Discussion


J.T. Chen\textsuperscript{a,}\textsuperscript{*}, I.L. Chen\textsuperscript{b}, Y.T. Lee\textsuperscript{a}

\textsuperscript{a}Department of Harbor and River Engineering, National Taiwan Ocean University, Keelung 20224, Taiwan, ROC
\textsuperscript{b}Department of Naval Architecture, National Kaohsiung Marine University, Kaohsiung 81157, Taiwan, ROC

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Abstract


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(1) Formulation: Based on the classical Helmholtz integral equation, the linear algebraic system is obtained as

$$[A][\Phi] = [B]\left\{ \frac{\partial \Phi}{\partial n}\right\}.$$  (1)
where $\Phi$ and $\partial \Phi/\partial n$ are boundary potential and its normal derivative, respectively. We have

$$\left\{ \frac{\partial \Phi}{\partial n} \right\} = [B]^{-1}[A]\{\Phi\} \quad (2)$$

and the stiffness matrix is

$$[K] = [B]^{-1}[A]. \quad (3)$$

Based on the method of fundamental solutions or BPM, we have

$$\Phi^*_T(p_j, q_j)c_j = \Phi(p_j), \quad (4)$$

$$\left\{ \frac{\partial \Phi^*_T(p_j, q_j)}{\partial n} \right\} c_j = \frac{\partial \Phi(p_j)}{\partial n}, \quad (5)$$

where $\Phi^*_T(p_j, q_j)$ is the fundamental solution for the response at $p_j$ due to a fabricated source at $q_j$ [1–7] and $c_j$ is the strength of the Chunky block. The matrix forms of Eqs. (4) and (5) are expressed as

$$[\Phi^*_T]_c = \{\Phi\}, \quad (6)$$

$$\left\{ \frac{\partial \Phi^*_T}{\partial n_p} \right\} c = \left\{ \frac{\partial \Phi}{\partial n} \right\}. \quad (7)$$

From Eq. (6), we can determine the unknown strength

$$\{c\} = [\Phi^*_T]^{-1}\{\Phi\}. \quad (8)$$

Substituting Eq. (8) into Eq. (7), we have

$$\left\{ \frac{\partial \Phi}{\partial n} \right\} = \left[ \frac{\partial \Phi^*_T}{\partial n} \right][\Phi^*_T]^{-1}\{\Phi\}. \quad (9)$$

The stiffness matrix is obtained in a new way

$$[K] = \left[ \frac{\partial \Phi^*_T}{\partial n} \right][\Phi^*_T]^{-1}. \quad (10)$$

Eq. (10) indicates that the strength of singularity is not required to be determined in advance to construct the stiffness matrix. For the same problem, there is the same stiffness matrix even though different methods are used. Therefore, we have

$$[B]^{-1}[A] = \left[ \frac{\partial \Phi^*_T}{\partial n} \right][\Phi^*_T]^{-1} \Rightarrow [A]^{-1}[B] = [\Phi^*_T]\left[ \frac{\partial \Phi^*_T}{\partial n} \right]^{-1}. \quad (11)$$

The result is the same to Eq. (6) of the paper [1]. We agree with Prof. Wu to point out that the intermediate process of derivation is not rigorous and easily misleads the readers. However, the final result of the paper by Zhang and Chen [1] is acceptable.

(2) Occurrence of fictitious frequencies: The authors in their paper claimed that the non-uniqueness problem appearing in the BEM is not present in the boundary point method. It may be not correct. It was demonstrated that irregular values also exist and shift to other positions for the method of fundamental solutions [8,9] and retracted BEM [10,11]. Since the boundary point method uses the fabricated source instead of concentrated source of MFS, it behaves like the retracted BEM [11]. It was theoretically proved and numerically demonstrated that the method of fundamental solutions [8] also encounter the irregular frequencies (non-uniqueness) as well as the boundary element method does [12]. The boundary point method and the method of fundamental solutions both are the meshless methods, which belong to the indirect BEM with concentrated source and fabricated sources distributing outside the domain, respectively. The main difference between the BPM and MFS is the singularity of a lumped source and a distributed fabricated source. In Refs. [8,9,11,13,14], the position of the irregular frequency depends on the location of source distribution and is different from that of direct BEM. The authors checked the irregular values of $k = 3.14$ and 6.28, 4.49 and 7.73 which are irregular values of direct BEM. This is not correct since the singularity is not located on the real boundary. It is expected that the non-uniqueness solutions were not found in the four locations
(3.14, 6.28, 4.49 and 7.73) in their paper. This cannot support the authors to claim that BPM is free of fictitious frequencies. The authors are encouraged to plot the response versus $k$ value to see the new irregular values and can explain why irregular values shift. The authors can find figures from Refs. [8,14] for reference. Many researchers of MFS have investigated the study of irregular values. Until 2006, Chen [8] extended the circulant idea of Chen and his coworkers [9,12,14,15] to prove the existence of irregular values in MFS theoretically and to demonstrate numerically. This finding points out the wrong statement of “Why the non-uniqueness solutions were not found in this paper, the reason is that a discrete set of source points does not define an internal surface uniquely” as quoted from Fairweather et al. [16]. It is obvious that the boundary point method must encounter the non-uniqueness problem, too. There are several approaches to deal with the non-uniqueness problems, e.g., CHIEF [12] and CHEEF [17] methods, SVD updating techniques [9,10,13,15] and mixed-potential method [8,11,18–21].

References


