



## LETTER TO THE EDITOR

### A NEW HYSTERETIC DAMPING MODEL?

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In [1] a new solution for the free vibration of a single mass and spring system with ideal hysteretic damping is proposed. The purported solution, which is constructed from piecewise harmonic motions, forms a smooth shrinking spiral in the phase plane. The segments in the first and third quadrants are free undamped vibrations of the same mass on a slightly stiffer spring while the segments in the second and third quadrants are free undamped vibrations of the same mass on a slightly softer spring. Nonzero motion derivatives of second and higher orders are discontinuous across the joints between segments. The segments of this ingenious solution are exact solutions to Eq.(9) of [1]. They are however unrelated to the original hysteretic system because Eq.(4), from which Eq.(9) follows, is not the time-domain equivalent of the frequency-domain equations in Eq.(2) and Eq.(5). The real parts of the complex phasors  $\bar{u}e^{i\omega t}$  and  $\bar{P}e^{i\omega t}$  which satisfy Eq.(2) and Eq.(5), do not satisfy Eq.(4).

The correct time-domain equivalent is provided by a Fourier transform of the frequency-domain relation as explained in [2] and [3]. The proposal in [1] continues a long line [4,5] of attempts to produce time-domain representations of ideal hysteric damping by heuristic reasoning. The free vibrations predicted by [1] share some similarities with the exact solution [6,7] for the hysteretically damped oscillator. The damped natural frequency is correct to first order in  $\eta$  but the first-order term in the decrement is too small by a factor of about  $2/\pi$ . The procedure of [1] provides no hint of the major flaw in the ideal hysteretic model: the noncausal response property. This property was discovered, independently, in the early sixties by Fraeijs de Veubeke [8], Caughey [9], and Crandall [6].

**References**

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