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Letter to the Editor

Comments on "Free vibration analysis of arbitrarily shaped plates with clamped edges using wave-type function"☆

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In Ref. [1], Kang and Lee presented a non-dimensional dynamic influence function (NDIF) method for plate vibration. This paper extended the NDIF method from membrane vibration [2] to plate vibration problems. Kang and Lee [3] also applied this method to solve the membrane vibration by using domain partition for multiply connected and concave problems. Although Laura and Bambill [4] commented that the considered problem is very simple, it really proposed an easy method for engineers. Since only boundary node is required, the approach is meshless. Many successful examples of the clamped boundary conditions were demonstrated. It seems that this method is very attractive. However, this method can be treated as one kind of the Trefftz method [5–7] or boundary collocation method [8]. Based on the dual formulation developed by Chen and Hong [9–11], the interpolation function is nothing but the imaginary-part of the fundamental solution ($W(s,x) = (\mathrm{i}/(8\lambda^2))\{H_0^{(2)}(\lambda r) + H_0^{(1)}(\mathrm{i}\lambda r)\}$), where Kang and Lee chose $J_0(\lambda r)$ and $I_0(\lambda r)$ as radial basis functions. The method proposed by Kang and Lee [1] can be treated as a special case of the imaginary-part dual BEM, and its occurrence of spurious eigenvalues has been verified in Refs. [12,13]. In addition, Chen et al. [14] and Kuo et al. [15] employed the theory of circulants to prove that spurious eigensolutions and ill-posed problems may occur in case of circular membrane. For general shape problems, the two drawbacks are also inherent. To overcome the problem of spurious eigensolutions, a net approach was proposed by Kang and Lee [16]. Another alternative to avoid the occurence of spurious eigensolution was also proposed by Chen et al. [12] using the double-layer approach. This singularity-free method also results in the ill-posed behavior when the number of degrees of freedom becomes large [14]. Kuo et al. [17] employed the generalized singular value decomposition (GSVD) method in conjunction with the Tikhonov regularization to deal with the ill-posed problem for the incomplete boundary element formulation. Until now, research of ill-posed problem is an active area. However, why the spurious eigenvalues occur in Ref. [1] for a circular plate was not studied analytically. In this letter to editor, we will prove it.

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1. Derivation of true and spurious eigensolutions for the plate vibration using degenerate kernels and circulants in the NDIF method

As mentioned earlier, spurious eigenvalues occur in the NDIF method. Here, we will derive analytically the true and spurious solutions in the discrete system for a circular clamped plate by using the NDIF method [1]. The degenerate kernels and circulants are employed to study the discrete system in an exact form. On the basis of the NDIF method, the displacement and slope solutions can be represented by

$$w(x_i) = \sum_{j=1}^{2N} W(\lambda r_{ij})A(s_j) + \sum_{j=1}^{2N} \Theta(\lambda r_{ij})B(s_j), \tag{1}$$

$$\theta(x_i) = \sum_{j=1}^{2N} \frac{\partial W(\lambda r_{ij})}{\partial n_{x_i}} A(s_j) + \sum_{j=1}^{2N} \frac{\partial \Theta(\lambda r_{ij})}{\partial n_{x_i}} B(s_j), \tag{2}$$

where λ is the frequency parameter, $W(\lambda r_{ij}) = \mathrm{J}_0(\lambda r_{ij})$ is the zeroth order Bessel function, $\Theta(\lambda r_{ij}) = \mathrm{I}_0(\lambda r_{ij})$ is the zeroth order modified Bessel function, $r_{ij} = |x_i - s_j|$, x_i is the ith observation point, s_j is the jth boundary point, w and $\theta = \partial w/\partial n_{x_i}$ are the transverse deflection and its slope along the normal direction, respectively, $A(s_j)$ and $B(s_j)$ are the generalized unknowns at s_j, $2N$ is the number of boundary points, and the four kernels can be expressed in terms of degenerate kernels as shown below [18]:

$$W(s,x) = \mathrm{J}_0(\lambda r) = \sum_{m=-\infty}^{\infty} \mathrm{J}_m(\lambda\rho)\,\mathrm{J}_m(\lambda\rho)\cos(m(\theta-\phi)), \tag{3}$$

$$\Theta(s,x) = \mathrm{I}_0(\lambda r) = \sum_{m=-\infty}^{\infty} (-1)^m\,\mathrm{I}_m(\lambda\rho)\,\mathrm{I}_m(\lambda\rho)\cos(m(\theta-\phi)), \tag{4}$$

$$\frac{\partial W(s,x)}{\partial n_x} = \sum_{m=-\infty}^{\infty} \lambda \mathrm{J}_m(\lambda\rho)\,\mathrm{J}'_m(\lambda\rho)\cos(m(\theta-\phi)), \tag{5}$$

$$\frac{\partial \Theta(s,x)}{\partial n_x} = \sum_{m=-\infty}^{\infty} \lambda(-1)^m \mathrm{I}'_m(\lambda\rho)\,\mathrm{I}_m(\lambda\rho)\cos(m(\theta-\phi)), \tag{6}$$

where r is the distance between x and s, and J_m and I_m denote the first kind of the mth order Bessel and modified Bessel functions, respectively, $x = (\rho, \phi)$ and $s = (\rho, \theta)$ in the polar co-ordinate. For simplicity, we consider the same problem of a clamped plate of circular domain [1]. The boundary conditions for the clamped plate are given by

$$w(x) = 0 \quad \text{and} \quad \theta(x) = 0, \quad x \text{ on the circular boundary.} \tag{7}$$

By matching the boundary conditions on the $2N$ circular points into Eqs. (1) and (2), we have

$$\{0\} = [W]\{A\} + [\Theta]\{B\}, \tag{8}$$

$$\{0\} = [W']\{A\} + [\Theta']\{B\}, \tag{9}$$

where $[W]$, $[\Theta]$, $[W']$ and $[\Theta']$ are the corresponding matrices of $W(s,x)$, $\Theta(s,x)$, $\partial W(s,x)/\partial n_x$ and $\partial\Theta(s,x)/\partial n_x$, respectively. $\{A\}$ and $\{B\}$ are the undetermined coefficients. Eq. (8) can be rearranged to

$$\{B\} = -[\Theta]^{-1}[W]\{A\}. \tag{10}$$

By substituting Eq. (10) into Eq. (9), we have

$$[W']\{A\} - [\Theta'][\Theta]^{-1}[W]\{A\} = \{0\}, \tag{11}$$

then, we obtain

$$[[W'] - [\Theta'][\Theta]^{-1}[W]]\{A\} = \{0\} \Rightarrow [SM_N]\{A\} = \{0\}, \tag{12}$$

where

$$[SM_N] = [[W'] - [\Theta'][\Theta]^{-1}[W]]. \tag{13}$$

For the existence of non-trivial solution for $\{A\}$, the determinant of the matrix must become zero, i.e.,

$$\det[SM_N] = 0. \tag{14}$$

Since the rotation symmetry is preserved for a circular boundary, the four influence matrices in Eqs. (1) and (2) are denoted by $[W]$, $[\Theta]$, $[W']$ and $[\Theta']$ of the circulants with the elements

$$K_{ij} = K(\rho, \theta_j; \rho, \phi_i) = a_{ij}, \tag{15}$$

where $[K]$ can be $[W]$, $[\Theta]$, $[W']$ or $[\Theta']$, ϕ_i and θ_j are the angles of observation and boundary points, respectively. By superimposing $2N$ lumped strength along the boundary, we have the influence matrices,

$$[K] = \begin{bmatrix} a_0 & a_1 & a_2 & \cdots & a_{2N-2} & a_{2N-1} \\ a_{2N-1} & a_0 & a_1 & \cdots & a_{2N-3} & a_{2N-2} \\ a_{2N-2} & a_{2N-1} & a_0 & \cdots & a_{2N-4} & a_{2N-3} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ a_1 & a_2 & a_3 & \cdots & a_{2N-1} & a_0 \end{bmatrix}, \tag{16}$$

where the elements of the first row can be obtained by

$$a_{j-i} = K(s_j, x_i), \tag{17}$$

where the kernel K can be W, Θ, $\partial W/\partial n$ or $\partial\Theta/\partial n$. The matrix $[K]$ in Eq. (16) is found to be a circulant [15] since the rotational symmetry for the influence coefficients is considered. By introducing the following bases for the circulants $I, (C_{2N})^1, (C_{2N})^2, \ldots, (C_{2N})^{2N-1}$, we can expand $[K]$ into

$$[K] = a_0 I + a_1 (C_{2N})^1 + a_2 (C_{2N})^2 + \cdots + a_{2N-1}(C_{2N})^{2N-1}, \tag{18}$$

where I is a unit matrix and

$$C_{2N} = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 0 & 1 \\ 1 & 0 & 0 & \cdots & 0 & 0 \end{bmatrix}_{2N\times 2N}. \tag{19}$$

Based on the circulant theory, the eigenvalues for the influence matrix, [K], can be easily found as follows:

$$\lambda_\ell = a_0 + a_1\alpha_\ell + a_2\alpha_\ell^2 + \cdots + a_{2N-1}\alpha_\ell^{2N-1}, \quad \ell = 0, \pm 1, \pm 2, \ldots, \pm(N-1), N, \tag{20}$$

where λ_ℓ and α_ℓ are the eigenvalues for [K] and [C_{2N}], respectively. It is easily found that the eigenvalues for the circulant [C_{2N}] are the roots of $\alpha^{2N} = 1$ as show below:

$$\alpha_\ell = \mathrm{e}^{\mathrm{i}(2\pi\ell/2N)}, \quad \ell = 0, \pm 1, \pm 2, \ldots, \pm(N-1), N \text{ or } \ell = 0, 1, 2, \ldots, 2N-1. \tag{21}$$

Substituting Eq. (21) into Eq. (20), we have

$$\lambda_\ell = \sum_{m=0}^{2N-1} a_m\alpha_\ell^m = \sum_{m=0}^{2N-1} a_m\mathrm{e}^{\mathrm{i}(2\pi/2N)m\ell}, \quad \ell = 0, \pm 1, \pm 2, \ldots, \pm(N-1), N. \tag{22}$$

According to the definition for a_m in Eq. (17), we have

$$a_m = a_{2N-m}, \quad m = 0, 1, 2, \ldots, 2N-1. \tag{23}$$

Substitution of Eq. (23) into Eq. (22) yields

$$\lambda_\ell = a_0 + (-1)^\ell a_N + \sum_{m=1}^{N-1} (\alpha_\ell^m + \alpha_\ell^{2N-m})a_m = \sum_{m=0}^{2N-1} \cos(m\ell\Delta\theta)a_m. \tag{24}$$

Putting Eq. (17) into Eq. (24) for choosing W for K, the Reimann sum of infinite terms reduces to the following integral:

$$\lambda_\ell = \sum_{m=0}^{2N-1} \cos(m\ell\Delta\theta)W(m\Delta\theta, 0) \approx \frac{1}{\rho\Delta\theta}\int_0^{2\pi} \cos(\ell\theta)W(\theta, 0)\rho\,\mathrm{d}\theta \tag{25}$$

as N approaches infinity, where $\Delta\theta = 2\pi/2N$. By using the degenerate kernel for $W(s, x)$ in Eq. (3), Eq. (25) reduces to

$$\begin{aligned}\lambda_\ell &= \frac{1}{\rho\Delta\theta}\int_0^{2\pi} \cos(\ell\theta) \sum_{m=-\infty}^{\infty} \mathrm{J}_m(\lambda\rho)\,\mathrm{J}_m(\lambda\rho)\cos(m\theta)\rho\,\mathrm{d}\theta \\ &= \frac{2}{\rho\Delta\theta}\pi\rho\,\mathrm{J}_\ell(\lambda\rho)\,\mathrm{J}_\ell(\lambda\rho) = 2N\,\mathrm{J}_\ell(\lambda\rho)\,\mathrm{J}_\ell(\lambda\rho), \quad \ell = 0, \pm 1, \pm 2, \ldots, \pm(N-1), N.\end{aligned} \tag{26}$$

Similarly, we have

$$\mu_\ell = 2N\,\mathrm{I}_\ell(\lambda\rho)I_\ell(\lambda\rho), \quad \ell = 0, \pm 1, \pm 2, \ldots, \pm(N-1), N, \tag{27}$$

$$v_\ell = 2N\lambda\,\mathrm{J}_\ell(\lambda\rho)\mathrm{J}'_\ell(\lambda\rho), \quad \ell = 0, \pm 1, \pm 2, \ldots, \pm(N-1), N, \tag{28}$$

$$\delta_\ell = 2N\lambda\,\mathrm{I}'_\ell(\lambda\rho)\mathrm{I}_\ell(\lambda\rho), \quad \ell = 0, \pm 1, \pm 2, \ldots, \pm(N-1), N, \tag{29}$$

where μ_ℓ, v_ℓ and δ_ℓ are the eigenvalues of $[\Theta]$, $[W']$ and $[\Theta']$ matrices, respectively. The determinants for the four matrices can be obtained by multiplying all the eigenvalues as shown below:

$$\det[W] = \lambda_0(\lambda_1\lambda_2\cdots\lambda_{N-1})^2\lambda_N, \tag{30}$$

$$\det[\Theta] = \mu_0(\mu_1\mu_2\cdots\mu_{N-1})^2\mu_N, \tag{31}$$

$$\det[W'] = v_0(v_1v_2\cdots v_{N-1})^2 v_N, \tag{32}$$

$$\det[\Theta'] = \delta_0(\delta_1\delta_2\cdots\delta_{N-1})^2\delta_N. \tag{33}$$

Since $[W]$, $[\Theta]$, $[W']$ and $[\Theta']$ are all symmetric circulants, they can be expressed by

$$[W] = \Phi \begin{bmatrix} \lambda_0 & 0 & 0 & \cdots & 0 & 0 & 0 \\ 0 & \lambda_1 & 0 & \cdots & 0 & 0 & 0 \\ 0 & 0 & \lambda_{-1} & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & \lambda_{(N-1)} & 0 & 0 \\ 0 & 0 & 0 & \cdots & 0 & \lambda_{-(N-1)} & 0 \\ 0 & 0 & 0 & \cdots & 0 & 0 & \lambda_N \end{bmatrix}_{2N\times 2N} \Phi^{-1}, \tag{34}$$

$$[\Theta] = \Phi \begin{bmatrix} \mu_0 & 0 & 0 & \cdots & 0 & 0 & 0 \\ 0 & \mu_1 & 0 & \cdots & 0 & 0 & 0 \\ 0 & 0 & \mu_{-1} & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & \mu_{(N-1)} & 0 & 0 \\ 0 & 0 & 0 & \cdots & 0 & \mu_{-(N-1)} & 0 \\ 0 & 0 & 0 & \cdots & 0 & 0 & \mu_N \end{bmatrix}_{2N\times 2N} \Phi^{-1}, \tag{35}$$

$$[W'] = \Phi \begin{bmatrix} v_0 & 0 & 0 & \cdots & 0 & 0 & 0 \\ 0 & v_1 & 0 & \cdots & 0 & 0 & 0 \\ 0 & 0 & v_{-1} & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & v_{(N-1)} & 0 & 0 \\ 0 & 0 & 0 & \cdots & 0 & v_{-(N-1)} & 0 \\ 0 & 0 & 0 & \cdots & 0 & 0 & v_N \end{bmatrix}_{2N\times 2N} \Phi^{-1}, \tag{36}$$

$$[\Theta'] = \Phi \begin{bmatrix} \delta_0 & 0 & 0 & \cdots & 0 & 0 & 0 \\ 0 & \delta_1 & 0 & \cdots & 0 & 0 & 0 \\ 0 & 0 & \delta_{-1} & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & \delta_{(N-1)} & 0 & 0 \\ 0 & 0 & 0 & \cdots & 0 & \delta_{-(N-1)} & 0 \\ 0 & 0 & 0 & \cdots & 0 & 0 & \delta_N \end{bmatrix}_{2N\times 2N} \Phi^{-1}, \tag{37}$$

where

$$\Phi = \frac{1}{\sqrt{2N}} \times \begin{bmatrix} 1 & 1 & 0 & \cdots & 1 & 0 & 1 \\ 1 & \cos(\frac{2\pi}{2N}) & \sin(\frac{2\pi}{2N}) & \cdots & \cos(\frac{2\pi(N-1)}{2N}) & \sin(\frac{2\pi(N-1)}{2N}) & \cos(\frac{2\pi N}{2N}) \\ 1 & \cos(\frac{4\pi}{2N}) & \sin(\frac{4\pi}{2N}) & \cdots & \cos(\frac{4\pi(N-1)}{2N}) & \sin(\frac{4\pi(N-1)}{2N}) & \cos(\frac{4\pi N}{2N}) \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 1 & \cos(\frac{2\pi(2N-2)}{2N}) & \sin(\frac{2\pi(2N-2)}{2N}) & \cdots & \cos(\frac{\pi(4N-4)(N-1)}{2N}) & \sin(\frac{\pi(4N-4)(N-1)}{2N}) & \cos(\frac{\pi(4N-4)(N)}{2N}) \\ 1 & \cos(\frac{2\pi(2N-1)}{2N}) & \sin(\frac{2\pi(2N-1)}{2N}) & \cdots & \cos(\frac{\pi(4N-2)(N-1)}{2N}) & \sin(\frac{\pi(4N-2)(N-1)}{2N}) & \cos(\frac{\pi(4N-2)(N)}{2N}) \end{bmatrix}_{2N\times 2N}. \tag{38}$$

By employing Eqs. (34)–(37) for Eq. (13), we have

$$[SM_N] = \Phi \begin{bmatrix} \sigma_0 & 0 & 0 & \cdots & 0 & 0 & 0 \\ 0 & \sigma_1 & 0 & \cdots & 0 & 0 & 0 \\ 0 & 0 & \sigma_{-1} & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & \sigma_{(N-1)} & 0 & 0 \\ 0 & 0 & 0 & \cdots & 0 & \sigma_{-(N-1)} & 0 \\ 0 & 0 & 0 & \cdots & 0 & 0 & \sigma_N \end{bmatrix} \Phi^{-1}, \tag{39}$$

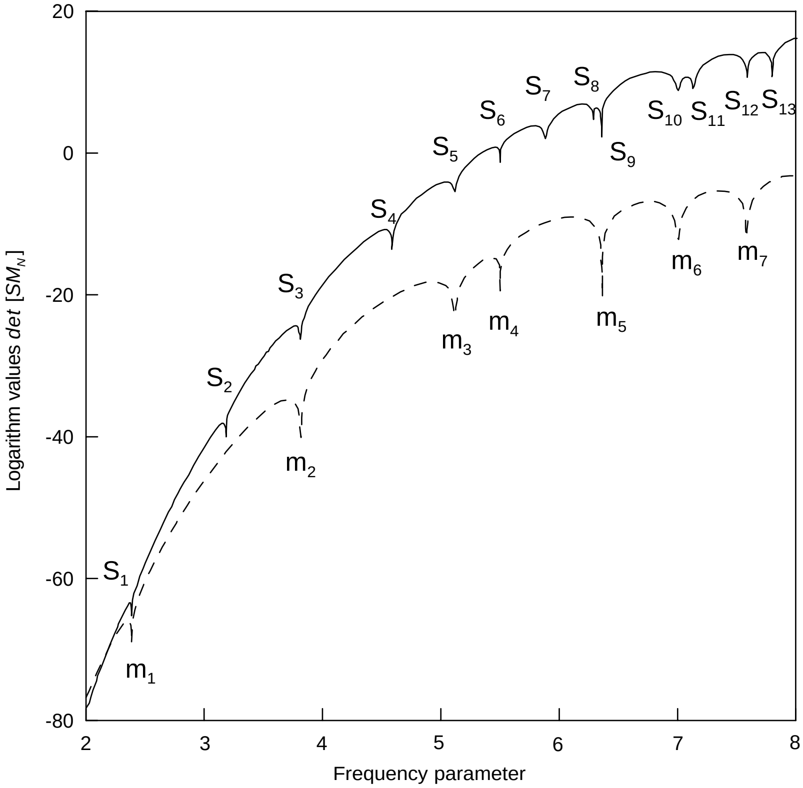

Fig. 1. Logarithm curves for $\det[SM_N]$ versus frequency parameter of the circular plate using the NDIF method [1].

where

$$\sigma_\ell = 2N\lambda \mathrm{J}_\ell(\lambda\rho)\mathrm{J}'_\ell(\lambda\rho) - \frac{2N\lambda\,\mathrm{I}_\ell(\lambda\rho)\mathrm{I}'_\ell(\lambda\rho)2N\,\mathrm{J}_\ell(\lambda\rho)\mathrm{J}_\ell(\lambda\rho)}{2N\,\mathrm{I}_\ell(\lambda\rho)\mathrm{I}_\ell(\lambda\rho)}$$
$$= 2N\lambda\frac{\mathrm{J}_\ell(\lambda\rho)[\mathrm{J}'_\ell(\lambda\rho)\mathrm{I}_\ell(\lambda\rho) - \mathrm{I}'_\ell(\lambda\rho)\mathrm{J}_\ell(\lambda\rho)]}{\mathrm{I}_\ell(\lambda\rho)}, \quad \ell = 0, \pm 1, \pm 2, \ldots, \pm(N-1), N. \tag{40}$$

According to Eqs. (39) and (40), we have

$$\det[SM_N] = \det|\Phi|\,\sigma_0(\sigma_1\sigma_2\cdots\sigma_{N-1})^2\sigma_N\,\det|\Phi^{-1}|$$
$$= \sigma_0(\sigma_1\sigma_2\cdots\sigma_{N-1})^2\sigma_N, \tag{41}$$

since $\det|\Phi| = \det|\Phi^{-1}| = 1$. By employing the differential formula for the Bessel and the modified Bessel functions, we can prove the following identity for any λ:

$$\mathrm{J}'_\ell(\lambda\rho)\mathrm{I}_\ell(\lambda\rho) - \mathrm{I}'_\ell(\lambda\rho)\mathrm{J}_\ell(\lambda\rho) = -[\mathrm{J}_\ell(\lambda\rho)\mathrm{I}_{\ell+1}(\lambda\rho) + \mathrm{I}_\ell(\lambda\rho)\mathrm{J}_{\ell+1}(\lambda\rho)]. \tag{42}$$

Zero determinant in Eq. (41) implies that the eigenequation is

$$\frac{\mathrm{J}_\ell(\lambda\rho)[\mathrm{J}_\ell(\lambda\rho)\mathrm{I}_{\ell+1}(\lambda\rho) + \mathrm{I}_\ell(\lambda\rho)\mathrm{J}_{\ell+1}(\lambda\rho)]}{\mathrm{I}_\ell(\lambda\rho)} = 0, \quad \ell = 0, \pm 1, \pm 2, \ldots, \pm(N-1), N. \tag{43}$$

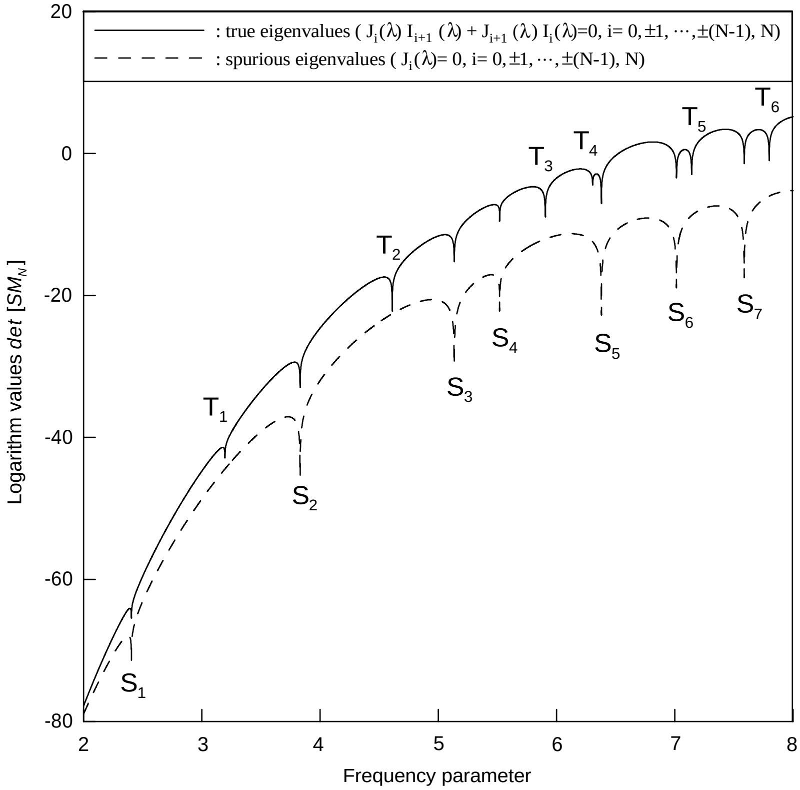

Fig. 2. Logarithm curves for det[SM_N] versus frequency parameter of the circular plate using the circulant method.

Since the denominator term of $I_\ell(\lambda\rho)$ is never zero for any value of $\lambda > 0$, eigenequation reduces to $J_\ell(\lambda\rho)[J_\ell(\lambda\rho)I_{\ell+1}(\lambda\rho) + I_\ell(\lambda\rho)J_{\ell+1}(\lambda\rho)] = 0,\ \ell = 0, \pm 1, \pm 2, \ldots, \pm(N-1), N$. Logarithm curves for det[SM_N] versus frequency parameter of the circular plate using the NDIF method are shown in Fig. 1 as quoted from Ref. [1]. By employing the circulant properties of Eq. (41), the similar results are shown in Fig. 2. This result in Fig. 2 matches well with that of the NDIF method in Fig. 1. After comparing with the exact solutions for the fixed circular membrane and the fixed circular plate, we find that the NDIF method results in the spurious eigenequation of $J_\ell(\lambda\rho) = 0,\ \ell = 0, \pm 1, \pm 2, \ldots, \pm(N-1), N$, which is the true eigensolution of membrane vibration and the true eigenequation is preserved $(I_\ell(\lambda\rho)I_{\ell+1}(\lambda\rho) + I_\ell(\lambda\rho)J_{\ell+1}(\lambda\rho) = 0,\ \ell = 0, \pm 1, \pm 2, \ldots, \pm(N-1), N)$ for plate vibration.

2. Concluding remarks

In this Letter to the Editor, we have proved that spurious eigensolutions were embedded in the NDIF method for the circular clamped plate. This can support the net approach which Kang and Lee proposed by dividing the spurious eigenequation. For the non-circular plate, only numerical experiments can be performed without the theoretical proof in the discrete system. Very recently, the authors became aware of relevant pioneer works by Chen [19,20] where he also utilized the imaginary-part kernel in his boundary knot method.

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