In the [R1], at the place before Eq. (3), the following statement was found : “ the series convergent to ln r since r is not zero”. In my comment, (1) “r is not zero” is true, however (2) “the series convergent to ln r” is questionable?

[R1] Journal: Journal of Mechanics Title: On the linkage between influence matrices in the BIEM and BEM to explain the mechanism of degenerate scale , by J. T. Chen

Comment for the statement “the series convergent to ln r”

In the paper [R1]. author introduced the following notations $z = \overrightarrow{AB}$, or $z = re^{i\gamma}$, and $\ln z = \ln\{re^{i\gamma}) = \ln r + i\gamma$ (Fig. 1). By using those notations, we can find

$$U(s,x) = \ln|x-s| = \ln r = \mathrm{Re}\{\ln z\} = \mathrm{Re}\{\ln((Re^{i\theta} - \rho e^{i\phi})\} \tag{1}$$

where

$$\ln((Re^{i\theta} - \rho e^{i\phi}) = \ln(Re^{i\theta}(1 - \frac{\rho}{R}e^{i(\theta-\phi)})) = \ln R + i\theta + \ln(1 - \frac{\rho}{R}e^{i(\theta-\phi)}) \tag{2}$$

$$\ln(1 - \frac{\rho}{R}e^{i(\theta-\phi)}) = -\sum_{m=1}^{\infty}\frac{1}{m}(\frac{\rho}{R})^m e^{im(\theta-\phi)}, \qquad (\text{for } \rho < R) \tag{3}$$

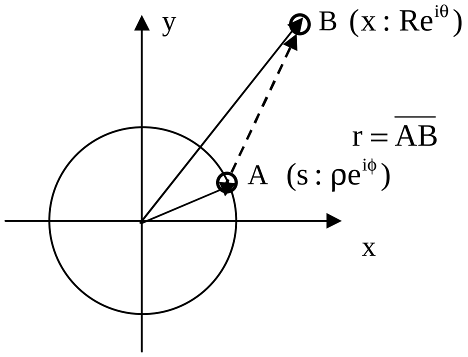

Fig. 1

From Eqs. (1), (2) and (3) we have

$$U(s,x) = \ln|x-s| = \ln r = \ln R - \sum_{m=1}^{\infty}\frac{1}{m}(\frac{\rho}{R})^m \cos m(\theta-\phi) \quad (\text{for } \rho < R) \tag{4}$$

Assume Eq. (4) is still valid for the case of $R = \rho$, and let $\beta = \theta - \phi$, from Eq. (4) we have

$$\ln r = \ln R - \sum_{m=1}^{\infty}\frac{1}{m}\cos m\beta \quad (\text{for } \rho = R) \tag{5}$$

Clearly, we have (Fig.2)

$$r = 2R\sin\frac{\beta}{2}, \qquad \ln r = \ln 2 + \ln R + \ln(\sin\frac{\beta}{2}) \tag{6}$$

Substituting Eq. (6) into (5) yields

$$\ln 2 + \ln(\sin\frac{\beta}{2}) = -\sum_{m=1}^{\infty}\frac{1}{m}\cos m\beta \quad (\text{for} \ \ \rho = R) \tag{7}$$

Or

$$\ln(\sin\frac{\beta}{2}) = -\ln 2 - \sum_{m=1}^{\infty}\frac{1}{m}\cos m\beta \quad (\text{for} \ \ \rho = R) \tag{8}$$

We can prove that the equality shown by Eq. (8) is generally impossible. In fact, for the case of $-2\pi < \beta < 0$, $\sin\frac{\beta}{2} = -h$ (h-positive, or $h = \left|\sin\frac{\beta}{2}\right|$). Thus, we have

$$\ln(\sin\frac{\beta}{2}) = \ln(-h) = \ln(he^{(2N+1)\pi i}) = \ln h + (2N+1)\pi i \tag{9}$$

Substituting Eq. (9) into (8) yields

$$\ln h + (2N+1)\pi i = -\ln 2 - \sum_{m=1}^{\infty}\frac{1}{m}\cos m\beta \quad (\text{for} \ \ \rho = R \text{ case}) \tag{10}$$

Since the term $(2N+1)\pi i$ is involved in the left hand side of Eq. (10), the equality shown by Eq. (10) is impossible.

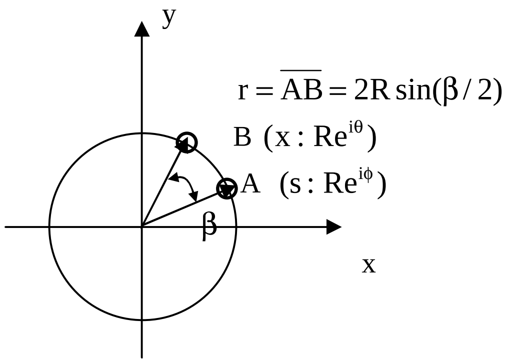

Fig. 2