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Dual integral formulation for determining the acoustic modes of a two-dimensional cavity with a degenerate boundary

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In this paper, the dual integral formulation for the Helmholtz equation used in solving the acoustic modes of a two-dimensional cavity with a degenerate boundary is derived. All the improper integrals for the kernel functions in the dual integral equations are reformulated into regular integrals by integrating by parts and are calculated by means of the Gaussian quadrature rule. The jump properties for the single layer potential, double layer potential and their directional derivatives are examined and the potential distributions are shown. To demonstrate the validity of the present formulation, the acoustic frequencies and acoustic modes of the two-dimensional cavity with an incomplete partition are determined by the developed dual BEM program. Also, the numerical results are compared with those of the ABAQUS program, FEM by Petyt and the dual multiple reciprocity method. Good agreement between the present formulation and measurements by Petyt is also shown. © 1998 Elsevier Science Ltd. All rights reserved

1 INTRODUCTION

The hypersingular integral equation was first formulated by Hadamard[1] to treat the cylindrical wave equation by spherical means of descent. In the meantime, Mangler derived the same mathematical form in solving the thin airfoil problem.[2] The improper integral was then defined by Tuck[3] as the 'Hadamard principal value'. In aerodynamics, it was termed the 'Mangler's principal value'.[2,4]. Such a non-integrable integral naturally arises in the dual integral formulation especially for problems with a degenerate boundary, e.g. crack problems in elasticity,[5–10] heat flow through a baffle,[11] Darcy flow around a cut-off wall,[12,13] the aerodynamic problem of a thin airfoil[4] and acoustic waves impinging on a screen.[14,15] The dual formulation also plays an important role in some other problems, e.g. the corner problem,[16] adaptive BEM,[17] and the exterior problem.[18] A general application of the hypersingular integral equation in mechanics was discussed in Ref. [19], and a review lecture on recent development of dual BEM was presented in Ref. [20]. Combining the conventional integral equation, e.g. Green's Identity or Somigliana Identity, with the hypersingular integral equation, we call the two equations 'dual integral equations' owing to the presence of the pair of continuous and discontinuous properties of the potential as the field point moves across the boundary.[21–23] From the above point of view, the definition of the dual integral equations is quite different from the conventional one used in crack elastodynamics by Buecker.[24] The dual equations in the present paper are independent with respect to each other for the undetermined coefficients of the complementary solution. The dual integral equations defined by Buecker resulted from the same equation, but by collocating different points. The present formulation totally has four kernel functions, which make possible a unified theory encompassing different schemes, various derivations and interpretations. For elasticity, a detailed derivation can be found in Ref. [5]. The singularity order of hypersingularity for the kernel in the normal derivative of the double layer potential is stronger than that of the Cauchy-type kernel by one. The paradox of the non-integrable kernel is introduced owing to the illegal change of the integral and trace operators from the point of view of the dual integral formulation.[21] In order to ensure a finite value, the Leibnitz rule should be considered as the derivative of Cauchy principal value ($C.P.V.$) so that the boundary term $2/\epsilon$ can be included to compensate for the minus infinity. In the literature, many researchers have paid attention to regularization

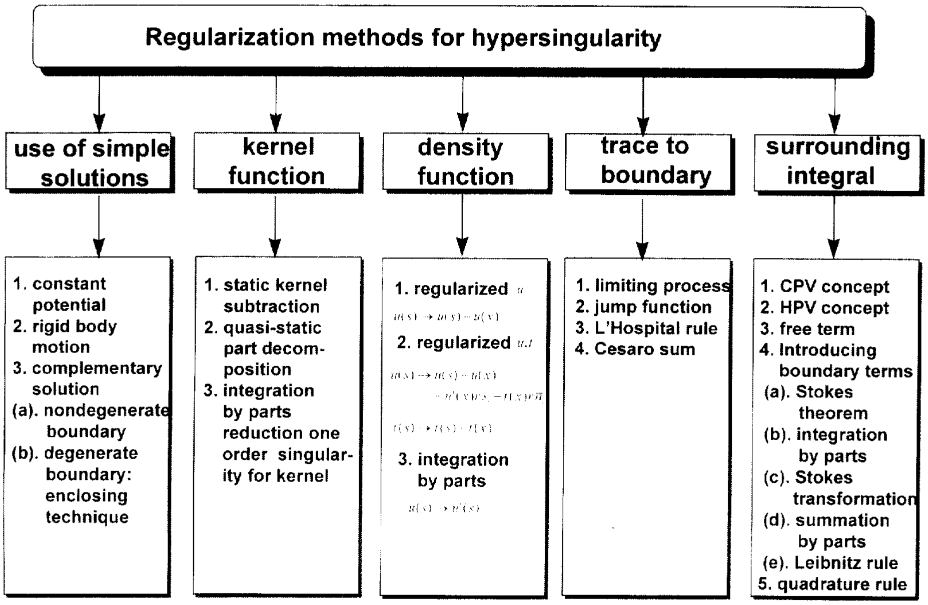


Fig. 1. Regularization methods for hypersingularity.

techniques[25] for hypersingularity and nearly hypersingular integrals. The available techniques are summarized in Fig. 1. Therefore, the value for the finite part can be determined by means of regularization techniques. Based on the theory of dual integral equations, the dual boundary element method can be implemented.[9,10] The dual integral representation for the Laplace equation was proposed in Ref. [22]. In the same way, the acoustic problem with a degenerate boundary also requires the dual integral formulation. In the literature, a large number of papers have focused on the non-physical solution for the exterior problem of the Helmholtz equation by using the integral equation method. Burton and Miller[26] first combined dual integral equations to deal with fictitious eigenvalues. Furthermore, the multiple reciprocity method (MRM) also encounters spurious eigenvalues for the interior problem of the Helmholtz equation.[27,32] Both cases, the exterior problem by BEM and the interior problem by MRM, have problems with non-uniqueness for the solution. However, for the interior problem with a degenerate boundary, conventional BEM also results in a singular system, and the problem of non-uniqueness also occurs. Terai[14] and Wu and Wan[15] solved the three-dimensional acoustic problem with a screen by using the dual integral formulation. To the authors' best knowledge, a detailed derivation on a two-dimensional acoustic cavity with an incomplete partition has not been found, although FEM results are readily available.[28,29]

In this paper, we extend the concept of the dual integral formulation for the Laplace equation[22] to the two-dimensional Helmholtz equation and examine the potential properties of the four kernel functions. After discretizing the dual integral equations, all the improper integrals are transformed into regular integrals and are calculated using the Gaussian quadrature rule. The transcendental eigenequation is constructed, and the eigenvalues are solved using the direct search method. A general dual BEM program is implemented to solve the acoustic frequencies and acoustic modes for an arbitrary two-dimensional cavity with or without incomplete partitions. An illustrative problem for the acoustic modes of a cavity with an incomplete partition is solved to show the validity of the present formulation for the acoustic problem with a degenerate boundary. Results are compared with other numerical methods and experiment data.

2 DUAL INTEGRAL FORMULATION FOR AN ACOUSTIC PROBLEM WITH A DEGENERATE BOUNDARY

Consider an acoustic problem which has the following governing equation:

$$\nabla^2\phi(x) + k^2\phi(x) = 0, \quad x \text{ in } D \tag{1}$$

where D is the domain of interest, x is the domain point, ϕ is the acoustic pressure and k is the wave number defined by the angular frequency divided by the sound speed. The homogeneous boundary conditions are shown as follows:

$$\phi(x) = f(x), \quad x \text{ on } B_1 \tag{2}$$

$$\frac{\partial\phi(x)}{\partial n_x} = g(x), \quad x \text{ on } B_2 \tag{3}$$

where B_1 is the essential boundary in which the acoustic pressure is prescribed, B_2 is the natural boundary where the normal derivative of the acoustic pressure in the n_x

direction is specified, and B_1 and B_2 construct the whole boundary of the domain D.

The first equation of the dual boundary integral equations for the domain point can be derived from Green's third identity:

$$2\pi\phi(x) = \int_B T(s, x)\phi(s)\,\mathrm{d}B(s) - \int_B U(s, x)\frac{\partial\phi(s)}{\partial n_s}\mathrm{d}B(s), \quad x \in D \tag{4}$$

where $T(s, x)$ is defined by

$$T(s, x) \equiv \frac{\partial U(s, x)}{\partial n_s} \tag{5}$$

in which n_s is the outnormal direction at the boundary point s, and $U(s, x)$ is the fundamental solution which satisfies

$$\nabla^2 U(x, s) + k^2 U(x, s) = \delta(x - s), \quad x \in D \tag{6}$$

In eqn (6), $\delta(x - s)$ is the Dirac-delta function. After taking the normal derivative with respect to eqn (4), the second equation of the dual boundary integral equations for the domain point can be derived:

$$2\pi\frac{\partial\phi(x)}{\partial n_x} = \int_B M(s, x)\phi(s)\,\mathrm{d}B(s) - \int_B L(s, x)\frac{\partial\phi(s)}{\partial n_s}\mathrm{d}B(s), \quad x \in D \tag{7}$$

where

$$L(s, x) \equiv \frac{\partial U(s, x)}{\partial n_x} \tag{8}$$

$$M(s, x) \equiv \frac{\partial^2 U(s, x)}{\partial n_x \partial n_s} \tag{9}$$

in which n_x and n_s represent the outnormal directions of x and s respectively. The explicit forms for the four kernel functions are shown in Table 1. By moving the field point x in eqns (4) and (7) to the boundary, the dual boundary integral equations for the boundary point can be obtained as follows:

$$\pi\phi(x) = C.P.V. \int_B T(s, x)\phi(s)\,\mathrm{d}B(s) - R.P.V. \int_B U(s, x)\frac{\partial\phi(s)}{\partial n_s}\mathrm{d}B(s), \quad x \in B \tag{10}$$

$$\pi\frac{\partial\phi(x)}{\partial n_x} = H.P.V. \int_B M(s, x)\phi(s)\,\mathrm{d}B(s) - C.P.V. \int_B L(s, x)\frac{\partial\phi(s)}{\partial n_s}\mathrm{d}B(s), \quad x \in B \tag{11}$$

where *R.P.V.* is the Riemann principal value and *H.P.V.* is the Hadamard (Mangler) principal value.

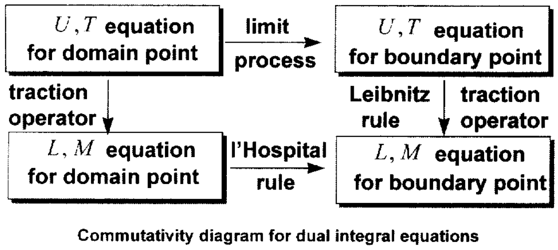


Fig. 2. Commutativity for the derivation of hypersingularity.

It must be noted that eqn (11) can be derived simply by applying a normal derivative operator with respect to eqn (10). Differentiation of the *C.P.V.* should be carried out carefully using Leibnitz's rule. The commutative property provides us with two alternatives for calculating the H.P.V., as shown in Fig. 2, in the same way used for crack problems.[5] For the problem including a normal boundary S and degenerate boundary $C^+ + C^-$, i.e. $B = S + C^+ + C^-$, eqns (10) and (11) can be reformulated as follows:For $x \in$ S, eqns (10) and (11) become

$$\pi\phi(x) = C.P.V. \int_S T(s, x)\phi(s)\,\mathrm{d}B(s) - R.P.V. \int_S U(s, x)\frac{\partial\phi(s)}{\partial n_s}\mathrm{d}B(s) + \int_{C^+} T(s, x)\Delta\phi(s)\,\mathrm{d}B(s) - \int_{C^+} U(s, x)\sum\frac{\partial\phi(s)}{\partial n_s}\mathrm{d}B(s) \tag{12}$$

$$\pi\frac{\partial\phi(x)}{\partial n_x} = H.P.V. \int_S M(s, x)\phi(s)\,\mathrm{d}B(s) - C.P.V. \int_S L(s, x)\frac{\partial\phi(s)}{\partial n_s}\mathrm{d}B(s) + \int_{C^+} M(s, x)\Delta\phi(s)\,\mathrm{d}B(s) - \int_{C^+} L(s, x)\sum\frac{\partial\phi(s)}{\partial n_s}\mathrm{d}B(s) \tag{13}$$

where

$$\Delta\phi(s) = \phi(s^+) - \phi(s^-) \tag{14}$$

$$\sum\frac{\partial\phi}{\partial n}(s) = \frac{\partial\phi}{\partial n}(s^+) + \frac{\partial\phi}{\partial n}(s^-) \tag{15}$$

For $x \in C^+$, eqns (10) and (11) reduce to

$$\pi\sum\phi(x) = C.P.V. \int_{C^+} T(s, x)\Delta\phi(s)\,\mathrm{d}B(s) - R.P.V. \int_{C^+} U(s, x)\sum\frac{\partial\phi(s)}{\partial n_s}\mathrm{d}B(s) + \int_S T(s, x)\phi(s)\,\mathrm{d}B(s) - \int_S U(s, x)\frac{\partial\phi(s)}{\partial n_s}\mathrm{d}B(s) \tag{16}$$

Table 1. The explicit form of kernel functions for the two-dimensional Helmholtz equation.

Kernel function	$U(s,x)$	$T(s,x)$	$L(s,x)$	$M(s,x)$
Order of singularity	weak ($\ln(r)$)	strong ($1/r$)	strong ($1/r$)	hypersingular ($1/r^2$)
Symmetry	$U(x,s)$	$L(x,s)$	$T(x,s)$	$M(x,s)$
Two-dimensional case	$\frac{-i\pi H_0^1(kr)}{2}$	$\frac{-ik\pi}{2}H_1^1(kr)\frac{y_i n_i}{r}$	$\frac{ik\pi}{2}H_1^1(kr)\frac{y_i \bar{n}_i}{r}$	$\frac{-ik\pi}{2}\left[-k\frac{H_2^1(kr)}{r^2}y_i y_j n_i \bar{n}_j+\frac{H_1^1(kr)}{r}n_i\bar{n}_i\right]$
Remark	$r^2=y_i y_i$	$n_i=n_i(s)$	$\bar{n}_i=n_i(x)$	$y_i=x_i-s_i$

$H_m^1(kr)$ denotes the first kind of mth-order Hankel function.

$$\pi\Delta\frac{\partial\phi(x)}{\partial n_x}=H.P.V.\int_{C^+}M(s,x)\Delta\phi(s)\,dB(s)-C.P.V.\int_{C^+}L(s,x)\sum\frac{\partial\phi(s)}{\partial n_s}dB(s)+\int_S M(s,x)\phi(s)\,dB(s)-\int_S L(s,x)\frac{\partial\phi(s)}{\partial n_s}dB(s)\tag{17}$$

where

$$\sum\phi(x)=\phi(x^+)+\phi(x^-)\tag{18}$$

$$\Delta\frac{\partial\phi}{\partial n}(x)=\frac{\partial\phi}{\partial n}(x^+)-\frac{\partial\phi}{\partial n}(x^-)\tag{19}$$

eqns (14), (15), (18) and (19) indicate that the unknowns on the degenerate boundary double, and that the additional hypersingular integral equation, eqn (17), is correspondingly necessary; i.e. the dual boundary integral equations can provide us with sufficient constraint relations for the doubled boundary unknowns on the degenerate boundary.

3 ON THE FOUR KERNEL FUNCTIONS AND THEIR POTENTIALS

The four kernel functions, $U(s,x)$, $T(s,x)$, $L(s,x)$ and $M(s,x)$, in the dual integral equations have different orders of singularity when x approaches s. The order of singularity and the symmetry properties for the four kernel functions are shown in Table 1. The continuous properties of the potentials across the boundary resulting from the four kernel functions are summarized in Table 2. In Table 2, not only the normal derivatives for the single- and double-layer potentials, but also the tangential derivatives are considered. For the regular elements, no special treatment is needed since the Gaussian quadrature rule can be employed. Without loss of generality, the four improper integrals for the singular elements obtained by using the constant element scheme after coordinate transformation in Fig. 3 can be formulated into the following regular integrals:

(1) $U(s,x)$ kernel:

$$\begin{aligned}U_{ii}&=\frac{-i\pi}{2}\lim_{\epsilon\to 0}\int_{-0.5l}^{0.5l}H_0^1(k\sqrt{s^2+\epsilon^2})\,ds\\&=\frac{-i\pi}{2}\lim_{\epsilon\to 0}\left[\int_{-0.5l}^{-\sqrt{\epsilon}}H_0^1(k|s|)\,ds+\int_{-\sqrt{\epsilon}}^{\sqrt{\epsilon}}i\frac{2}{\pi}\ln\left(\frac{k}{2}\sqrt{s^2+\epsilon^2}\right)ds+\int_{\sqrt{\epsilon}}^{0.5l}H_0^1(ks)\,ds\right]\\&=\frac{-i\pi}{2}\lim_{\epsilon\to 0}\left[\int_{-0.5l}^{-\sqrt{\epsilon}}H_0^1(k|s|)\,ds+0+\int_{\sqrt{\epsilon}}^{0.5l}H_0^1(ks)\,ds\right]\\&=\frac{-i\pi}{2}\left[H_0^1\left(\frac{kl}{2}\right)l+k\int_{-0.5l}^{0.5l}H_1^1(k|s|)|s|\,ds\right]\end{aligned}$$

(i no sum) (20)

Table 2. The properties of the single potential, the double layer potential and their directional derivatives

Kernel function $K(s,x)$	$U(s,x)$	$T(s,x)$	$L(s,x)$	$M(s,x)$	$L'(s,x)$	$M'(s,x)$
Density function $\mu(s)$	$\partial\phi/\partial n$	ϕ	$\partial\phi/\partial n$	ϕ	$\partial\phi/\partial n$	ϕ
Potential type $\int K(s,x)\mu(s)\,ds$	single layer	double layer	normal derivative of single layer potential	normal derivative of double layer potential	tangential derivative of single layer potential	tangential derivative of double layer potential
Continuity across boundary	continuous	discontinuous	discontinuous	psuedo continuous	continuous	discontinuous
Jump value	no jump	$2\pi\phi$	$-2\pi(\partial\phi/\partial n)$	no jump	no jump	$2\pi\frac{\partial\phi}{\partial t}$

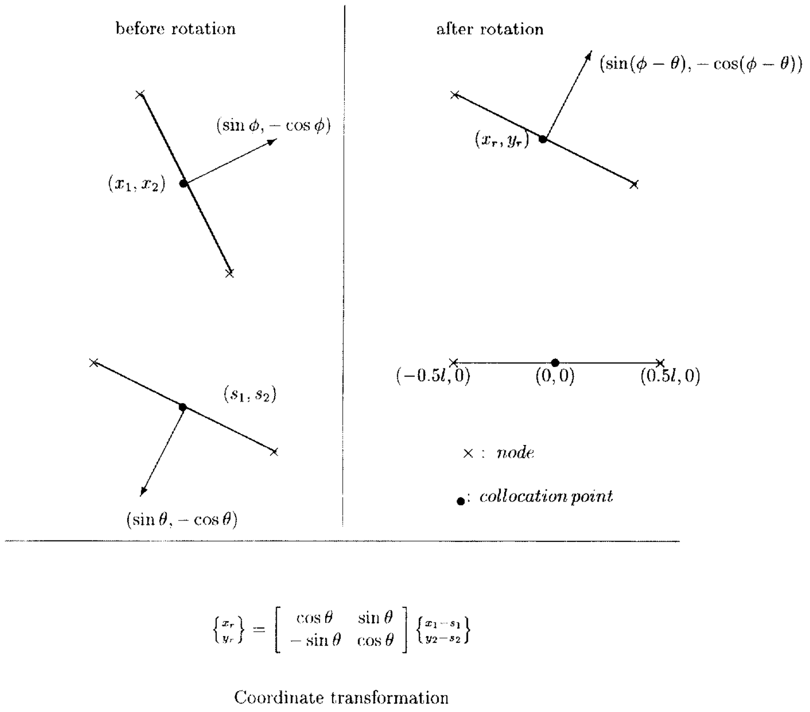


Fig. 3. Coordinate transformation for improper integrals.

where $H_0^1(ks)$ is the first kind of the zeroth-order Hankel function, l is the element length and the coordinate of the collocation point is (0, 0).

(2) $T(s, x)$ kernel:

$$\begin{aligned} T_{ii} &= \frac{i\pi k}{2}\lim_{\epsilon\to 0}\int_{-0.5l}^{0.5l} H_1^1(k\sqrt{s^2+\epsilon^2})\,\frac{\epsilon}{\sqrt{s^2+\epsilon^2}}\,ds \\ &= \frac{i\pi k}{2}\lim_{\epsilon\to 0}\int_{-\sqrt[4]{\epsilon}}^{\sqrt[4]{\epsilon}} \frac{i(-2)}{\pi k\sqrt{s^2+\epsilon^2}}\,\frac{\epsilon}{\sqrt{s^2+\epsilon^2}}\,ds \\ &= \lim_{\epsilon\to 0}\arctan.\frac{s}{\epsilon}\Bigg|_{-\sqrt[4]{\epsilon}}^{\sqrt[4]{\epsilon}} \\ &= \pi \ (\mathbf{i}\text{ no sum}) \end{aligned} \tag{21}$$

where $H_1^1(ks)$ is the first kind of the first-order Hankel function.

(3) $L(s, x)$ kernel:

$$\begin{aligned} L_{ii} &= \frac{i\pi k}{2}\lim_{\epsilon\to 0}\int_{-0.5l}^{0.5l} H_1^1(k\sqrt{s^2+\epsilon^2})\,\frac{-\epsilon}{\sqrt{s^2+\epsilon^2}}\,ds \\ &= \lim_{\epsilon\to 0}\frac{-i\pi k}{2}\int_{-\sqrt[4]{\epsilon}}^{\sqrt[4]{\epsilon}} \frac{i(-2)}{\pi k\sqrt{s^2+\epsilon^2}}\,\frac{\epsilon}{\sqrt{s^2+\epsilon^2}}\,ds \\ &= \pi \ (\mathbf{i}\text{ no sum}) \end{aligned} \tag{22}$$

(4) $M(s, x)$ kernel:

$$\begin{aligned} M_{ii} &= \frac{-i\pi k}{2}\lim_{\epsilon\to 0}\int_{-0.5l}^{0.5l}\left\{-k\frac{H_2^1(k\sqrt{s^2+\epsilon^2})}{s^2+\epsilon^2}(-\epsilon)(-\epsilon) + \frac{H_1^1(k\sqrt{s^2+\epsilon^2})}{\sqrt{s^2+\epsilon^2}}\,ds\right\} \\ &= \frac{-i\pi k}{2}\left\{-2H_1^1\left(\frac{kl}{2}\right) + k\left[H_0^1\left(\frac{kl}{2}\right) + k\int_{-0.5l}^{0.5l} H_1^1(k|s|)|s|\,ds\right]\right\} \ (\mathbf{i}\text{ no sum}) \end{aligned} \tag{23}$$

where $H_1^2(ks)$ is the first kind of the second-order Hankel function. After the above manipulations, the improper integrals, including weak ($U(s, x)$), strong ($T(s, x)$, $L(s, x)$) and superstrong ($M(s, x)$) singularities, are reduced to regular integrals and can be calculated using the Gaussian quadrature rule.

The potentials of the six kernel functions in Table 2, $U(s, x)$, $T(s, x)$, $L(s, x)$, $M(s, x)$, $L^t(s, x)$ and $M^t(s, x)$, induced by the constant singularity source distributed along the boundary from $s = (-0.5, 0)$ to $s = (0.5, 0)$ are shown in Figs 4–6 for different values of $k = 0.01$, 1 and 2 respectively. The behavior of the single layer potential ($U(s, x)$ kernel), the double layer potential ($T(s, x)$ kernel), the normal derivative of the single layer potential ($L^n(s, x)$ kernel), the normal derivative of the double layer potential

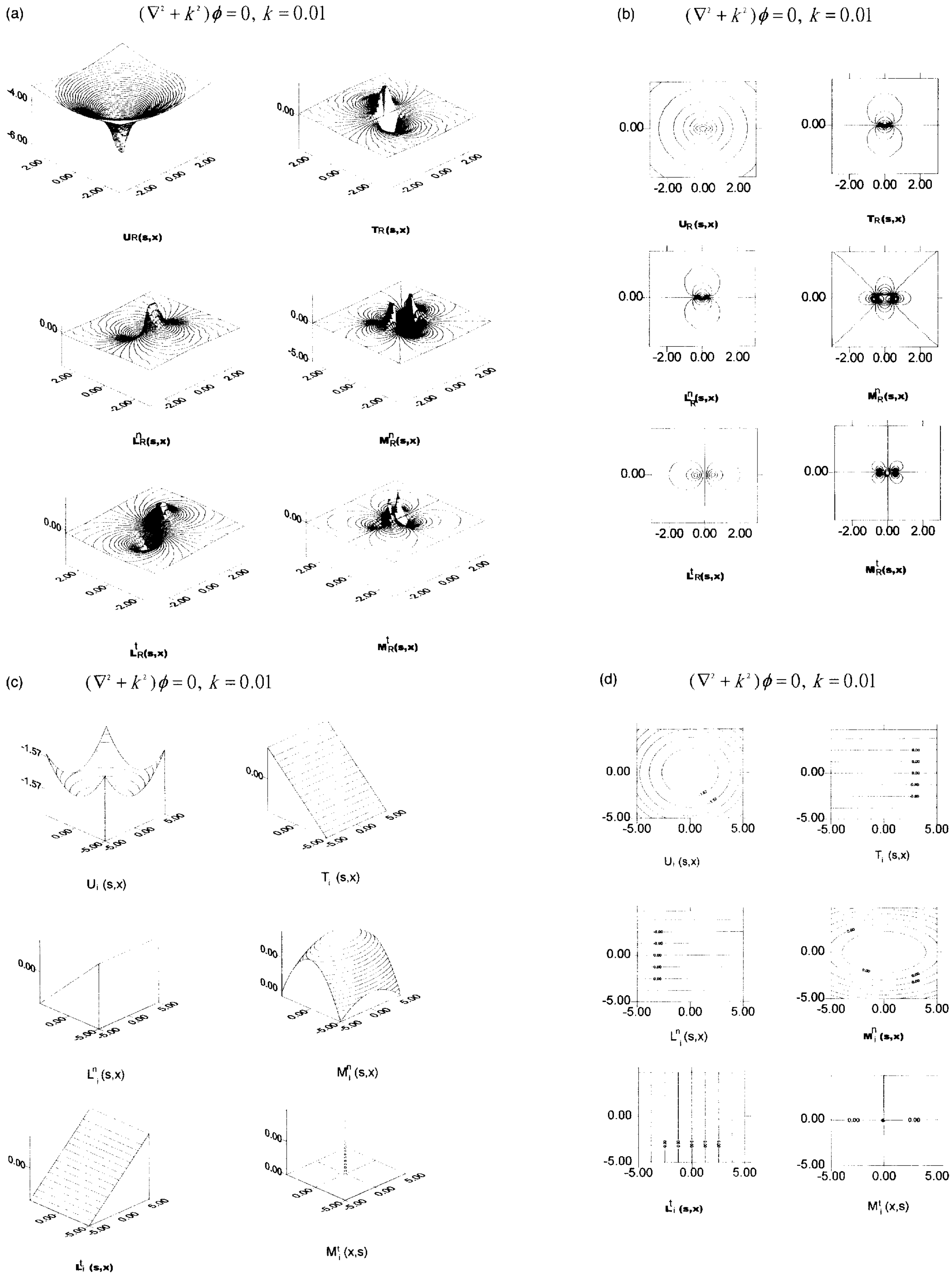


Fig. 4. (a) Pressure distributions of the real potentials resulting from the six kernel functions for the case of $k = 0.01$. (b) Pressure contour of the real potentials resulting from the six kernel functions for the case of $k = 0.01$. (c) Pressure distributions of the imaginary potentials resulting from the six kernel functions for the case of $k = 0.01$. (d) Pressure contour of the imaginary potentials resulting from the six kernel functions for the case of $k = 0.01$.

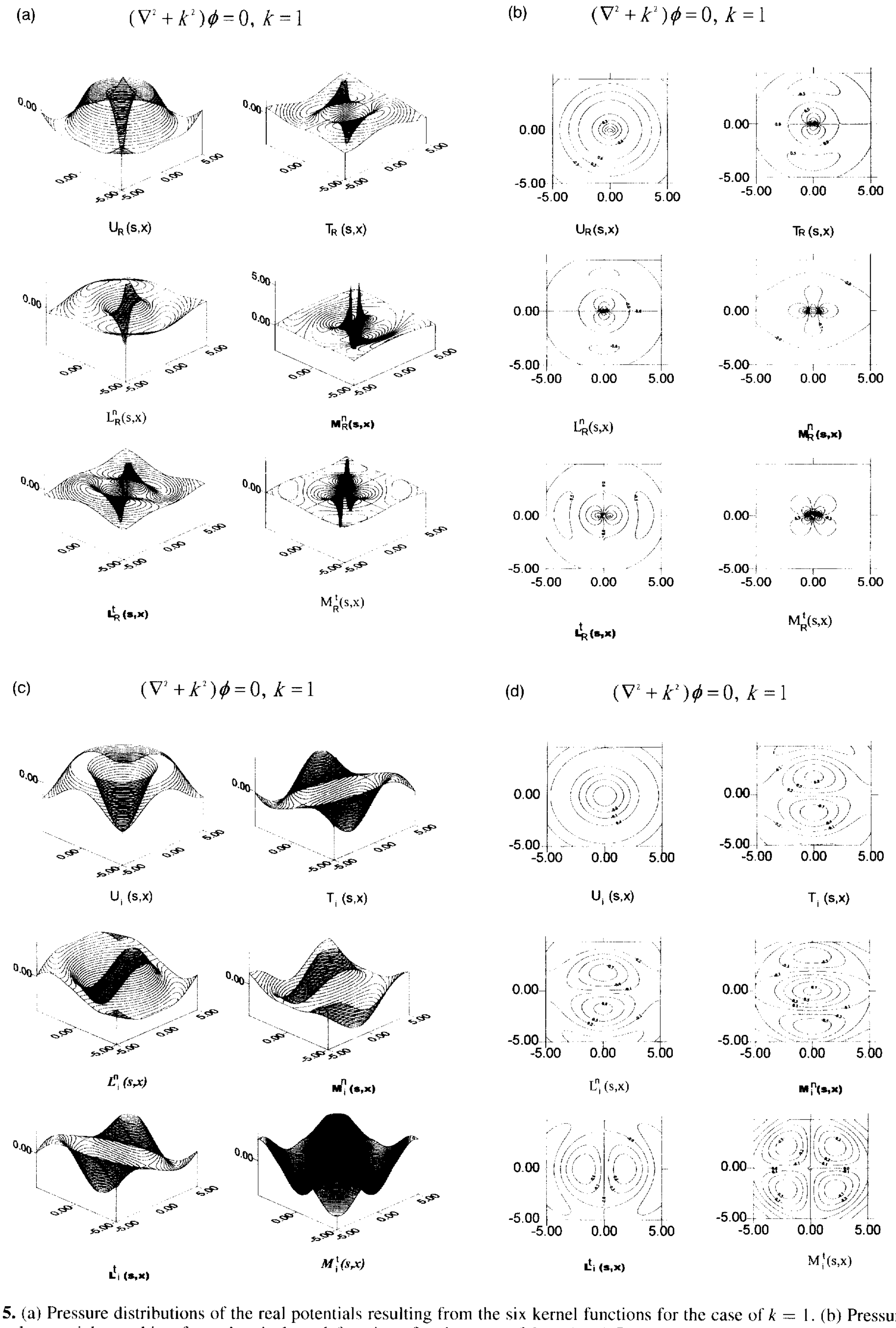


Fig. 5. (a) Pressure distributions of the real potentials resulting from the six kernel functions for the case of $k = 1$. (b) Pressure contour of the real potentials resulting from the six kernel functions for the case of $k = 1$. (c) Pressure distributions of the imaginary potentials resulting from the six kernel functions for the case of $k = 1$. (d) Pressure contour of the imaginary potentials resulting from the six kernel functions for the case of $k = 1$.

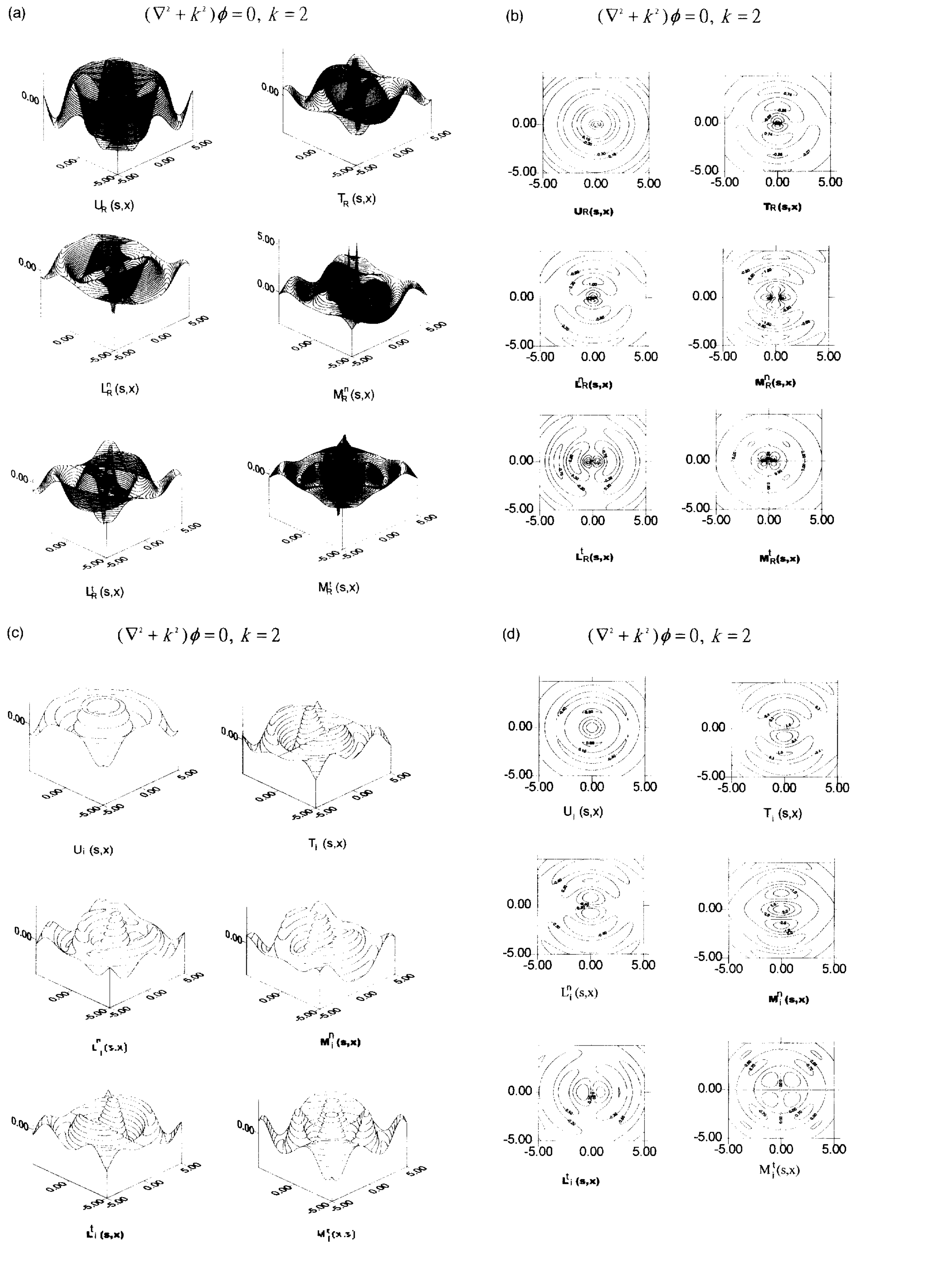


Fig. 6. (a) Pressure distributions of the real potentials resulting from the six kernel functions for the case of $k = 2$. (b) Pressure contour of the real potentials resulting from the six kernel functions for the case of $k = 2$. (c) Pressure distributions of the imaginary potentials resulting from the six kernel functions for the case of $k = 2$. (d) Pressure contour of the imaginary potentials resulting from the six kernel functions for the case of $k = 2$.

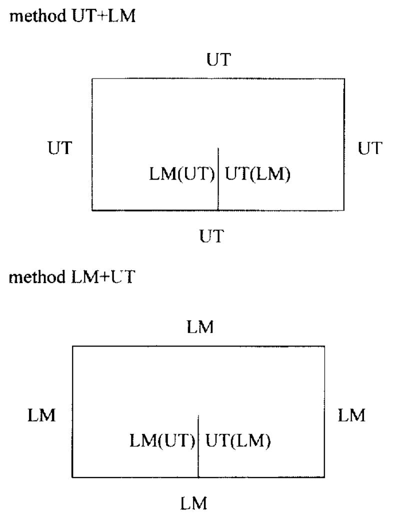
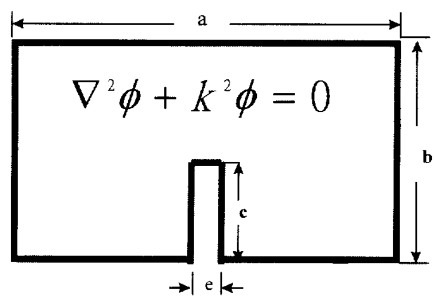


Fig. 7. Two alternative approaches.

($M^n(s, x)$ kernel), the tangential derivative of the single layer potential ($L^t(s, x)$ kernel) and the tangential derivative of the double layer potential ($M^t(s, x)$ kernel) are shown in the figures. The subscript indexes of 'R' and 'i' in the kernels represent the real part and imaginary parts respectively. Both the real part and imaginary part are included in the cases of (a,b) and (c,d) respectively. It is found that in the case of Fig. 4(a) and Fig. 4(b) for $k = 0.01$, the asymptotic behavior of the real part of the kernels for the Helmholtz equation is similar to that of the Laplace equation in Refs [21,22] as expected. The continuous behaviors of the single layer potential ($U(s, x)$ kernel) and the normal derivative of the double layer potential ($M(s, x)$ kernel) are displayed in this figure. The jump behaviors across the boundary along $s = (-0.5, 0)$ to $s = (0.5, 0)$ can be observed for the double layer potential ($T(s, x)$ kernel) and the normal derivative of the single layer potential ($L(s, x)$ kernel). Also, the dipole and quadrupole source structures can be found. Based on the singular solutions, the strength of the singularity can be determined by satisfying the boundary conditions.



a=0.112, b=0.236, c=0.056 m, e=0

Fig. 8. A cavity with an incomplete partition.

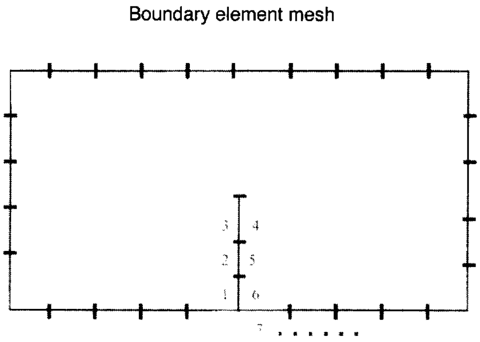


Fig. 9. The boundary element mesh.

4 EIGENEQUATION FOR THE ACOUSTIC CAVITY WITH A DEGENERATE BOUNDARY

For simplicity, the Neumann problem is considered in this paper. After determining the influence coefficients, we can obtain the transcendental equation as follows:

$$[\bar{\bar{T}}_{ij}(k)]\{\phi_j\} = 0$$

$$[M_{ij}(k)]\{\phi_j\} = 0$$

where $\{\phi_j\}$ is the boundary mode of acoustic pressure and

$$T_{ij}(k) = -2\pi\delta_{ij} + \int_{B_j} T(s_j, x_i)\, dB(s_j)$$

$$M_{ij}(k) = \int_{B_j} M(s_j, x_i)\, dB(s_j)$$

in which B_j denotes the jth boundary element and the eigenvalue k is imbedded in the elements of the matrix. After combining the dual equations on the degenerate boundary when x collocates on C^+ or C^-, the non-trivial eigensolution only exists when the determinant of the influence matrix is zero. Since either one of the two equations, *UT*

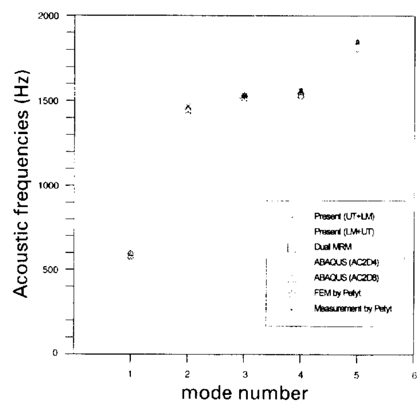


Fig. 10. Comparisons of the acoustic frequencies for the former five modes using dual BEM, dual MRM, FEM by Petyt, FEM by the ABAQUS program and experiments.

Table 3. The former five acoustic frequencies (Hz) by dual BEM, the other numerical methods and experiments

	Mode 1	Mode 2	Mode 3	Mode 4	Mode 5
Present (*UT* + *LM*)	584	1439	1518	1537	1818
Present (*LM* + *UT*)	584	1439	1518	1534	1818
Dual MRM	577	1444	1529	1534	NA
ABAQUS (AC2D4)	618	1421	1496	1527	1780
ABAQUS (AC2D8)	605	1458	1536	1563	1851
FEM by Petyt	591	1478	1540	1570	1861
Measurement by Petyt	570	1470	1534	1555	1840

or *LM*, for the outer boundary *S* can be selected, two alternative approaches in Fig. 7, *UT* + *LM* and *LM* + *UT*, are proposed as follows.

The *UT* + *LM* method has the eigenequation

$$\begin{bmatrix} T_{i_S j_S} & T_{i_S j_{C^+}} & T_{i_S j_{C^-}} \\ T_{i_{C^+} j_S} & T_{i_{C^+} j_{C^+}} & T_{i_{C^+} j_{C^-}} \\ M_{i_{C^+} j_S} & M_{i_{C^+} j_{C^+}} & M_{i_{C^+} j_{C^-}} \end{bmatrix} \begin{Bmatrix} \phi_{j_S} \\ \phi_{j_{C^+}} \\ \phi_{j_{C^-}} \end{Bmatrix} = \{0\} \tag{26}$$

where i_S and i_{C^+} denote the collocation points on the *S* and C^+ boundaries respectively, and j_S and j_{C^+} denote the element ID on the *S* and C^+ boundaries respectively.

The *LM* + *UT* method has the eigenequation

$$\begin{bmatrix} M_{i_S j_S} & M_{i_S j_{C^+}} & M_{i_S j_{C^-}} \\ T_{i_{C^+} j_S} & T_{i_{C^+} j_{C^+}} & T_{i_{C^+} j_{C^-}} \\ M_{i_{C^+} j_S} & M_{i_{C^+} j_{C^+}} & M_{i_{C^+} j_{C^-}} \end{bmatrix} \begin{Bmatrix} \phi_{j_S} \\ \phi_{j_{C^+}} \\ \phi_{j_{C^-}} \end{Bmatrix} = \{0\} \tag{27}$$

To solve for the eigenequation, a direct search method is employed.

5 AN ILLUSTRATIVE EXAMPLE

To demonstrate the validity of the dual integral formulation,

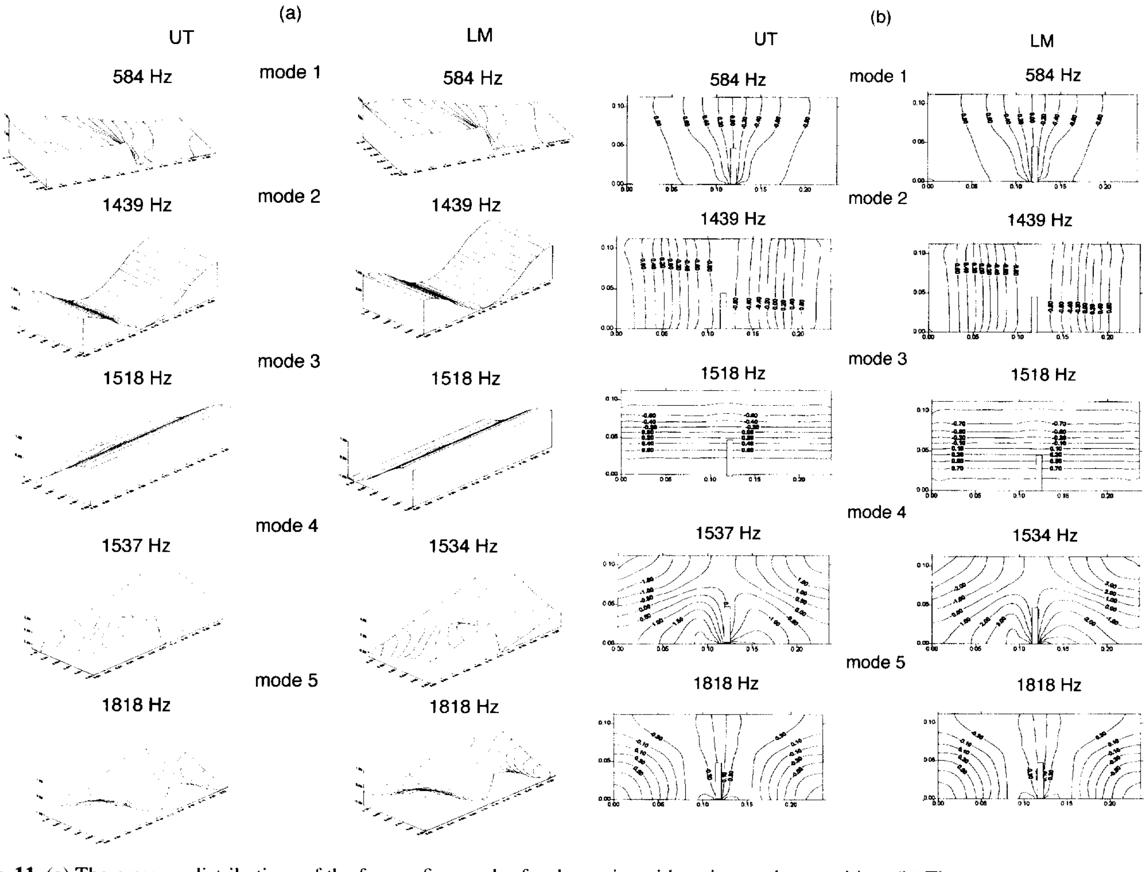


Fig. 11. (a) The pressure distributions of the former five modes for the cavity with an incomplete partition. (b) The pressure contours of the former five modes for the cavity with an incomplete partition.

an example given by Petyt[28,29] is considered. A two-dimensional cavity enclosed by rigid walls is shown in Fig. 8. The cavity is a rectangle, 236 mm long and 113 mm high, and contains a rigid partition located halfway along the longer side of the cavity. The thickness of the partition is modeled as zero thickness; i.e. the boundary of partition is degenerate. The partition extends from one side of the cavity halfway across to the other wall. The cavity is filled with an acoustic fluid whose density is 1.0 kg m^{-3} and whose bulk modulus is 0.1183 MPa. The boundary element mesh is shown in Fig. 9. The former five acoustic frequencies given in Table 3 were solved using dual BEM, and the results were compared with those of dual MRM,[30] the ABAQUS program[30,31] and FEM by Petyt.[28,29] Two types of element in the ABAQUS program, AC2D4 and AC2D8, were considered. As shown in Fig. 7, two methods, $UT + LM$ and $LM + UT$, were employed. There was fairly close agreement between the two solutions and all the other results as shown in Fig. 10. It was also found that the two methods match very well, as shown in Table 3 and Fig. 11(a) and Fig. 11(b). Although no mesh convergence studies have been performed, the close agreement between the acoustic frequencies and the acoustic modes of the present results in coarse mesh and those given by Petyt and coworkers[28,29] suggest that the mesh is adequate. Also, the pressure distributions and contour plot for the acoustic modes are shown in Fig. 11(a) and Fig. 11(b) respectively. The present results are also in better agreement with the experimental data obtained by Petyt than they are with the data obtained using other numerical methods.

6 CONCLUSION

The general formulation of the dual integral equations of the boundary value problem for the two-dimensional Helmholtz equation with a degenerate boundary has been derived in this paper. The properties of the potentials resulting from the four kernel functions in the dual integral equations have been examined, and their potential distributions have also been given. A general dual BEM program has been implemented to solve for the acoustic frequencies and acoustic modes for an arbitrary two-dimensional cavity with or without incomplete partitions. An illustrative example has been successfully solved using the proposed dual BEM, and the results compare well with those obtained using other numerical methods and experiments.

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