Reviewers' comments ASME Rev.

Comment on the paper
" Review of dual integral representations with emphasis on hypersingularity and divergent series"

Boundary Element Method (BEM) or Boundary Integral Equations Method is based on the integral representations for the solution of the boundary value problem or initial-boundary value problem. The first important step for BEM is to formulate the related boundary integral equations (BIE) which can be carried on different approaches. The same boundary value problem may be transformed into several forms of BIE. According to the way by which the BIE formed and to the meaning of the unknowns in the BIE, boundary element methods are usually classified into two groups: The direct and indirect method. In the direct method, the BIE used were primarily formulated from the Green's formula, the unknown boundary data are the actual physical variables of the problem. In engineering practice, the direct BIE are usually derived from weighted residual method or by its physical meaning, which are no other techniques than application of generalized Green's theorem. For example, in elastostatics, the BIE which is known as Somigliana's identity is obtained from Betti's theorem, from mathematical point of view, this is a special form of Green's theorem related to elastostatics. In the indirect method, the boundary integral formulation is based on the potential theory, the BIE derived from simple potential, double potential or their combination depends on the properties of potential and their derivation The unknowns in the indirect BIE are usually called density functions.

The direct and indirect methods are essentially equivalent by the following fact, If we introduce the complementary problem defined in the complement of the region, then consider the interior and the exterior boundary value problems simultaneously, we can establish the connections between the direct and indirect boundary integral formulations. There is no essential difference between them theoretically. What we care from mathematical point of view is whether the BIE is the first, or second kind or the type with hypersingularity which may be interpreted as the finite part integration in the sense of distribution and can be considered as pseudodifferential operators

I think that the so called 'dual integral representations' is a generalization and summation of all type of BIE, but lay stress on the form with hypersingularity which is derived by formally differentiation to simple or double layer potential. In fact, the original idea came from the applications of the continuous and discontinuous properties of the simple and double layer potential and their derivations when the field point approaching or passing through the boundary. These properties are classical results and the so called "dual integral equations" appeared in many literatures (there are known as Calderon projection), although they may have interpretations by modern mathematical tools.

Many researchers on BEM found and studied the four basic integral forms in "dual integral equations" and used to solve the problems with degenerate boundary such as crack, screen problems. They did not term the formulation "dual" because in numerical computing, only one or two proper integral forms is chosen to buildup the integral equation which is independent of the other forms in numerical treatment procedures. I am not sure if the term "dual integral representations" should be introduced necessarily, which is no other than the sum-up of the existing integral representations. However the paper is a complete review of the different admissible integral representations and their relations. Putting the focal point of the dual integral equations on the hypersingularity and divergent series is useful for the solution of the problems with geometrically degenerate boundary. By my knowledge, from 1970's, several mathematicians Nedelec, Hsiao, Kleinman, Wendland did a series studies on solving the boundary integrals with hypersingularity and applied to many fields. Feng Kang and Yu De-hao in their study on "canonical integral equations" or "natural boundary element method" focus on the solution of the integral equations with hypersingular kernel in which the Green's function instead of the fundamental solution is used. The paper lists many works on this subject so that the authors of the paper fit together the many scattered contributions into a comprehensive account. The summary in the paper on the regularization methods for the hypersingular integrals is thoughtful, which are the key techniques for implementation of BEM, of course, further researches are expected. In general, this paper is a good review

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