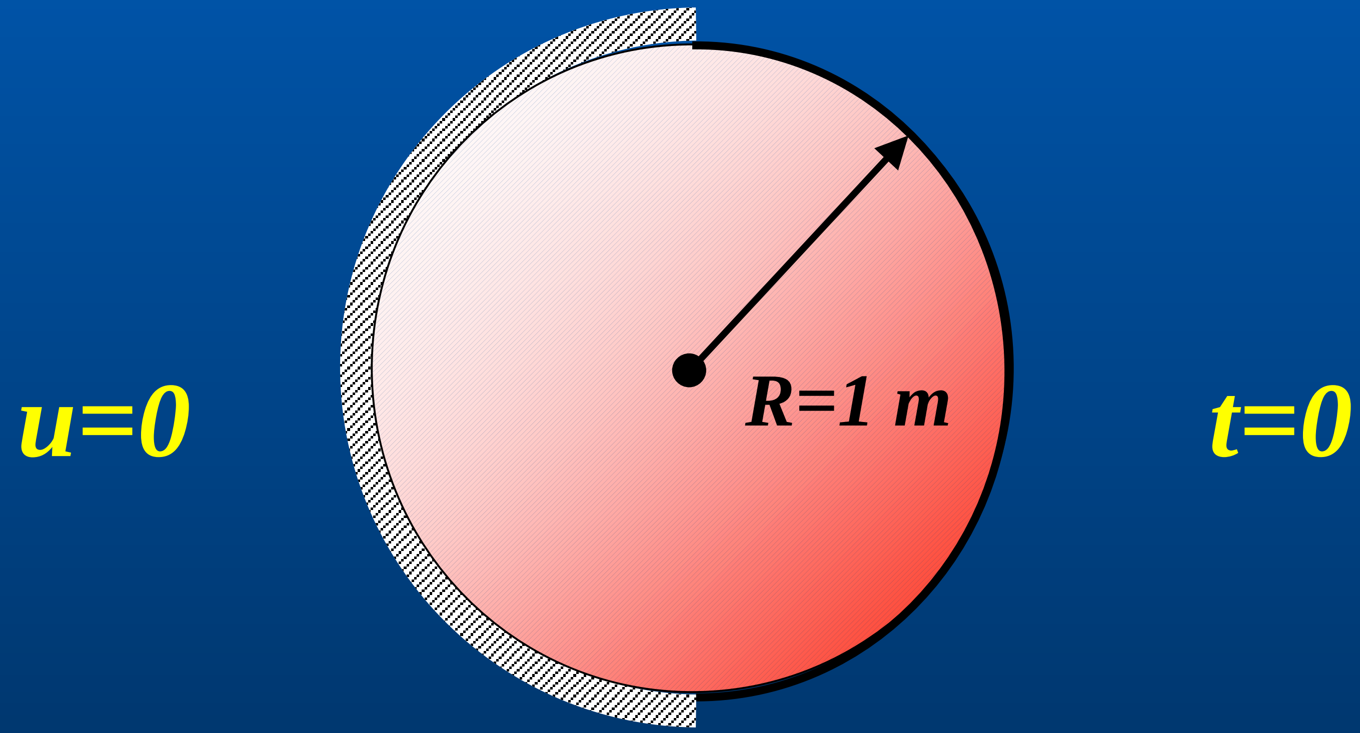
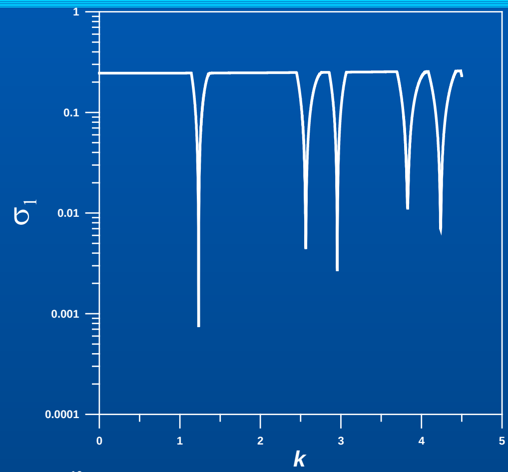
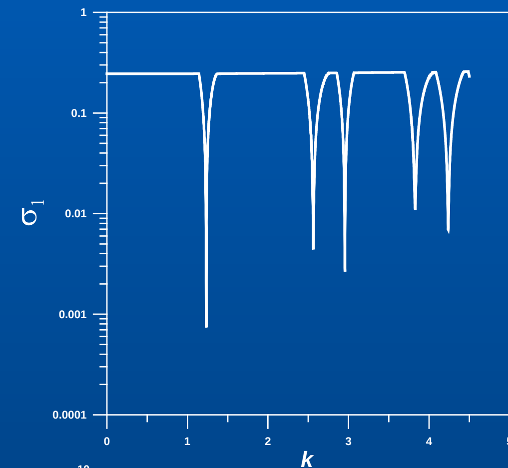
Circular cavity



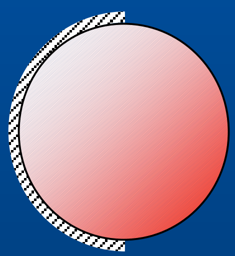
Detection the eigenvalues using the complex-valued BEM

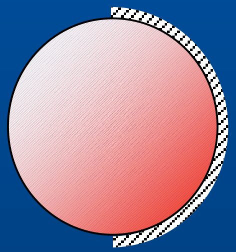
$[A]$



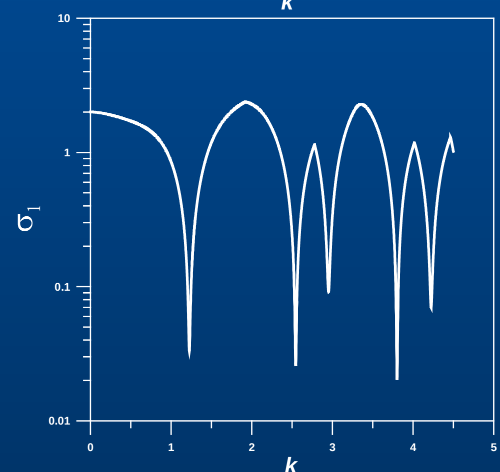


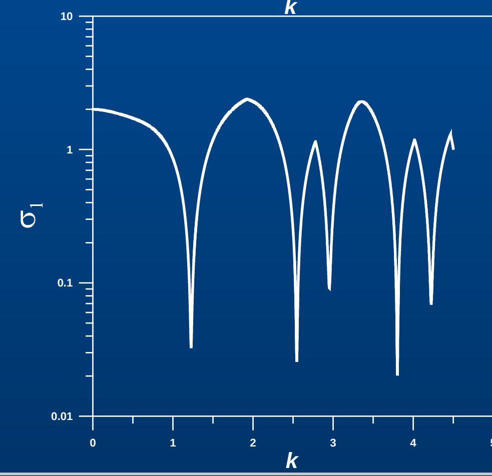
$[B]$





$[C]$



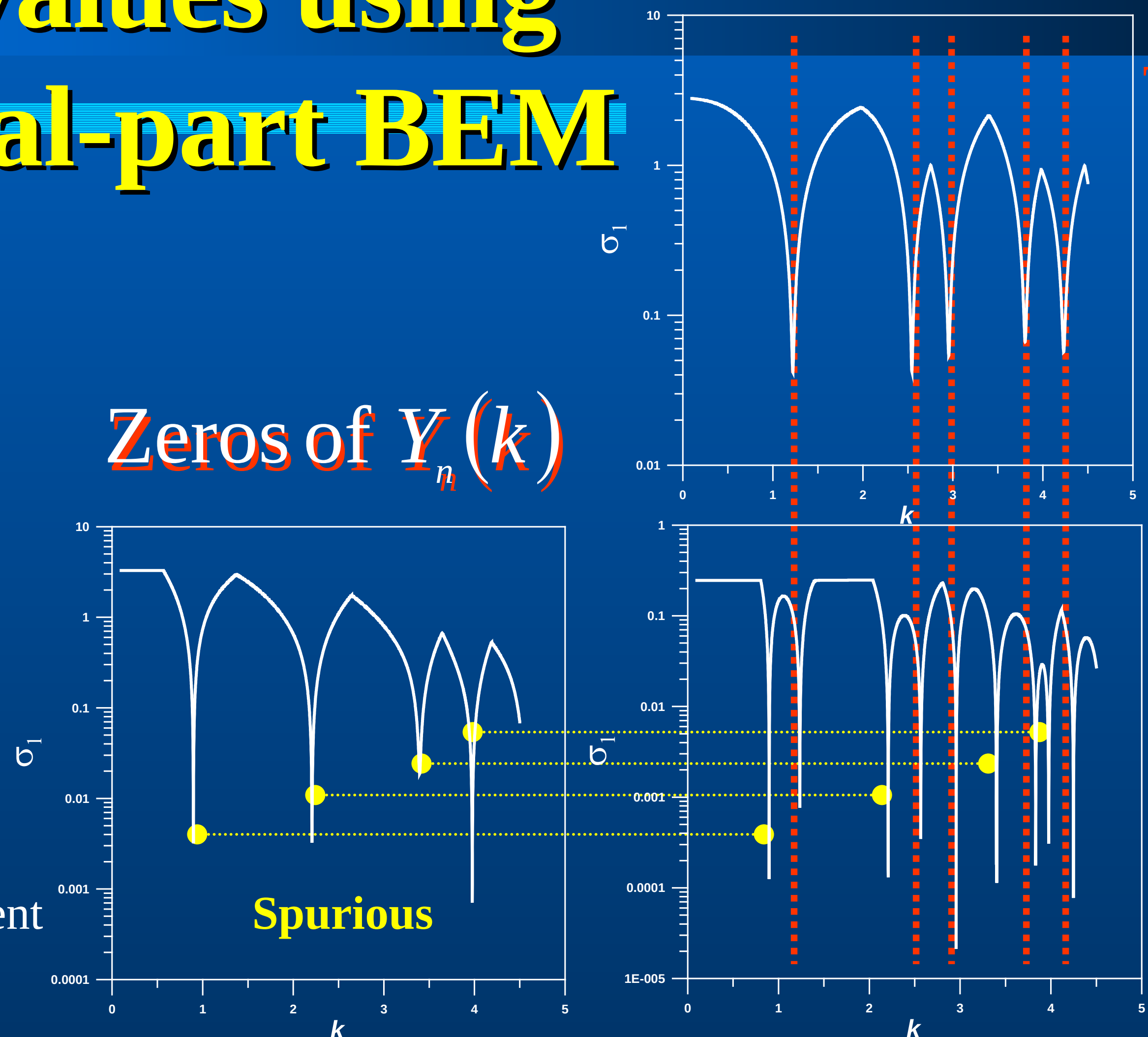


$[D]$

Detection of true and spurious eigenvalues using the real-part BEM

$$\begin{bmatrix} A \\ C \end{bmatrix}$$

Updating term

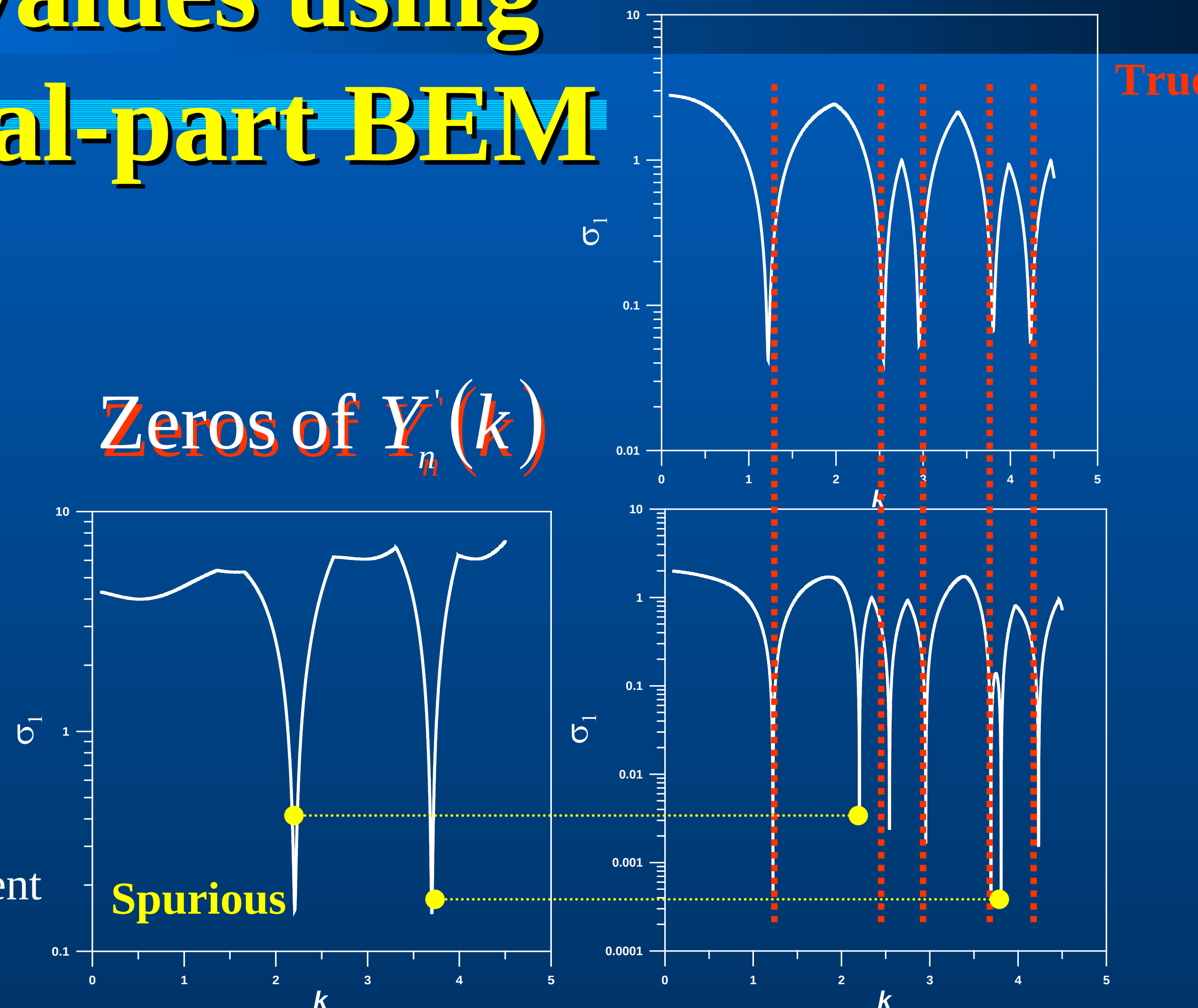


$$\begin{bmatrix} A & B \end{bmatrix}$$

Updating document

$$\begin{bmatrix} A \end{bmatrix}$$

Detection of true and spurious eigenvalues using the real-part BEM



$\begin{bmatrix} A \\ C \end{bmatrix}$

Updating term

$\begin{bmatrix} C & D \end{bmatrix}$

Updating document

$\begin{bmatrix} C \end{bmatrix}$

The comparison for the diagrams with different B.C.

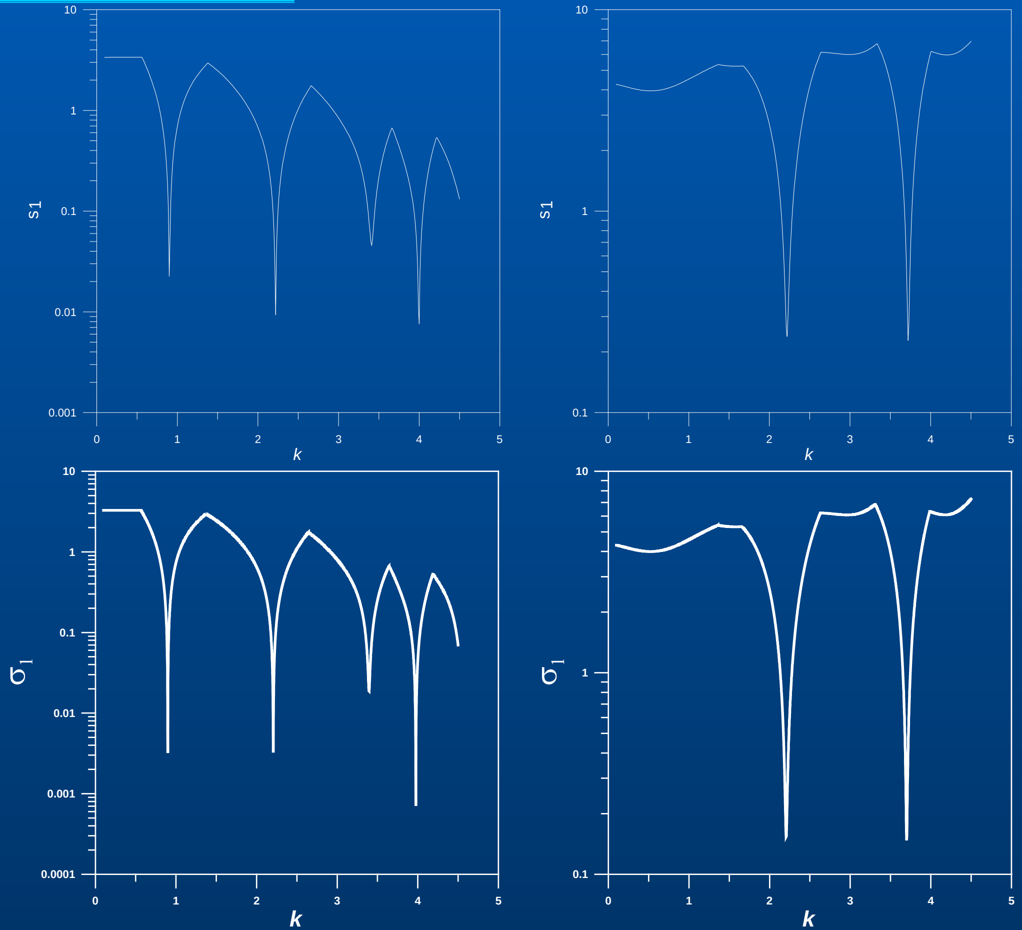
Neumann or

Dirichlet

Zeros of $Y_n(k)$

Mixed-typed

Zeros of $Y_n'(k)$



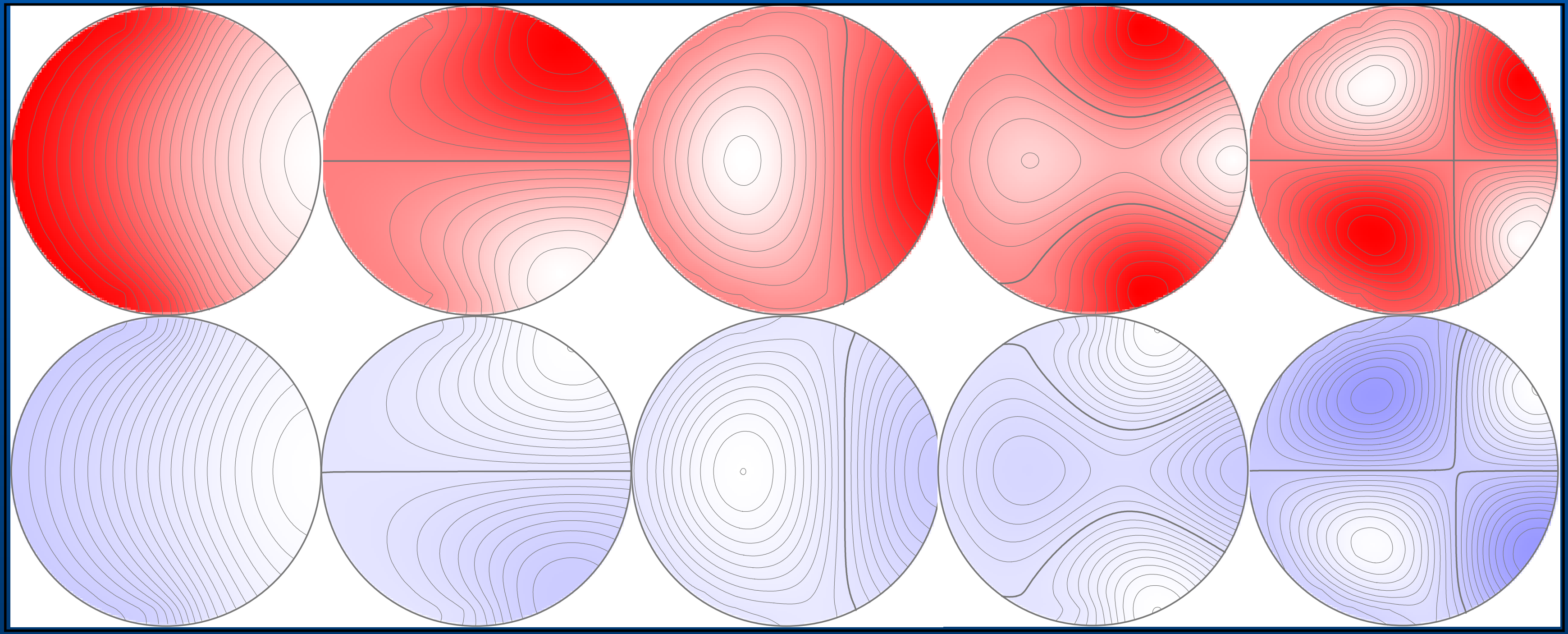
The spurious eigenvalues using different methods

	Complex-valued BEM	Real-part BEM	Imaginary-part BEM	MRM	FEM (ABAQUS)
Singular formulation	-	Y_n	J_n	$\overline{Y}_n$*	-
Hyperingular formulation	-	Y_n'	J_n'	$\overline{Y}_n'$	-

$$\overline{Y}_n = Y_n^*(kr) - \frac{2}{\pi}[\ln\frac{k}{2} + \gamma]J_n(kr)$$

The comparison for the former five eigenmodes

Real –part BEM



FEM

The comparison for the former five eigenvalues

	k_1	k_2	k_3	k_4	k_5
Real –part BEM	**1.222**	**2.544**	**2.954**	**3.802**	**4.231**
FEM(ABAQUS)	**1.254**	**2.593**	**2.934**	**3.842**	**4.194**

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We have successfully detected the spurious and true eigensolutions with the mixed-type B.C. by using five numerical methods.

The spurious eigenvalues depend on the representation no matter what the given types of B.C. for the problem are specified.

報告完畢

敬請指教