ON THE IRREGULAR EIGENVALUES IN WAVE RADIATION SOLUTIONS USING DUAL BOUNDARY ELEMENT METHOD

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ABSTRACTS

This paper presents the mechanism for the irregular frequencies appearing in the wave radiation problem using the dual BEM. The relation between the matrices of influence coefficients for interior and exterior acoustic problems is examined. Also, the irregular (fictitious) frequencies (eigenvalues) embedded in the singular or hypersingular integral equations are discussed, respectively. It is found that the irregular values depend on the kernels in the integral representation for the solution. Numerical experiments using dual formulation program are conducted to check the validity in comparison with the theoretical proof of the independence of boundary conditions which have been shown by Chen using the degenerate kernels. A two-dimensional dual BEM program for the exterior acoustic problems was developed. Numerical examples are demonstrated by using the dual BEM program. Two cases, including the exterior Dirichlet and Neumann problems, show that the singular integral equation produces the fictitious eigenvalues which are associated with the eigenfrequencies of interior Dirichlet problem, while the hypersingular integral equation produces the fictitious eigenvalues which are associated with the interior Neumann problem.

Keywords: dual BEM, radiation, fictitious eigenvalues and exterior acoustic problem

INTRODUCTION

Integral equation method has been used to solve exterior acoustic problems (radiation and scattering) for many years. It is well known that fictitious eigenvalues stem from the numerical resonance instead of the physical resonance. Man references including commercial software claimed that th integral solution does not have a solution at certain eigenfre quencies of an associated interior problem. However, thei conclusions are not consistent. Chen (1988) drawed the cor clusion that the positions of fictitious eigenvalues are inde pendent of the boundary conditions once the method is cho sen by using dual series model. To demonstrate the mecha nism why fictitious eigenvalues occur, Chen and Hong (1992 and Chen (1998) showed that the positions where fictitiou eigenvalues occur depend on the kernels in the integral repre sentation for the solution by using a one-dimensional sem infinite example. From the numerical point of view, thi nonunique problem can be seen as the indefinite form of zer divided by zero. If L'hospital's rule can be employed ar alytically, no fictitious eigenvalues should occur. Howeve L'hospital's rule can not be applied in the numerical compu tation.

In this paper, the dual boundary element program wa developed to verify the conclusion by Chen (1998). The dua BEM program is based on the theory of dual integral equa tions. A detailed study on dual BEM can be found by Che and Hong (1999). The relations of the influence matrices be tween the interior and exterior acoustic problems are exam ined. Two examples, including the Dirichlet and Neuman radiation problems, are illustrated to show the mechanisn of fictitious eigenvalues. It shows that boundary conditior can not change the positions of fictitious eigenvalues onc the integral representation for the solution is chosen. Som misleading statements in the literature will become clear an will be corrected after the theoretical proof (Chen, 1998) an the present numerical studies.

DUAL INTEGRAL FORMULATION FOR AN EX

TERIOR ACOUSTIC PROBLEM

The governing equation for an exterior acoustic problem is the Helmholtz equation as follows:

$$(\nabla^2 + k^2)u(x_1, x_2) = 0, \ \ (x_1, x_2) \in D,$$

where ∇^2 is the Laplacian operator, D is the domain of the cavity and k is the wave number, which is angular frequency over the speed of sound. For simplicity, radiation problem is considered only. The boundary conditions can be either the Neumann or Dirichlet type.

Based on the dual integral equations (Chen and Hong, 1999), the dual equations for the boundary points are

$$\pi u(x) = C.P.V. \int_B T(s,x)u(s)dB(s) - R.P.V. \int_B U(s,x)t(s)dB(s), \ x \in B \quad (1)$$

$$\pi t(x) = H.P.V. \int_B M(s,x)u(s)dB(s) - C.P.V. \int_B L(s,x)t(s)dB(s), \ x \in B \quad (2)$$

where $C.P.V.$, $R.P.V.$ and $H.P.V.$ denote the Cauchy principal value, the Riemann principal value and the Hadamard principal value, $t(s) = \frac{\partial u(s)}{\partial n_s}$, B denotes the boundary enclosing D and the explicit forms of the four kernels, U, T, L and M, can be found in Chen and Hong (1999).

RELATIONS OF THE INFLUENCE MATRICES BETWEEN INTERIOR AND EXTERIOR PROBLEMS USING DUAL BEM

The linear algebraic equations for an interior problem discretized from the dual boundary integral equations Eqs.1 and 2 can be written as

$$[T^i_{pq}]\{u_q\} = [U^i_{pq}]\{t_q\} \quad (3)$$

$$[M^i_{pq}]\{u_q\} = [L^i_{pq}]\{t_q\}, \quad (4)$$

where the superscript "i" denotes the interior problem, $\{u_q\}$ and $\{t_q\}$ are the boundary potential and flux, and the subscripts p and q correspond to the labels of the collocation element and integration element, respectively.

For the interior problem, the influence coefficients of the four square matrices $[U], [T], [L]$ and $[M]$ can be represented as

$$U^i_{pq} = R.P.V. \int_{B_q} U(s_q, x_p)dB(s_q) \quad (5)$$

$$T^i_{pq} = \bar{T}_{pq} - 2\pi\delta_{pq} = -\pi\delta_{pq} + C.P.V. \int_{B_q} T(s_q, x_p)dB(s_q) \quad (6)$$

$$L^i_{pq} = \bar{L}_{pq} + 2\pi\delta_{pq} = \pi\delta_{pq} + C.P.V. \int_{B_q} L(s_q, x_p)dB(s_q) \quad (7)$$

$$M^i_{pq} = H.P.V. \int_{B_q} M(s_q, x_p)dB(s_q), \quad (8)$$

where B_q denotes the q^{th} element and $\delta_{pq} = 1$ if $p = q$ otherwise it is zero. T_{pq} and $\bar{T}_{pq}$ differ by a jump term $-2\pi\delta_{pq}$ while L_{pq} and $\bar{L}_{pq}$ differ by a jump term $2\pi\delta_{pq}$.

For the exterior problem, we have

$$[T^e_{pq}]\{u_q\} = [U^e_{pq}]\{t_q\} \quad (9)$$

$$[M^e_{pq}]\{u_q\} = [L^e_{pq}]\{t_q\}. \quad (10)$$

where the superscript "e" denotes the exterior problem.

According to the dependence of the outnormal vector in these four kernel functions for the interior and exterior problems, their relationship can be easily found as shown below (Chen et al. 1995) :

$$U^i_{pq} = U^e_{pq} \quad (11)$$

$$M^i_{pq} = M^e_{pq} \quad (12)$$

$$T^i_{pq} = \begin{cases} -T^e_{pq}, & if \ p \neq q, \\ T^e_{pq}, & if \ p = q \end{cases} \quad (13)$$

$$L^i_{pq} = \begin{cases} -L^e_{pq}, & if \ p \neq q, \\ L^e_{pq}, & if \ p = q. \end{cases} \quad (14)$$

Based on the relations for the influence matrices between the interior and exterior problems, the dual BEM program can be easily extended to exterior problems. For comparison with analytical solutions, a circular domain is considered. The absolute value for the determinant of the eight matrices, $U^i_{pq}, T^i_{pq}, L^i_{pq}, M^i_{pq}$ $U^e_{pq}, T^e_{pq}, L^e_{pq}$ and M^e_{pq} versus the wave number k are plotted in Fig.1. It is found that the characteristic values match the same for the four kernels $U^i_{pq}, L^i_{pq}, U^e_{pq}$ and T^e_{pq} which are the eigenvalues of the associated interior Dirichlet problem as shown in Table 1. On the other hand, the four kernels $T^i_{pq}, M^i_{pq}, L^e_{pq}$, and M^e_{pq} have the same characteristic values which are the eigenvalues of the associated interior Neumann problem as shown in Table 2.

NUMERICAL EXAMPLES FOR FICTITIOUS FREQUENCIES USING THE DUAL BEM

Two examples, including the Dirichlet and Neumann boundary conditions, are provided (Harari et al., 1998) and are shown in Fig.2. In the two cases, the same exact solutions are designed as shown in Fig.3. For the specified value one for k, the numerical solutions $u(r,\theta)$ for the Dirichlet and Neumann exterior problems by using the dual BEM are shown in Figs.4 and 5, respectively. Good agreement can be made. The Neumann exterior problem by using the UT formulation has the fictitious eigenvalues near $k = 2.41$ as shown in Fig.6

However, the Dirichlet exterior problem by using the UT formulation has the same fictitious eigenvalues near $k = 2.40$ as shown in Fig.7. This indicates that the type of boundary conditions (Dirichlet or Neumann) for the exterior problem can not change the position of the fictitious eigenvalues once the integral representation for the solution is chosen, *e,g.*, in this case UT formulation is adopted. Also, it is found that the fictitious eigenvalues resulted from the UT formulation corresponds to the associated Dirichlet problem since $k = 2.40$ can be found in Table 1. In a similar way, the Neumann exterior problem by using the LM formulation has the fictitious eigenvalues at $k = 3.8355$ as shown in Fig.8. Also, the Dirichlet exterior problem by using the LM formulation has the same fictitious eigenvalues near $k = 3.83$ as shown in Fig.9. This also indicates that the type of boundary conditions (Dirichlet or Neumann) for the exterior problem can not change the position of the fictitious eigenvalues. The numerical experiments for the fictitious eigenvalues match well with the theoretical derivation in Chen (1998).

CONCLUDING REMARKS

The occurring mechanism of fictitious eigenvalues in direct BEM has been examined using the dual BEM by considering the relations between the influence matrices of interior and exterior problems. It is found that the irregular values depend on the first (UT) or second (LM) equation used in the dual integral equations no matter what the types of specified boundary conditions are. Two examples have been given to verify this point of view. Both examples show that the first UT equation results in fictitious eigenvalues which are associated with the interior eigenfrequency with essential homogeneous boundary conditions, while the second LM equation produces fictitious eigenvalues which are associated with the interior eigenfrequency with natural homogeneous boundary conditions. The numerical results are consistent with the analytical derivations.

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Table 1 Characteristic solutions for the interior Helmholt
equation with the Dirichlet boundary conditions

No. (n)	eigenvalues (k_n)	eigen equation	eigenmode
1	2.4048(2.4070)	$J_0(ka) = 0$	$J_0(2.40$
2,3	3.8317(3.8342)	$J_1(ka) = 0$	$J_1(3.831$
4,5	5.1356(5.1388)	$J_2(ka) = 0$	$J_2(5.1356$
6	5.5201(5.5223)	$J_0(ka) = 0$	$J_0(5.52$

Note that data in parenthesis are obtained by using du
BEM.

Table 2 Characteristic solutions for the interior Helmholt
equation with the Neumann boundary conditions

No. (n)	eigenvalues (k_n)	eigen equation	eigenmode
1	0.0000(0.0000)	$J_0'(ka) = 0$	$J_0(0.00$
2,3	1.8412(1.8436)	$J_1'(ka) = 0$	$J_1(1.841$
4,5	3.0542(3.0586)	$J_2'(ka) = 0$	$J_2(3.0542$
6	3.8317(3.8364)	$J_0'(ka) = 0$	$J_0(3.83$

Note that data in parenthesis are obtained by using du
BEM.

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