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Regularized meshless method for antiplane piezoelectricity problems with multiple inclusions

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Abstract: In this paper, solving antiplane piezoelectricity problems with multiple inclusions are attended by using the regularized meshless method (RMM). This is made possible that the troublesome singularity in the MFS disappears by employing the subtracting and adding-back technique. The governing equations for linearly electro-elastic medium are reduced to two uncoupled Laplace's equations. The representations of two solutions of the two uncoupled system are obtained by using the RMM. By matching interface conditions, the linear algebraic system is obtained. Finally, typical numerical examples are presented and discussed to demonstrate the accuracy of the solutions.

Keywords: antiplane shear, piezoelectricity, regularized meshless method, method of fundamental solutions, subtracting and adding-back techniques, electric field, displacement field, inclusion.

1 Introduction

In recent years, the significant progress in the development of piezoelectric materials or structures has been made by the research community [Bleustein (1968), Chung and Ting (1996), Honein; Honein and Herrmann (1992), Honein and Honein (1995), Pak (1992), Sladek; Sladek and Zhang (2007), Sladek; Sladek; Zhang; Garcia-Sanche and W " u nsche (2006), Sze; Jin; Sheng and Li (2003), Wu and Syu (2006)]. It is well known that piezoelectric materials undergo deformation when subject to electric field because of the electro-mechanical coupling phenomenon. Bleustein (1968) investigated the antiplane piezoelectric dynamics problem and discovered the existence of Bleustein wave. Pak (1992) has considered a more

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general case by introducing a piezoelectric inclusion, which in the limiting case of vanishing elastic and piezoelectric constants, become a permeable hole containing free space with electric fields. He obtained an analytical solution by using the alternative method. Later, Honein and Honein (1995) have visited the problem of two circular piezoelectric fibers subjected to out-of-plane displacement and in-plane electric fields. On the other hand, Chung and Ting (1996) have used basic solution [Stroh (1962)] approach for solved the problem of an elliptic hole in a solid of anisotropic material. Zhong and Meguid (1997) employ the complex variable method to treat the partially-debonded circular inhomogeneity problems in materials under antiplane shear and inplane electric field. In 1997, Chen and Chiang solved for 2D problems of an infinite piezoelectric medium containing a solitary cavity or rigid inclusion of arbitrary shape, subjected to a coupled antiplane mechanical and inplane electric load at the matrix by using the conformal mapping technique. In recent years, Chao and Chang (1999) studied the stress concentration and tangential stress distribution on double piezoelectric inclusions by using the complex variable theory and the method of successive approximations. Wu; Chen and Meng (2000) employ conformal mapping and the theorem of analytic continuation to solve the problem of two piezoelectric circular cylindrical inclusions in the infinite piezoelectric medium. Based on the method of fundamental solutions (MFS) [Alves and Antunes (2005), Godinho; Tadeu and Amado (2007), Chen; Golberg and Hon (1998), Fairweather and Karageorghis (1998), Kupradze and Aleksidze (1964), Poullikkas; Karageorghis and Georgiou (1998), Reutskiy (2005), Tsangaris; Smyrlis and Karageorghis (2004) Young; Tsai; Lin and Chen (2006)], we will develop a novel meshless method to solve antiplane piezoelectricity problems with multiple inclusions without the troublesome singularity is embedded in the linear algebraic system.

The MFS is one important method of the meshless methods [Atluri; Liu and Han (2006), Han and Atluri (2004), Li and Atluri (2008), Liu; Han; Rajendran and Atluri (2008), Sladek; Sladek and Atluri (2004), Sladek; Sladek; Solek and Wen (2008), Sladek; Sladek; Solek; Wen and Atluri (2008), Sze; Jin; Sheng and Li (2003)] and belongs to a boundary method of boundary value problems, which can be viewed as a discrete type of indirect boundary element method. The method is relatively easy to implement. It is adaptive in the sense that it can take into account sharp changes in the solution and in the geometry of the domain [Chen; Kuo; Chen and Cheng (2000), Chen; Chen; Chen; Lee and Yeh (2004)] and can easily treat with complex boundary conditions [Karageorghis and Georgiou (1998)]. A survey of the MFS and related methods over the last thirty years has been found [Kupradze and Aleksidze (1964)]. However, the MFS is still not a popular method because of the debatable artificial boundary distance of source location in numerical implementation especially for a complicated geometry. The diagonal coefficients of influence matrices are divergent in the conventional case when the fictitious boundary is far away from the physical boundary. It results in an ill-posed problem when the fictitious boundary approaches the physical boundary since the condition number for the influence matrix becomes very large.

We have developed a modified MFS, namely regularized meshless method (RMM), to overcome the drawback of MFS [Chen; Kao; Chen; Young and Lu (2006), Young Chen and Lee (2006)]. The method eliminates the well-known drawback of equivocal artificial boundary. The subtracting and adding-back techniques [Chen; Kao; Chen; Young and Lu (2006), Young; Chen and Lee (2005), Young; Chen and Lee (2006)] can regularize the singularity and hypersingularity of the kernel functions. This method can simultaneously distribute the observation and source points on the physical boundary even using the singular kernels instead of non-singular kernels [Chen; Chang; Chen and Lin (2002), Chen; Chang; Chen and Chen (2002)]. The diagonal terms of the influence matrices can be extracted out by using the proposed technique. Recently, a simple approach to derive the analytical formula of the diagonal elements of the interpolation matrix of the regularized meshless method (RMM) for regular and irregular domain problems have been studied [Chen and Song (2009), Song and Chen (2009)].

This paper is an extension work of the paper [Chen; Chen and Kao (2008)] for solving the antiplane elasticity problem. The RMM is extended to solve the antiplane piezoelectricity problem and multiple inclusions with arbitrary shape are embedded in an infinite matrix in this paper. A general-purpose program was developed to solve antiplane piezoelectricity problems with arbitrary number of inclusions. The results are compared with analytical solutions and those of the method of successive approximations [Chao and Chang (1999)]. Furthermore, the tangential electric field distribution and stress concentration for different ratios of piezoelectric module will be studied through several examples to show the validity of our method.

2 Governing equation and boundary conditions

Consider piezoelectric inclusions embedded in an infinite domain as shown in Fig. 1. The inclusions and matrix have different material properties. The matrix is subjected to a remote antiplane shear, $\sigma_{zy} = \tau_\infty$, and a remote inplane electric field, $E_y = E_\infty$. A uniform electric field can be induced in piezoelectric material by applying a potential field $E = E_\infty$.

For this problem, the out-of-plane elastic displacement w and the electric potential ϕ are only functions of x and y, such that

$$w = w(x,y), \quad \phi = \phi(x,y). \tag{1}$$

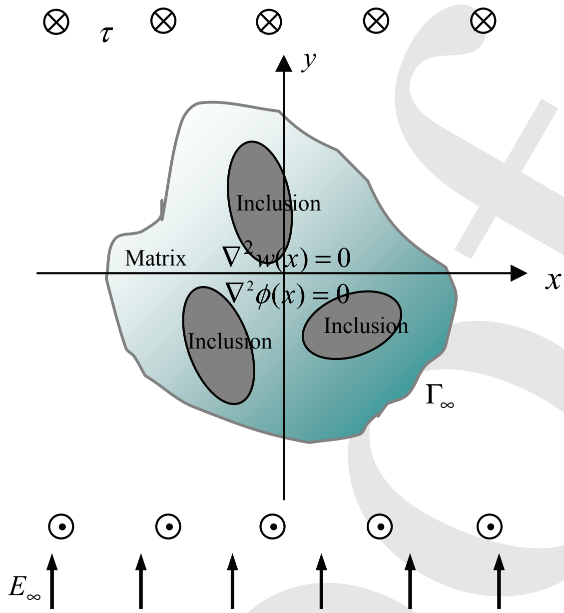

Figure 1: Problem sketch

The equilibrium equations [Chao and Chang (1999)] for the stresses and the electric displacements are

$$\partial \sigma_{zx}/\partial x + \partial \sigma_{zy}/\partial y = 0, \quad \partial D_x/\partial x + \partial D_y/\partial y = 0, \tag{2}$$

where σ_{zx} and σ_{zy} are the shear stresses, while D_x and D_y are the electric displacements. For linear piezoelectric materials, the constitutive relations [Chao and Chang (1999)] are written as

$$\begin{aligned} \sigma_{zx} &= c_{44}\gamma_{zx} - e_{15}E_x, \quad \sigma_{zy} &= c_{44}\gamma_{zy} - e_{15}E_y, \\ D_x &= e_{15}\gamma_{zx} + \varepsilon_{11}E_x, \quad D_y &= e_{15}\gamma_{zy} + \varepsilon_{11}E_y, \end{aligned} \tag{3}$$

in which γ_{zx} and γ_{zy} are the shear strains, E_x and E_y are the electric fields, c_{44} is the elastic modulus, e_{15} denotes the piezoelectric modulus and ε_{11} represents the dielectric modulus. The shear strains γ_{zx} and γ_{zy} and the electric fields E_x and E_y are obtained by taking gradient of the displacement potential w and the electric potential ϕ by the following relations:

$$\begin{aligned} &\gamma_{zx} = \partial w/\partial x, \quad \gamma_{zy} = \partial w/\partial y, \\ &E_x = -\partial \phi/\partial x, \quad E_y = -\partial \phi/\partial y. \end{aligned} \tag{4}$$

Substituting Eqs. (3) and (4) into (2), we can obtain the following governing equations:

$$\begin{cases} c_{44}\nabla^2 w + e_{15}\nabla^2\phi = 0 \\ e_{15}\nabla^2 w - \varepsilon_{11}\nabla^2\phi = 0 \end{cases} \tag{5}$$

From Eq. (5), we can obtain the equations as

$$\nabla^2 w = 0, \quad \nabla^2\phi = 0, \tag{6}$$

where ∇^2 is the Laplacian operator. The continuity conditions across the matrix-inclusion interface are written as

$$w^i = w^m, \quad \sigma^i_{zr} = \sigma^m_{zr}, \tag{7}$$

$$\phi^i = \phi^m, \quad D^i_r = D^m_r, \tag{8}$$

where the superscripts i and m denote the inclusion and material, respectively. The loading is remote shear.

3 Review of conventional method of fundamental solutions

By employing the RBF technique [Chen and Tanaka (2002), Cheng (2000)], the representation of the solution in Eq. (6) for multiple inclusions problem as shown in Fig. 1, can be approximated in terms of the strengths α_j of the singularities at s_j as

$$u(x_i) = \sum_{j=1}^{N} T(s_j,x_i)\alpha_j = \sum_{j=1}^{N_1} T(s_j,x_i)\alpha_j + \sum_{j=N_1+1}^{N_1+N_2} T(s_j,x_i)\alpha_j + \cdots + \sum_{j=N_1+N_2+\cdots+N_{m-1}+1}^{N} T(s_j,x_i)\alpha_j, \tag{9}$$

and

$$t(x_i) = \sum_{j=1}^{N} M(s_j,x_i)\alpha_j = \sum_{j=1}^{N_1} M(s_j,x_i)\alpha_j + \sum_{j=N_1+1}^{N_1+N_2} M(s_j,x_i)\alpha_j + \cdots + \sum_{j=N_1+N_2+\cdots+N_{m-1}+1}^{N} M(s_j,x_i)\alpha_j, \tag{10}$$

where $u(x_i)$ can be denoted as $w(x_i)$ or $\phi(x_i)$, $t(x_i) = \partial u(x_i)/\partial n_x$, $T(s_j,x_i)$ is RBF, x_i and s_j represent ith observation point and jth source point, respectively, α_j

are the jth unknown coefficients (strength of the singularity), $N_1, N_2, \cdots, N_m$ are the numbers of source points on m numbers of boundaries of inclusions, respectively, while N is the total numbers of source points $(N = N_1 + N_2 + \cdots + N_m)$ and $M(s_j, x_i) = \partial T(s_j, x_i)/\partial n_{x_i}$. After BCs are satisfied at the boundary points, the coefficients $\{\alpha_j\}_{j=1}^{N}$ are determined. The chosen bases are the double layer potentials [Chen; Kao; Chen; Young and Lu (2006), Young; Chen and Lee (2005)] as

$$T(s_j, x_i) = \frac{-<(x_i - s_j), n_j>}{r_{ij}^2}, \tag{11}$$

$$M(s_j, x_i) = \frac{2<(x_i - s_j), n_j><(x_i - s_j), \overline{n_i}>}{r_{ij}^4} - \frac{<n_j, \overline{n_i}>}{r_{ij}^2}, \tag{12}$$

where $< , >$ is the inner product of two vectors, r_{ij} is $|s_j - x_i|$, n_j is the normal vector at s_j, and $\overline{n_i}$ is the normal vector at x_i.

It is noted that the double layer potentials have both singularity and hypersingularity when source and field points coincide, which lead to difficulty in the conventional MFS. The fictitious distance between the fictitious (auxiliary) boundary and the physical boundary, d, needs to be chosen deliberately. To overcome the abovementioned shortcoming, s_j is distributed on the physical boundary, by using the proposed regularized technique as written in Section 4.

4 Regularized meshless method

The antiplane piezoelectricity problem with multiple inclusions is decomposed into two parts as shown in Fig. 2.

One is the exterior problem for matrix with hole subjected to the far-displacement field and far-electric field, the other is the interior problem for each inclusion. The two boundary data of matrix and inclusion satisfy the interface conditions in Eqs. (7) and (8). Furthermore, the exterior problem for matrix with holes subjected to a far-displacement field and far-electric field can be superimposed by two systems as shown in Fig. 3.

One is an infinite domain with no hole subjected to a far-displacement field and far-electric field, the other is the matrix with holes. The representations of the two solutions for the interior problem ($w(x_i^I)$ and $\phi(x_i^I)$) and exterior problem ($w(x_i^O)$ and $\phi(x_i^O)$) are formulated by using the RMM as follows:

4.1 Interior problem

When the collocation point x_i approaches the source point s_j, the kernels in Eqs. (9) and (10) become singular. Eqs. (9) and (10) for the multiple-inclusions problem

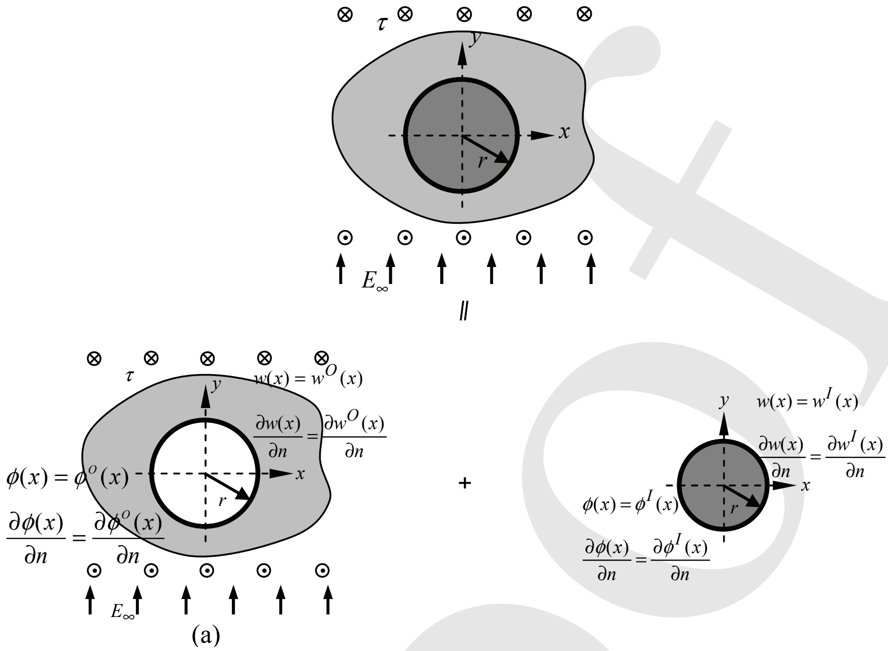

Figure 2: Decomposition of the problem

need to be regularized by using the regularization of subtracting and adding-back techniques [Chen; Kao; Chen; Young and Lu (2006), Young; Chen and Lee (2005)] as follows:

$$u(x_i^I) = \sum_{j=1}^{N_1} T(s_j^I, x_i^I)\alpha_j + \cdots + \sum_{j=N_1+\cdots+N_{P-1}+1}^{N_1+\cdots+N_p} T(s_j^I, x_i^I)\alpha_j + \cdots$$
$$+ \sum_{j=N_1+\cdots+N_{m-2}+1}^{N_1+\cdots+N_{m-1}} T(s_j^I, x_i^I)\alpha_j + \sum_{j=N_1+\cdots+N_{m-1}+1}^{N} T(s_j^I, x_i^I)\alpha_j$$
$$- \sum_{j=N_1+\cdots+N_{P-1}+1}^{N_1+\cdots+N_p} T(s_j^I, x_i^I)\alpha_i, \quad x_i^I \in B_p,\ p = 1,2,3,\cdots,m \tag{13}$$

where $u(x_i^I)$ can be denoted as $w(x_i^I)$ and $\phi(x_i^I)$ in which the superscript I denotes the interior domain, x_i^I is located on the boundaries B_p ($p = 1,2,3,\cdots,m$), and

$$\sum_{j=N_1+\cdots+N_{p-1}+1}^{N_1+\cdots+N_p} T(s_j^I, x_i^I) = 0, \quad x_i^I \in B_p,\ p = 1,2,3,\cdots,m. \tag{14}$$

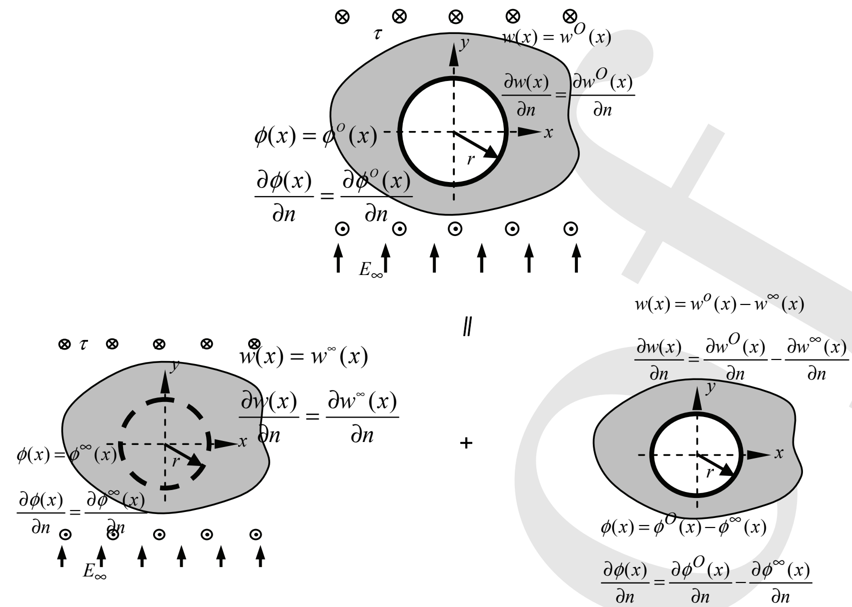

Figure 3: Decomposition of the problem of Fig. 2 (a)

The detailed derivations of Eq. (14) are given in the reference [Young; Chen and Lee (2005)]. Therefore, we can obtain

$$
\begin{aligned}
u(x_i^I) = & \sum_{j=1}^{N_1} T(s_j^I,x_i^I)\alpha_j + \cdots + \sum_{j=N_1+\cdots+N_{p-1}+1}^{i-1} T(s_j^I,x_i^I)\alpha_j + \sum_{j=i+1}^{N_1+\cdots+N_p} T(s_j^I,x_i^I)\alpha_j + \cdots \\
& + \sum_{j=N_1+\cdots+N_{m-2}+1}^{N_1+\cdots+N_{m-1}} T(s_j^I,x_i^I)\alpha_j + \sum_{j=N_1+\cdots+N_{m-1}+1}^{N} T(s_j^I,x_i^I)\alpha_j \\
& - \left[\sum_{j=N_1+\cdots+N_{P-1}+1}^{N_1+\cdots+N_p} T(s_j^I,x_i^I) - T(s_i^I,x_i^I) \right] \alpha_i, \quad x_i^I \in B_p,\ p = 1,2,3,\cdots,m. \qquad (15)
\end{aligned}
$$

Similarly, the boundary flux is obtained as

$$
\begin{aligned}
t(x_i^I) = {} & \sum_{j=1}^{N_1} M(s_j^I, x_i^I)\alpha_j + \cdots + \sum_{j=N_1+\cdots+N_{p-1}+1}^{N_1+\cdots+N_p} M(s_j^I, x_i^I)\alpha_j + \cdots \\
& + \sum_{j=N_1+\cdots+N_{m-2}+1}^{N_1+\cdots+N_{m-1}} M(s_j^I, x_i^I)\alpha_j + \sum_{j=N_1+\cdots+N_{m-1}+1}^{N} M(s_j^I, x_i^I)\alpha_j \\
& - \sum_{j=N_1+\cdots+N_{P-1}+1}^{N_1+\cdots+N_p} M(s_j^I, x_i^I)\alpha_i, \quad x_i^I \in B_p,\ p = 1,2,3,\cdots,m. \qquad (16)
\end{aligned}
$$

where $t(x_i^I) = \partial u(x_i^I)/\partial n_{x_i}$ and

$$
\sum_{j=N_1+\cdots+N_{p-1}+1}^{N_1+\cdots+N_p} M(s_j^I, x_i^I) = 0, \quad x_i^I \in B_p, \quad p = 1,2,3,\cdots,m. \qquad (17)
$$

The detailed derivations of Eq. (14) are also given in the reference [Young; Chen and Lee (2005)]. Therefore, we obtain

$$
\begin{aligned}
t(x_i^I) = {} & \sum_{j=1}^{N_1} M(s_j^I, x_i^I)\alpha_j + \cdots + \sum_{j=N_1+\cdots+N_{p-1}+1}^{i-1} M(s_j^I, x_i^I)\alpha_j \\
& + \sum_{j=i+1}^{N_1+\cdots+N_p} M(s_j^I, x_i^I)\alpha_j + \cdots + \sum_{j=N_1+\cdots+N_{m-2}+1}^{N_1+\cdots+N_{m-1}} M(s_j^I, x_i^I)\alpha_j \\
& + \sum_{j=N_1+\cdots+N_{m-1}+1}^{N} M(s_j^I, x_i^I)\alpha_j - \left[\sum_{j=N_1+\cdots+N_{P-1}+1}^{N_1+\cdots+N_p} M(s_j^I, x_i^I) - M(s_i^I, x_i^I) \right] \alpha_i, \\
& x_i^I \in B_p,\ p = 1,2,3,\cdots,m. \qquad (18)
\end{aligned}
$$

4.2 Exterior problem

When the observation point x_i^O locates on the boundaries B_p $(p = 1,2,3,\cdots,m)$, Eq. (13) becomes

$$
\begin{aligned}
u(x_i^O) = {} & \sum_{j=1}^{N_1} T(s_j^O, x_i^O)\alpha_j + \cdots + \sum_{j=N_1+\cdots+N_{P-1}+1}^{N_1+\cdots+N_p} T(s_j^O, x_i^O)\alpha_j + \cdots \\
& + \sum_{j=N_1+\cdots+N_{m-2}+1}^{N_1+\cdots+N_{m-1}} T(s_j^O, x_i^O)\alpha_j + \sum_{j=N_1+\cdots+N_{m-1}+1}^{N} T(s_j^O, x_i^O)\alpha_j \\
& - \sum_{j=N_1+\cdots+N_{P-1}+1}^{N_1+\cdots+N_p} T(s_j^I, x_i^I)\alpha_i, \quad x_i^{OorI} \in B_p,\ p = 1,2,3,\cdots,m, \qquad (19)
\end{aligned}
$$

where $u(x_i^O)$ can be denoted as $w(x_i^O)$ and $\phi(x_i^O)$ in which the superscript O denotes the exterior domain, x_i^O is also located on the boundaries B_p ($p = 1,2,3,\cdots,m$). Hence, we obtain

$$\begin{aligned}
u(x_i^O) = &\sum_{j=1}^{N_1} T(s_j^O,x_i^O)\alpha_j + \cdots + \sum_{j=N_1+\cdots+N_{p-1}+1}^{i-1} T(s_j^O,x_i^O)\alpha_j \\
&+ \sum_{j=i+1}^{N_1+\cdots+N_p} T(s_j^O,x_i^O)\alpha_j + \cdots + \sum_{j=N_1+\cdots+N_{m-2}+1}^{N_1+\cdots+N_{m-1}} T(s_j^O,x_i^O)\alpha_j \\
&+ \sum_{j=N_1+\cdots+N_{m-1}+1}^{N} T(s_j^O,x_i^O)\alpha_j - \left[\sum_{j=N_1+\cdots+N_{P-1}+1}^{N_1+\cdots+N_p} T(s_j^I,x_i^I) - T(s_i^O,x_i^O)\right]\alpha_i, \\
&\qquad x_i^{OorI} \in B_p,\ p = 1,2,3,\cdots,m. \qquad (20)
\end{aligned}$$

Similarly, the boundary flux is obtained as

$$\begin{aligned}
t(x_i^O) = &\sum_{j=1}^{N_1} M(s_j^O,x_i^O)\alpha_j + \cdots + \sum_{j=N_1+\cdots+N_{p-1}+1}^{N_1+\cdots+N_p} M(s_j^O,x_i^O)\alpha_j + \cdots \\
&+ \sum_{j=N_1+\cdots+N_{m-2}+1}^{N_1+\cdots+N_{m-1}} M(s_j^O,x_i^O)\alpha_j + \sum_{j=N_1+\cdots+N_{m-1}+1}^{N} M(s_j^O,x_i^O)\alpha_j \\
&- \sum_{j=N_1+\cdots+N_{P-1}+1}^{N_1+\cdots+N_p} M(s_j^I,x_i^I)\alpha_i, \quad x_i^{OorI} \in B_p,\ p = 1,2,3,\cdots,m, \qquad (21)
\end{aligned}$$

where $t(x_i^O) = \partial u(x_i^O)/\partial n_{x_i}$. Hence, we obtain

$$\begin{aligned}
t(x_i^O) = &\sum_{j=1}^{N_1} M(s_j^O,x_i^O)\alpha_j + \cdots + \sum_{j=N_1+\cdots+N_{p-1}+1}^{i-1} M(s_j^O,x_i^O)\alpha_j \\
&+ \sum_{j=i+1}^{N_1+\cdots+N_p} M(s_j^O,x_i^O)\alpha_j + \cdots + \sum_{j=N_1+\cdots+N_{m-2}+1}^{N_1+\cdots+N_{m-1}} M(s_j^O,x_i^O)\alpha_j \\
&+ \sum_{j=N_1+\cdots+N_{m-1}+1}^{N} M(s_j^O,x_i^O)\alpha_j - \left[\sum_{j=N_1+\cdots+N_{P-1}+1}^{N_1+\cdots+N_p} M(s_j^I,x_i^I) - M(s_i^O,x_i^O)\right]\alpha_i, \\
&\qquad x_i^{OorI} \in B_p,\ p = 1,2,3,\cdots,m. \qquad (22)
\end{aligned}$$

According to the dependence of the normal vectors for inner and outer boundaries [Young; Chen and Lee (2005)], their relationships are

$$\begin{cases} T(s_j^I,x_i^I) = -T(s_j^O,x_i^O), & i \neq j \\ T(s_j^I,x_i^I) = T(s_j^O,x_i^O), & i = j \end{cases} \qquad (23)$$

$$\begin{cases} M(s_j^I, x_i^I) = M(s_j^O, x_i^O), & i \neq j \\ M(s_j^I, x_i^I) = M(s_j^O, x_i^O), & i = j \end{cases} \tag{24}$$

where the left and right hand sides of the equal sign in Eqs. (23) and (24) denote the kernels for observation and source point with the inward and outward normal vectors, respectively.

By using the proposed technique, the singular terms in Eqs. (9) and (10) have been transformed into regular terms ($-\left[\sum_{j=N_1+N_2+\cdots+N_{p-1}+1}^{N_1+N_2+\cdots+N_p} T(s_j^I, x_i^I) - T(s_i^{I \text{ or } O}, x_i^{I \text{ or } O})\right]$ and $-\left[\sum_{j=N_1+\cdots+N_{p-1}+1}^{N_1+\cdots+N_p} M(s_j^I, x_i^I) - M(s_i^{I \text{ or } O}, x_i^{I \text{ or } O})\right]$) in Eqs. (15), (18), (20) and (22), respectively, where $p = 1, 2, 3, \cdots, m$. The terms of $\sum_{j=N_1+\cdots+N_{p-1}+1}^{N_1+\cdots+N_p} T(s_j^I, x_i^I)$ and $\sum_{j=N_1+\cdots+N_{p-1}+1}^{N_1+\cdots+N_p} M(s_j^I, x_i^I)$ are the adding-back terms and the terms of $T(s_i^{I \text{ or } O}, x_i^{I \text{ or } O})$ and $M(s_i^{I \text{ or } O}, x_i^{I \text{ or } O})$ are the subtracting terms in the two brackets for regularization. After using the abovementioned method of regularization of subtracting and adding-back techniques [Chen; Kao; Chen; Young and Lu (2006), Young; Chen and Lee (2005)], we are able to remove the singularity and hypersingularity of the kernel functions.

5 Derivation of influence matrices for arbitrary domain problems

5.1 Interior problem (Inclusion)

From Eqs. (15) and (18), the linear algebraic system can be obtained as:

$$\begin{Bmatrix} u_1 \\ \vdots \\ u_N \end{Bmatrix} = \begin{bmatrix} [T_{11}^I] & \cdots & [T_{1N}^I] \\ \vdots & \ddots & \vdots \\ [T_{N1}^I] & \cdots & [T_{NN}^I] \end{bmatrix} \begin{Bmatrix} \alpha_1 \\ \vdots \\ \alpha_N \end{Bmatrix}, \quad q \in w \text{ or } \phi, \tag{25}$$

$$\begin{Bmatrix} t_1 \\ \vdots \\ t_N \end{Bmatrix} = \begin{bmatrix} [M_{11}^I] & \cdots & [M_{1N}^I] \\ \vdots & \ddots & \vdots \\ [M_{N1}^I] & \cdots & [M_{NN}^I] \end{bmatrix} \begin{Bmatrix} \alpha_1 \\ \vdots \\ \alpha_N \end{Bmatrix}, \quad q \in w \text{ or } \phi, \tag{26}$$

where w and ϕ denote the out-of-plane elastic displacement and in-of-plane electric potential, respectively, and

$$\left[T_{11}^I\right]=\begin{bmatrix} A_{11} & T(s_2^I,x_1^I) & \cdots & T(s_{N_1}^I,x_1^I) \\ T(s_1^I,x_2^I) & A_{22} & \cdots & T(s_{N_1}^I,x_2^I) \\ \vdots & \vdots & \ddots & \vdots \\ T(s_1^I,x_{N_1}^I) & T(s_2^I,x_{N_1}^I) & \cdots & A_{NN} \end{bmatrix}_{N_1\times N_1}, \tag{27}$$

where

$$A_{11}=-\left[\sum_{j=1}^{N_1} T(s_j^I,x_1^I)-T(s_1^I,x_1^I)\right],$$
$$A_{22}=-\left[\sum_{j=1}^{N_1} T(s_j^I,x_2^I)-T(s_2^I,x_2^I)\right],$$
$$A_{NN}=-\left[\sum_{j=1}^{N_1} T(s_j^I,x_{N_1}^I)-T(s_{N_1}^I,x_{N_1}^I)\right].$$

$$\left[T_{1N}^I\right]=\begin{bmatrix} T(s_{N_1+\cdots+N_{m-1}+1}^I,x_1^I) & T(s_{N_1+\cdots+N_{m-1}+2}^I,x_1^I) & \cdots & T(s_N^I,x_1^I) \\ T(s_{N_1+\cdots+N_{m-1}+1}^I,x_2^I) & T(s_{N_1+\cdots+N_{m-1}+2}^I,x_2^I) & \cdots & T(s_N^I,x_2^I) \\ \vdots & \vdots & \ddots & \vdots \\ T(s_{N_1+\cdots+N_{m-1}+1}^I,x_{N_1}^I) & T(s_{N_1+\cdots+N_{m-1}+2}^I,x_{N_1}^I) & \cdots & T(s_N^I,x_{N_1}^I) \end{bmatrix}_{N_1\times N_m}, \tag{28}$$

$$\left[T_{N1}^I\right]=\begin{bmatrix} T(s_1^I,x_{N_1+\cdots+N_{m-1}+1}^I) & T(s_2^I,x_{N_1+\cdots+N_{m-1}+1}^I) & \cdots & T(s_{N_1}^I,x_{N_1+\cdots+N_{m-1}+1}^I) \\ T(s_1^I,x_{N_1+\cdots+N_{m-1}+2}^I) & T(s_2^I,x_{N_1+\cdots+N_{m-1}+2}^I) & \cdots & T(s_{N_1}^I,x_{N_1+\cdots+N_{m-1}+2}^I) \\ \vdots & \vdots & \ddots & \vdots \\ T(s_1^I,x_N^I) & T(s_2^I,x_N^I) & \cdots & T(s_{N_1}^I,x_N^I) \end{bmatrix}_{N_m\times N_1}, \tag{29}$$

$$\left[T_{NN}^I\right]=\begin{bmatrix} A_{11} & \cdots & T(s_{N_1+\cdots+N_{m-1}+1}^I,x_N^I) \\ \vdots & \ddots & \vdots \\ T(s_N^I,x_{N_1+\cdots+N_{m-1}+1}^I) & \cdots & A_{NN} \end{bmatrix}_{N_m\times N_m}, \tag{30}$$

where

$$A_{11} = -\left[\sum_{j=N_1+\cdots N_{m-1}+1}^{N} T(s_j^I, x_{N_1+\cdots+N_{m-1}+1}^I) - T(s_{N_1+\cdots+N_{m-1}+1}^I, x_{N_1+\cdots+N_{m-1}+1}^I)\right],$$

$$A_{NN} = -\left[\sum_{j=N_1+\cdots N_{m-1}+1}^{N} T(s_j^I, x_N^I) - T(s_N^I, x_N^I)\right].$$

$$\left[M_{11}^I\right] = \begin{bmatrix} A_{11} & M(s_2^I, x_1^I) & \cdots & M(s_{N_1}^I, x_1^I) \\ M(s_1^I, x_2^I) & A_{22} & \cdots & M(s_{N_1}^I, x_2^I) \\ \vdots & \vdots & \ddots & \vdots \\ M(s_1^I, x_{N_1}^I) & M(s_2^I, x_{N_1}^I) & \cdots & A_{NN} \end{bmatrix}_{N_1 \times N_1}, \tag{31}$$

where

$$A_{11} = -\left[\sum_{j=1}^{N_1} M(s_j^I, x_1^I) - M(s_1^I, x_1^I)\right],$$

$$A_{22} = -\left[\sum_{j=1}^{N_1} M(s_j^I, x_2^I) - M(s_2^I, x_2^I)\right],$$

$$A_{NN} = -\left[\sum_{j=1}^{N_1} M(s_j^I, x_{N_1}^I) - M(s_{N_1}^I, x_{N_1}^I)\right].$$

$$\left[M_{1N}^I\right] = \begin{bmatrix} M(s_{N_1+\cdots+N_{m-1}+1}^I, x_1^I) & M(s_{N_1+\cdots+N_{m-1}+2}^I, x_1^I) & \cdots & M(s_N^I, x_1^I) \\ M(s_{N_1+\cdots+N_{m-1}+1}^I, x_2^I) & M(s_{N_1+\cdots+N_{m-1}+2}^I, x_2^I) & \cdots & M(s_N^I, x_2^I) \\ \vdots & \vdots & \ddots & \vdots \\ M(s_{N_1+\cdots+N_{m-1}+1}^I, x_{N_1}^I) & M(s_{N_1+\cdots+N_{m-1}+2}^I, x_{N_1}^I) & \cdots & M(s_N^I, x_{N_1}^I) \end{bmatrix}_{N_1 \times N_m}, \tag{32}$$

$$\left[M_{N1}^I\right] = \begin{bmatrix} M(s_1^I, x_{N_1+\cdots+N_{m-1}+1}^I) & M(s_2^I, x_{N_1+\cdots+N_{m-1}+1}^I) & \cdots & M(s_{N_1}^I, x_{N_1+\cdots+N_{m-1}+1}^I) \\ M(s_1^I, x_{N_1+\cdots+N_{m-1}+2}^I) & M(s_2^I, x_{N_1+\cdots+N_{m-1}+2}^I) & \cdots & M(s_{N_1}^I, x_{N_1+\cdots+N_{m-1}+2}^I) \\ \vdots & \vdots & \ddots & \vdots \\ M(s_1^I, x_N^I) & M(s_2^I, x_N^I) & \cdots & M(s_{N_1}^I, x_N^I) \end{bmatrix}_{N_m \times N_1}, \tag{33}$$

$$\left[M_{NN}^{I}\right]=\begin{bmatrix} A_{11} & \cdots & M(s_{N_1+\cdots+N_{m-1}+1}^{I},x_{N}^{I}) \\ \vdots & \ddots & \vdots \\ M(s_{N}^{I},x_{N_1+\cdots+N_{m-1}+1}^{I}) & \cdots & A_{NN} \end{bmatrix}_{N_m\times N_m}, \qquad (34)$$

where

$$A_{11}=-\left[\sum_{j=N_1+\cdots N_{m-1}+1}^{N} M(s_j^{I},x_{N_1+\cdots+N_{m-1}+1}^{I})-M(s_{N_1+\cdots+N_{m-1}+1}^{I},x_{N_1+\cdots+N_{m-1}+1}^{I})\right],$$

$$A_{NN}=-\left[\sum_{j=N_1+\cdots N_{m-1}+1}^{N} M(s_j^{I},x_N^{I})-M(s_N^{I},x_N^{I})\right].$$

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Eqs. (20) and (22) yield

$$\begin{Bmatrix} u_1 \\ \vdots \\ u_N \end{Bmatrix}=\begin{bmatrix} \left[T_{11}^{O}\right] & \cdots & \left[T_{1N}^{O}\right] \\ \vdots & \ddots & \vdots \\ \left[T_{N1}^{O}\right] & \cdots & \left[T_{NN}^{O}\right] \end{bmatrix}\begin{Bmatrix} \alpha_1 \\ \vdots \\ \alpha_N \end{Bmatrix}, \quad q\in w \text{ or } \phi, \qquad (35)$$

$$\begin{Bmatrix} t_1 \\ \vdots \\ t_N \end{Bmatrix}=\begin{bmatrix} \left[M_{11}^{O}\right] & \cdots & \left[M_{1N}^{O}\right] \\ \vdots & \ddots & \vdots \\ \left[M_{N1}^{O}\right] & \cdots & \left[M_{NN}^{O}\right] \end{bmatrix}\begin{Bmatrix} \alpha_1 \\ \vdots \\ \alpha_N \end{Bmatrix}, \quad q\in w \text{ or } \phi, \qquad (36)$$

in which

$$\left[T_{11}^{O}\right]=\begin{bmatrix} A_{11} & T(s_2^{O},x_1^{O}) & \cdots & T(s_{N_1}^{O},x_1^{O}) \\ T(s_1^{O},x_2^{O}) & A_{22} & \cdots & T(s_{N_1}^{O},x_2^{O}) \\ \vdots & \vdots & \ddots & \vdots \\ T(s_1^{O},x_{N_1}^{O}) & T(s_2^{O},x_{N_1}^{O}) & \cdots & A_{NN} \end{bmatrix}_{N_1\times N_1}, \qquad (37)$$

where

$$A_{11}=-\left[\sum_{j=1}^{N_1} T(s_j^{I},x_1^{I})-T(s_1^{O},x_1^{O})\right],$$

$$A_{22}=-\left[\sum_{j=1}^{N_1} T(s_j^{I},x_2^{I})-T(s_2^{O},x_2^{O})\right],$$

$$A_{NN}=-\left[\sum_{j=1}^{N_1} T(s_j^{I},x_{N_1}^{I})-T(s_{N_1}^{O},x_{N_1}^{O})\right].$$

$$\left[T_{1N}^{O}\right]=\begin{bmatrix} T(s_{N_1+\cdots+N_{m-1}+1}^{O},x_1^{O}) & T(s_{N_1+\cdots+N_{m-1}+2}^{O},x_1^{O}) & \cdots & T(s_N^{O},x_1^{O}) \\ T(s_{N_1+\cdots+N_{m-1}+1}^{O},x_2^{O}) & T(s_{N_1+\cdots+N_{m-1}+2}^{O},x_2^{O}) & \cdots & T(s_N^{O},x_2^{O}) \\ \vdots & \vdots & \ddots & \vdots \\ T(s_{N_1+\cdots+N_{m-1}+1}^{O},x_{N_1}^{O}) & T(s_{N_1+\cdots+N_{m-1}+2}^{O},x_{N_1}^{O}) & \cdots & T(s_N^{O},x_{N_1}^{O}) \end{bmatrix}_{N_1\times N_m}, \tag{38}$$

$$\left[T_{N1}^{O}\right]=$$
$$\begin{bmatrix} T(s_1^{O},x_{N_1+\cdots+N_{m-1}+1}^{O}) & T(s_2^{O},x_{N_1+\cdots+N_{m-1}+1}^{O}) & \cdots & T(s_{N_1}^{O},x_{N_1+\cdots+N_{m-1}+1}^{O}) \\ T(s_1^{O},x_{N_1+\cdots+N_{m-1}+2}^{O}) & T(s_2^{O},x_{N_1+\cdots+N_{m-1}+2}^{O}) & \cdots & T(s_{N_1}^{O},x_{N_1+\cdots+N_{m-1}+2}^{O}) \\ \vdots & \vdots & \ddots & \vdots \\ T(s_1^{O},x_N^{O}) & T(s_2^{O},x_N^{O}) & \cdots & T(s_{N_1}^{O},x_N^{O}) \end{bmatrix}_{N_m\times N_1}, \tag{39}$$

$$\left[T_{NN}^{O}\right]=\begin{bmatrix} A_{11} & \cdots & T(s_{N_1+\cdots+N_{m-1}+1}^{O},x_N^{O}) \\ \vdots & \ddots & \vdots \\ T(s_N^{O},x_{N_1+\cdots+N_{m-1}+1}^{O}) & \cdots & A_{NN} \end{bmatrix}_{N_m\times N_m}, \tag{40}$$

where

$$A_{11}=-\left[\sum_{j=N_1+\cdots N_{m-1}+1}^{N} T(s_j^{I},x_{N_1+\cdots+N_{m-1}+1}^{I})-T(s_{N_1+\cdots+N_{m-1}+1}^{O},x_{N_1+\cdots+N_{m-1}+1}^{O})\right],$$

$$A_{NN}=-\left[\sum_{j=N_1+\cdots N_{m-1}+1}^{N} T(s_j^{I},x_N^{I})-T(s_N^{O},x_N^{O})\right].$$

$$\left[M_{11}^{O}\right]=\begin{bmatrix} A_{11} & M(s_2^{O},x_1^{O}) & \cdots & M(s_{N_1}^{O},x_1^{O}) \\ M(s_1^{O},x_2^{O}) & A_{22} & \cdots & M(s_{N_1}^{O},x_2^{O}) \\ \vdots & \vdots & \ddots & \vdots \\ M(s_1^{O},x_{N_1}^{O}) & M(s_2^{O},x_{N_1}^{O}) & \cdots & A_{NN} \end{bmatrix}_{N_1\times N_1}, \tag{41}$$

where

$$A_{11}=-\left[\sum_{j=1}^{N_1} M(s_j^{I},x_1^{I})-M(s_1^{O},x_1^{O})\right],$$

$$A_{22}=-\left[\sum_{j=1}^{N_1} M(s_j^{I},x_2^{I})-M(s_2^{O},x_2^{O})\right],$$

$$A_{NN}=-\left[\sum_{j=1}^{N_1} M(s_j^{I},x_{N_1}^{I})-M(s_{N_1}^{O},x_{N_1}^{O})\right]$$

$$\left[M_{1N}^{O}\right] = \begin{bmatrix} M(s^{O}_{N_1+\cdots+N_{m-1}+1}, x^{O}_{1}) & M(s^{O}_{N_1+\cdots+N_{m-1}+2}, x^{O}_{1}) & \cdots & M(s^{O}_{N}, x^{O}_{1}) \\ M(s^{O}_{N_1+\cdots+N_{m-1}+1}, x^{O}_{2}) & M(s^{O}_{N_1+\cdots+N_{m-1}+2}, x^{O}_{2}) & \cdots & M(s^{O}_{N}, x^{O}_{2}) \\ \vdots & \vdots & \ddots & \vdots \\ M(s^{O}_{N_1+\cdots+N_{m-1}+1}, x^{O}_{N_1}) & M(s^{O}_{N_1+\cdots+N_{m-1}+2}, x^{O}_{N_1}) & \cdots & M(s^{O}_{N}, x^{O}_{N_1}) \end{bmatrix}_{N_1\times N_m}, \tag{42}$$

$$\left[M_{N1}^{O}\right] = \begin{bmatrix} M(s^{O}_{1}, x^{O}_{N_1+\cdots+N_{m-1}+1}) & M(s^{O}_{2}, x^{O}_{N_1+\cdots+N_{m-1}+1}) & \cdots & M(s^{O}_{N_1}, x^{O}_{N_1+\cdots+N_{m-1}+1}) \\ M(s^{O}_{1}, x^{O}_{N_1+\cdots+N_{m-1}+2}) & M(s^{O}_{2}, x^{O}_{N_1+\cdots+N_{m-1}+2}) & \cdots & M(s^{O}_{N_1}, x^{O}_{N_1+\cdots+N_{m-1}+2}) \\ \vdots & \vdots & \ddots & \vdots \\ M(s^{O}_{1}, x^{O}_{N}) & M(s^{O}_{2}, x^{O}_{N}) & \cdots & M(s^{O}_{N_1}, x^{O}_{N}) \end{bmatrix}_{N_m\times N_1}, \tag{43}$$

$$\left[M_{NN}^{O}\right] = \begin{bmatrix} A_{11} & \cdots & M(s^{O}_{N_1+\cdots+N_{m-1}+1}, x^{O}_{N}) \\ \vdots & \ddots & \vdots \\ M(s^{O}_{N}, x^{O}_{N_1+\cdots+N_{m-1}+1}) & \cdots & A_{NN} \end{bmatrix}_{N_m\times N_m}, \tag{44}$$

where

$$A_{11} = -\left[\sum_{j=N_1+\cdots N_{m-1}+1}^{N} M(s^{I}_{j}, x^{I}_{N_1+\cdots+N_{m-1}+1}) - M(s^{O}_{N_1+\cdots+N_{m-1}+1}, x^{O}_{N_1+\cdots+N_{m-1}+1})\right],$$

$$A_{NN} = -\left[\sum_{j=N_1+\cdots N_{m-1}+1}^{N} M(s^{I}_{j}, x^{I}_{N}) - M(s^{O}_{N}, x^{O}_{N})\right].$$

6 Derivation of influence matrices for piezoelectricity problems

Substituting Eqs. (25), (26), (35) and (36) into Eqs. (7) and (8), the linear algebraic system for antiplane piezoelectricity problem can be obtained as:

$$\begin{bmatrix} -\left[T_w^I\right] & \left[T_w^O\right] & 0 & 0 \\ 0 & 0 & -\left[T_\phi^I\right] & \left[T_\phi^O\right] \\ -\frac{c_{44}^i}{c_{44}^m}\left[M_w^I\right] & -\left[M_w^O\right] & -\frac{e_{15}^i}{c_{44}^m}\left[M_\phi^I\right] & -\frac{e_{15}^m}{c_{44}^m}\left[M_\phi^O\right] \\ -\left[M_w^I\right] & -\frac{e_{15}^m}{e_{15}^i}\left[M_w^O\right] & \frac{\varepsilon_{11}^i}{e_{15}^i}\left[M_\phi^I\right] & \frac{\varepsilon_{11}^m}{e_{15}^i}\left[M_\phi^O\right] \end{bmatrix}_{4\times 4} \begin{Bmatrix} \left\{\alpha_w^i\right\} \\ \left\{\alpha_w^m\right\} \\ \left\{\alpha_\phi^i\right\} \\ \left\{\alpha_\phi^m\right\} \end{Bmatrix}_{4\times 1} = \begin{Bmatrix} -\left\{w^\infty\right\} \\ -\left\{\phi^\infty\right\} \\ \left\{\frac{\partial w}{\partial n}^\infty\right\} + \frac{e_{15}^m}{c_{44}^m}\left\{\frac{\partial \phi}{\partial n}^\infty\right\} \\ \frac{e_{15}^m}{e_{15}^i}\left\{\frac{\partial w}{\partial n}^\infty\right\} - \frac{\varepsilon_{11}^m}{e_{15}^i}\left\{\frac{\partial \phi}{\partial n}^\infty\right\} \end{Bmatrix}_{4\times 1}, \quad (45)$$

where w and ϕ denote the out-of-plane elastic displacement and electric potential, respectively. The unknown densities ($\{\alpha_w^i\}$, $\{\alpha_w^m\}$, $\left\{\alpha_\phi^i\right\}$, $\left\{\alpha_\phi^m\right\}$) in Eq. (45) can be obtained by implementing the linear algebraic solver and the stress concentration can be solved by using Eq. (3). To express clearly, the solution procedures is listed in Fig. 4.

7 Numerical examples

In order to show the accuracy and validity of the proposed method, the antiplane piezoelectricity problems with multiple inclusions subjected to the remote shear and the far-electric field are considered. Two examples contain single piezoelectric inclusion and two piezoelectric inclusions under antiplane shear, respectively.

7.1 Single piezoelectric inclusion

The single piezoelectric inclusion in a piezoelectric matrix is shown in Fig. 5.

In this case, the remote shear, shear modulus, piezoelectric modulus, dielectric modulus and elastic modulus are $\tau = 5\times 10^7$ Nm^{-2}, $e_{15}^i = 10.0$ Cm^{-2}, $\varepsilon_{11}^m = \varepsilon_{11}^i = 1.51\times 10^{-8}$ CV^{-1}m^{-1}and $c_{44}^m = c_{44}^i = 3.53\times 10^{10}$ Nm^{-2}, respectively. Stress concentrations versus different piezoelectric modulus ratio are shown in Figs. 6 and 7, respectively. When $E = -10^6$V/m and $e_{15}^m/e_{15}^i = -10$ for negative poling direction, the negative maximum stress concentration occurs in the matrix of $\theta = 0$ as shown in Fig. 6. However, the positive maximum stress concentration occurs in the matrix of $\theta = \pi/2$ as shown in Fig. 7. Contours of electric potential ϕ and shear

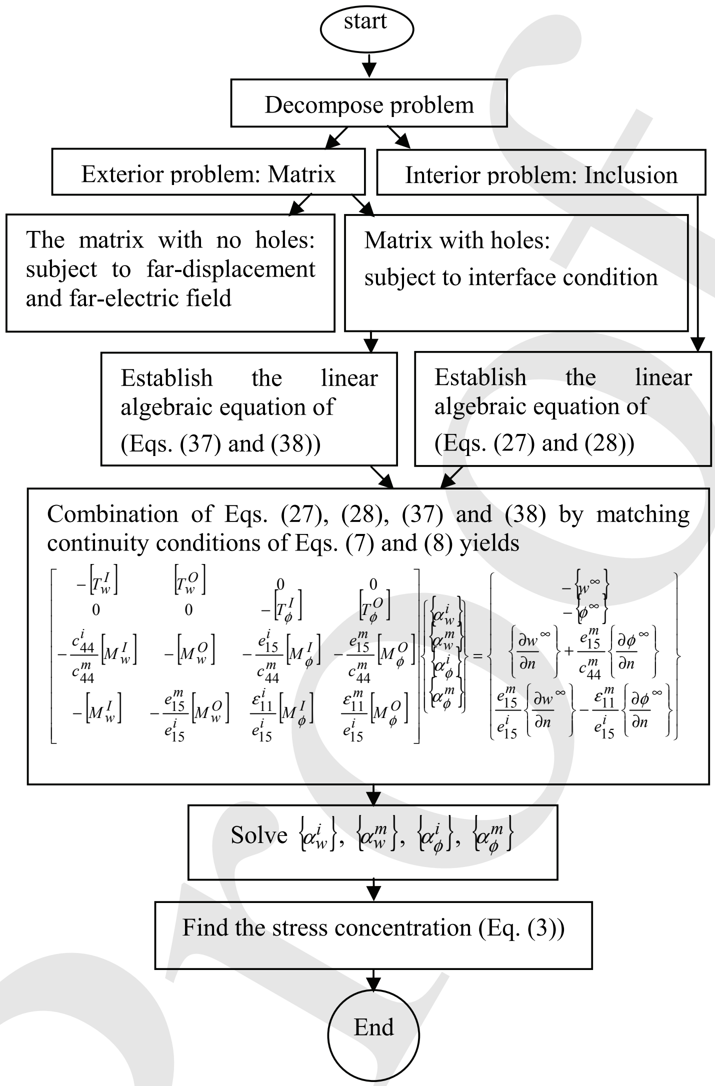

Figure 4: Flowchart of solution procedures

stress σ_{zy}^{m} are plotted in Fig. 8 (a)$\sim$(b), respectively. Good agreement is made after comparing with the analytical solution [Honein and Honein (1995)].

7.2 Two piezoelectric inclusions

Two piezoelectric inclusions in piezoelectric matrix are shown in Fig. 9.

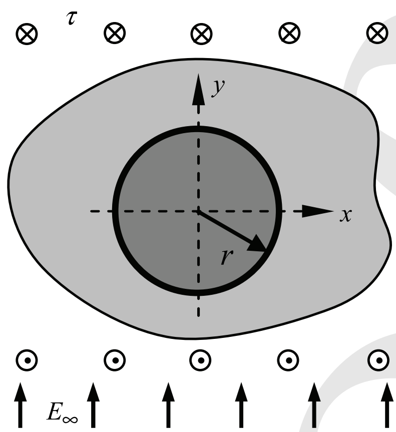

Figure 5: Problem sketch of single piezoelectric inclusion

The remote loading and material constants are $\tau = 5\times10^7\text{Nm}^{-2}$, $c_{44}^m = c_{44}^i = 3.53\times10^{10}\text{Nm}^{-2}$, $\varepsilon_{11}^m = \varepsilon_{11}^i = 1.51\times10^{-8}\text{CV}^{-1}\text{m}^{-1}$ and $e_{15}^i = 10.0\text{Cm}^{-2}$, respectively. Stress concentrations $\sigma_{z\theta}^m/\tau$ versus different piezoelectric modulus ratios are plotted in Fig. 10. On the other hand, stress concentrations σ_{zr}^m/τ versus different piezoelectric modulus ratios are respectively plotted in Fig. 11. The negative maximum stress concentration occurs in the matrix of $\theta = 0$ and $\beta = \pi/2$ as shown in Fig. 10 when $E = -10^6$v/m and $e_{15}^m/e_{15}^i = -10$. However, the maximum stress concentration occurs in the matrix at $\theta = \pi/2$ and $\beta = \pi/2$ as shown in Fig. 11.

When $E = 10^6$v/m, $e_{15}^m/e_{15}^i = -5$ and $\beta = \pi/2$, the tangential electric field along the boundaries of the matrix distribution function of the different ratios d/r_1 are shown in Fig. 12 (a)$\sim$(c).

It is interesting to find that the tangential electric field is not continuous at $\theta = \pi/2$, when the inclusion approaches another inclusion. Stress concentrations of the different ratios of d/r_1 at $\beta = 0$ versus piezoelectric modulus ratio are shown in Fig. 13. It is found that the stress concentration factor becomes larger, when the two inclusions approach each other inclusion. The results are well compared with those of the method of successive approximations [Chao and Chang (1999)].

8 Conclusions

In this study, we employ the RMM to solve piezoelectricity problems with multiple inclusions under antiplane shear and in-plane electric field. Only the boundary nodes on the physical boundary are required. The major difficulty of the coincidence of the source and collocation points in the conventional MFS is then circum-

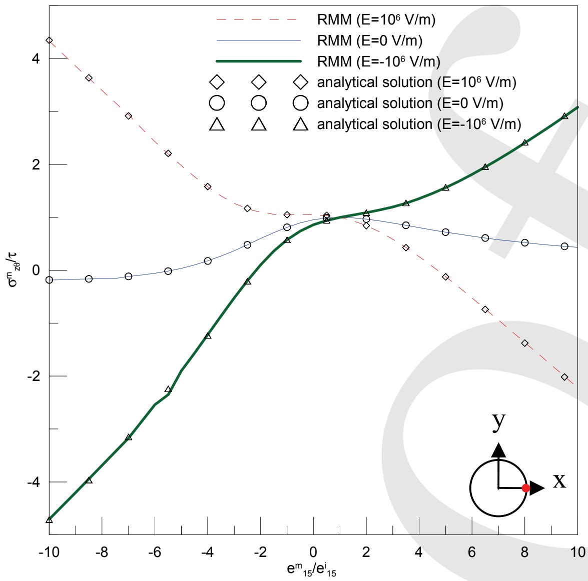

Figure 6: Stress concentration $\sigma^m_{z\theta}/\tau$ result of single piezoelectric inclusion in piezoelectric matrix for different piezoelectric module ratios and electric field

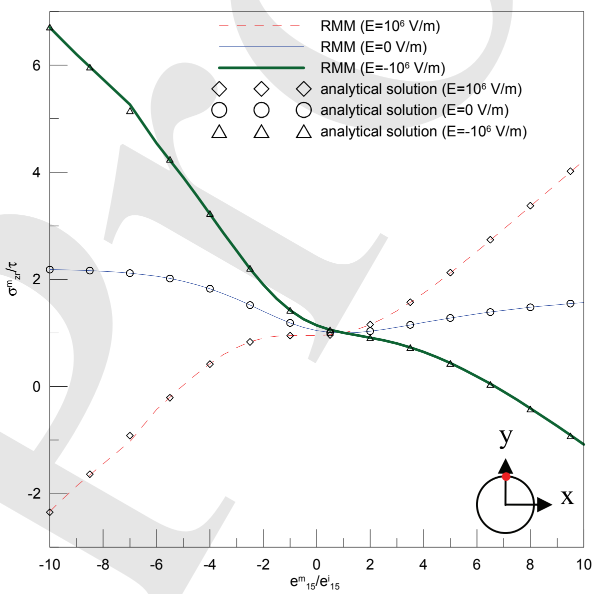

Figure 7: Stress concentration σ^m_{zr}/τ result of single piezoelectric inclusion in piezoelectric matrix for different piezoelectric module ratios and electric field

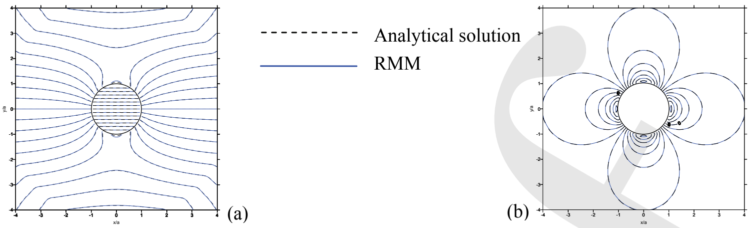

Figure 8: Contours result of single piezoelectric inclusion in piezoelectric matrix, (a) contours of constant for electric potential ϕ, (b) contours of constant for shear stress σ_{zy}^{m}

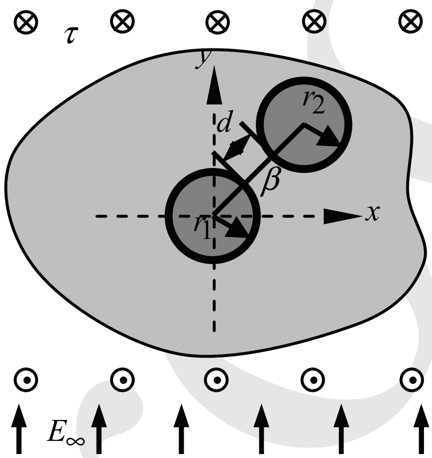

Figure 9: Problem sketch of two piezoelectric inclusions

vented. Furthermore, the controversy of the fictitious boundary outside the physical domain by using the conventional MFS no longer exists. Although it results in the singularity and hypersingularity due to the use of double layer potential, the finite values of the diagonal terms for the influence matrices have been determined by employing the regularization technique. The numerical results were obtained by applying the developed program to solve piezoelectricity problems through two examples. Numerical results agreed very well with the analytical solution [Honein and Honein (1995)] and those of the method of successive approximations [Chao and Chang (1999)].

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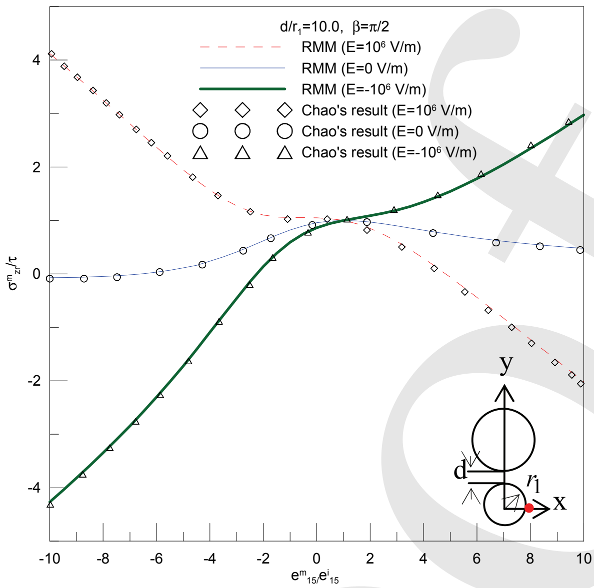

Figure 10: Stress concentration $\sigma^m_{z\theta}/\tau$ result of double piezoelectric inclusions in piezoelectric matrix for different piezoelectric module ratios and electric field

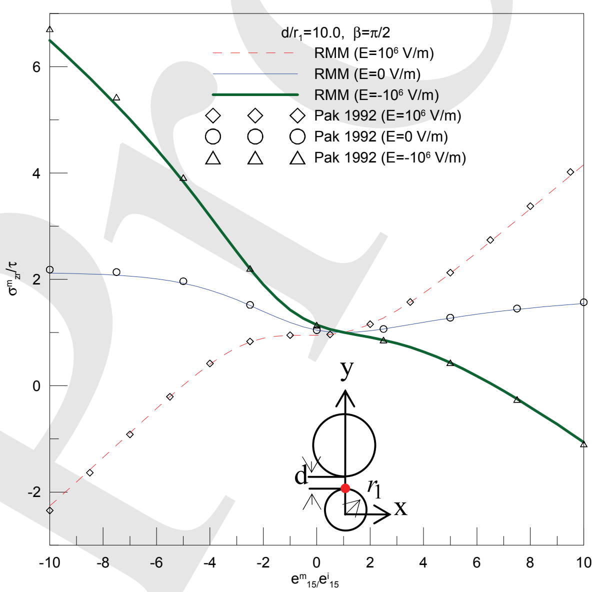

Figure 11: Stress concentration σ^m_{zr}/τ result of double piezoelectric inclusions in piezoelectric matrix for different piezoelectric module ratios and electric field

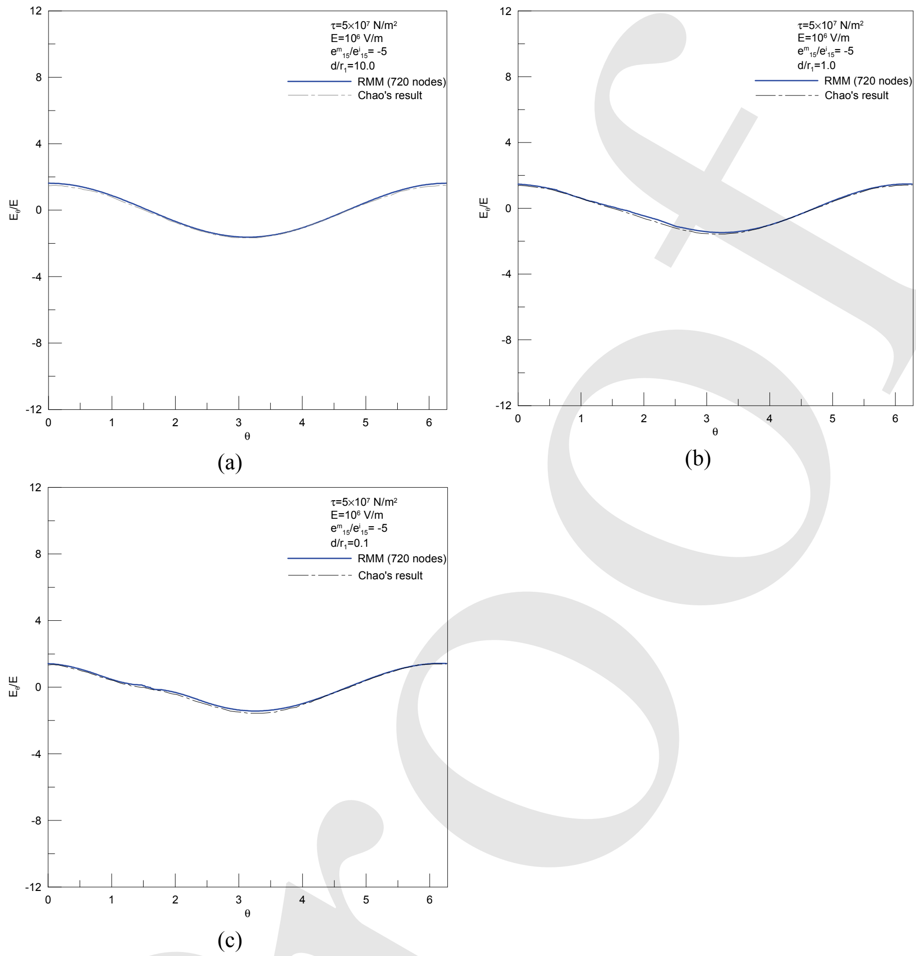

Figure 12: T

angential electric field distribution along the boundaries of first inclusion for different ratios d/r_1 with $\beta = \pi/2$, (a) $d/r_1 = 10.0$, (b) $d/r_1 = 1.0$, (c) $d/r_1 = 0.1$

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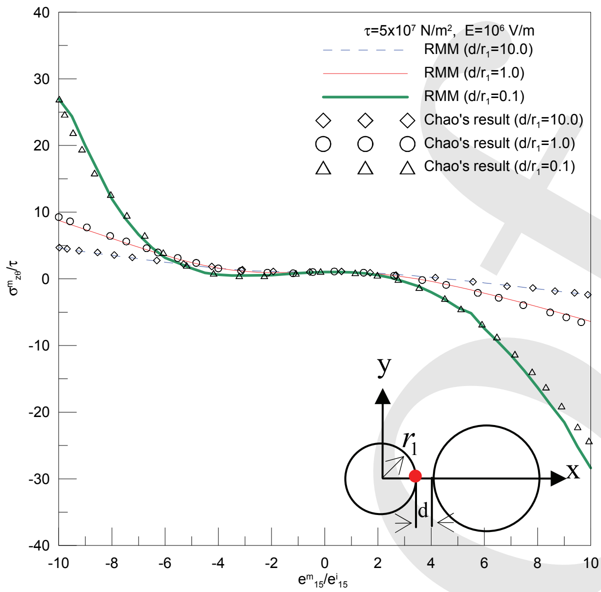

Figure 13: Stress concentration for different ratios d/r_1 of piezoelectric constants with $\beta = 0$

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