

## On the nonuniqueness of BIEM/BEM using SVD

Jeng-Tzong Chen

*Department of Harbor and River Engineering, National Taiwan Ocean University, 2 Pei-Ning Road, Keelung, 202-24, Taiwan*

### Abstract

The nonuniqueness of BIEM/BEM are studied by using the SVD technique. A nonuniqueness solution stems from the degenerate boundary, degenerate scale, spurious eigenvalue and fictitious frequency. After employing the SVD technique with respect to the four influence matrices in the dual BEM, it is found that true (physics) information is imbedded in the right unitary vector while the spurious or fictitious (numerics) information appears in the left unitary vector.

### Introduction

It is well known that BEM is based on the use of fundamental solutions to solve partial differential equations. In some situations, the singular or hypersingular boundary integral equation is not sufficient to ensure a unique solution, e.g. degenerate boundary, degenerate scale, fictitious frequency and spurious eigenvalue. The applications of singular and hypersingular equations have been summarized in the review article of Chen and Hong and a keynote lecture by Chen in APCOM'07 meeting. In this paper, we propose a unified point of view to understand the nonuniqueness solution in the integral formulation by using SVD.

### Dual boundary integral formulation—conventional one

Based on the dual boundary integral formulation for the domain point, we have

$$2\pi u(x) = \int_B T(s, x)u(s)dB(s) - \int_B U(s, x)t(s)dB(s), \quad x \in \Omega, \quad (1)$$

$$2\pi \frac{\partial u(x)}{\partial n_x} = \int_B M(s, x)u(s)dB(s) - \int_B L(s, x)t(s)dB(s), \quad x \in \Omega, \quad (2)$$

where  $s$  and  $x$  are the source and field points, respectively,  $B$  is the boundary,  $n_x$  denotes the outward normal vector at field point  $x$ , and the kernel function  $U(s, x)$  is the fundamental solution and the other kernel functions can be found. By moving the field point  $x$  to the boundary, the dual boundary integral equations for the boundary point can be obtained as follows:

$$\pi u(x) = C.P.V. \int_B T(s, x)u(s)dB(s) - R.P.V. \int_B U(s, x)t(s)dB(s), \quad x \in B, \quad (3)$$

$$\pi \frac{\partial u(x)}{\partial n_x} = H.P.V. \int_B M(s, x)u(s)dB(s) - C.P.V. \int_B L(s, x)t(s)dB(s), \quad x \in B, \quad (4)$$

where the R.P.V., C.P.V. and H.P.V. are the Riemann, Cauchy and Hadamard (or called Mangler) principal value. The dual null-field integral equations are

$$0 = \int_B T(s, x)u(s)dB(s) - \int_B U(s, x)t(s)dB(s), \quad x \in \Omega^c, \quad (5)$$

$$0 = \int_B M(s, x)u(s)dB(s) - \int_B L(s, x)t(s)dB(s), \quad x \in \Omega^c, \quad (6)$$

when the field point  $x$  is moved to the complementary domain, and the superscript “c” denotes the complementary domain.

### Dual null-field integral formulation — the present version

By introducing the degenerate kernels, the collocation point can be exactly located on the real boundary free of facing singularity. Therefore, the representations of integral equations including the boundary point can be written as

$$2\pi u(x) = \int_B T(s, x)u(s)dB(s) - \int_B U(s, x)t(s)dB(s), \quad x \in \Omega \cup B, \quad (7)$$

$$2\pi \frac{\partial u(x)}{\partial n_x} = \int_B M(s, x)u(s)dB(s) - \int_B L(s, x)t(s)dB(s), \quad x \in \Omega \cup B, \quad (8)$$

and

$$0 = \int_B T(s, x)u(s)dB(s) - \int_B U(s, x)t(s)dB(s), \quad x \in \Omega^c \cup B, \quad (9)$$

$$0 = \int_B M(s, x)u(s)dB(s) - \int_B L(s, x)t(s)dB(s), \quad x \in \Omega^c \cup B, \quad (10)$$

where the kernel must be expressed in term of an appropriate degenerate form. After discretizing the boundary in the BIE, we have the linear algebraic equation as follows:

$$[T]\{u\} = [U]\{t\} \quad (11)$$

$$[M]\{u\} = [L]\{t\} \quad (12)$$

It is found that the influence matrices are singular for the problem with degenerate boundary, degenerate scale, spurious eigenvalue and fictitious frequency.

### SVD structure for the four influence matrices

After employing the SVD technique for the four influence matrices, the results are shown in Table 1

Table 1 The SVD structure of the four influence matrices in the dual BEM

|  |  |   |   |
|--|--|---|---|
| <div style="text-align: center;"> <p><b>The same</b><br/>(spurious)</p> </div> |  |   |   |
|  |  | U | T |
|  |  | L | M |

It is found that true (physics) information is imbedded in the right unitary vector while the spurious or fictitious (numerics) information appears in the left unitary vector with respect to the corresponding zero singular values.

### Conclusions

Nonuniqueness problems in the BIEM and BEM were revisited by using the SVD technique. Rank deficiency in the influence matrices can be detected by the zero singular value. The meaning of left and right unitary vectors indicate the true (physics) mode and spurious (numerics) mode, respectively. SVD updating terms and updating document can be employed to deal with the nonuniqueness solution.

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