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FREE VIBRATION OF A SDOF SYSTEM WITH HYSTERETIC DAMPING

L. Y. Chen[1], J. T. Chen[2], C. H. Chen[1] and H.-K. Hong[1]
[1]Department of Civil Engineering, National Taiwan University, Taipei, Taiwan
[2]Department of Harbor and River Engineering, National Ocean University, Keelung, Taiwan

Corresponding address : P.O. Box 23-36, Taipei, Taiwan

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Introduction

The damping characteristic is often utilized to suppress the vibration level using various energy dissipation mechanisms. Damping models, e.g., viscous, Coulomb and hysteresis damping, have been discussed in detail in the literature of structural dynamics and viscoelasticity. A great deal of effort has been focused on the frequency domain approach, especially for the hysteretic damping model. However, free vibration of a single degree of freedom(SDOF) system with damping of hysteretic type has not been exactly solved yet in the time domain to the authors' knowledge. Many researchers are of the opinion that the problem is still challenging us now [1,2,3,4].

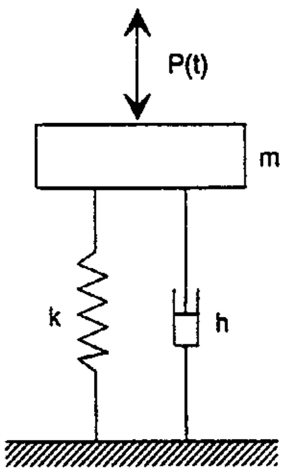

FIG.1 Hysteretic damping model of a single degree of freedom

The governing equation in the time domain of the SDOF system shown in FIG.1 has been formulated as[5]

$$m\ddot{u} + \frac{h}{\omega}\dot{u} + ku = \bar{P}e^{i\omega t} \tag{1}$$

where m, h and k represent mass, hysteretic damping coefficient and stiffness, respectively. $\bar{P}$ and ω are the amplitude of harmonic loading and exciting frequency, respectively. To make the transfer functions conjugate for $-\omega$ and ω, the governing equation has been modified to be[6]

$$m\ddot{u} + \frac{h}{|\omega|}\dot{u} + ku = \bar{P}e^{i\omega t} \tag{2}$$

Although good for harmonic motion,Eq.(2) is invalid for free vibration since, when ($\bar{P}e^{-i\omega t} = 0$) is set to vanish, the presence of $|\omega|$ in Eq.(2) is ambiguous. In this paper,the governing equation for free vibration in the time domain is rewritten free of frequency and solved analytically by the concept of phase plane. The decrement ratio of the maximum response and the damped period of free vibration are also derived. Two examples, one subjected to initial disturbance of displacement and the other initial velocity disturbance, are illustrated to show the validity of the present formulation.

Formulation

The definition of hysteretic damping has been defined by Clough and Penzien[7] where the damping force is proportional to the amplitude of the displacement and is in phase with the velocity. Therefore, the damping force, f_d, of a SDOF system as shown in FIG.1 can be expressed as

$$f_d \equiv h\frac{|u|}{|\dot{u}|}\dot{u} \tag{3}$$

The governing equation can be derived as

$$m\ddot{u} + h\frac{|u|}{|\dot{u}|}\dot{u} + ku = P(t) \tag{4}$$

When $P(t)$ is set to be $\bar{P}e^{i\omega t}$, the steady state solution, $u = \bar{u}e^{i\omega t}$, is expected. Eq.(4) can be reformulated in frequency domain as follows:

$$-m\omega^2\bar{u} + k(1 \pm i\eta)\bar{u} = \bar{P}, \ + \ when \ \omega > 0, - \ when \ \omega < 0 \tag{5}$$

where $k(1 \pm i\eta)$ denotes complex stiffness k^* with η denoting the loss factor by

$$\eta = \frac{h}{k} \tag{6}$$

When $\omega > 0$, k^* reduces to $k(1 + i\eta)$, which is the conventional complex stiffness. Nevertheless, the conjugacy of complex stiffness for positive and negative frequencies is crucial. Since ω is not present, Eqs.(4) and (5) can be viewed as the governing equations of the SDOF system with the hysteretic damping in the time and the frequency domains, respectively. In the case of free vibration $P(t) = 0$, Eq.(5) is not applicable, and Eq.(4) becomes

$$m\ddot{u} + h\frac{|u|}{|\dot{u}|}\dot{u} + ku = 0 \tag{7}$$

If we set the undamped ($h = 0$) natural frequency ω_n to be

$$\omega_n^2 = \frac{k}{m} \tag{8}$$

then, Eq.(7) can be written as

$$\ddot{u} + \omega_n^2 \eta \frac{|u|}{|\dot{u}|}\dot{u} + \omega_n^2 u = 0 \tag{9}$$

which is subject to the following initial conditions:

$$u(0) = u_0 \tag{10}$$

$$\dot{u}(0) = \dot{u}_0 \tag{11}$$

By using the phase plane method, the analytical solution in each quadrant of the phase plane of Eq.(9), with the prescribed initial state $(u_0, \dot{u}_0)$ and $0.0 < \eta < 1.0$, is

$$u(t) = u_0 \cos\bar{\omega}t + \frac{\dot{u}_0}{\bar{\omega}} \sin\bar{\omega}t \tag{12}$$

where

$$\bar{\omega}^2 = \begin{cases} \omega_n^2(1+\eta) & (u_0, \dot{u}_0) \text{ in the 1st or 3rd quadrant} \\ \omega_n^2(1-\eta) & (u_0, \dot{u}_0) \text{ in the 2nd or 4th quadrant} \end{cases} \tag{13}$$

The term $\bar{\omega}$ is the effective natural frequency, which is denoted as $\bar{\omega}_1$ in the 1st and 3rd quadrants and $\bar{\omega}_2$ in the 2nd and 4th quadrants as shown in FIG.2.

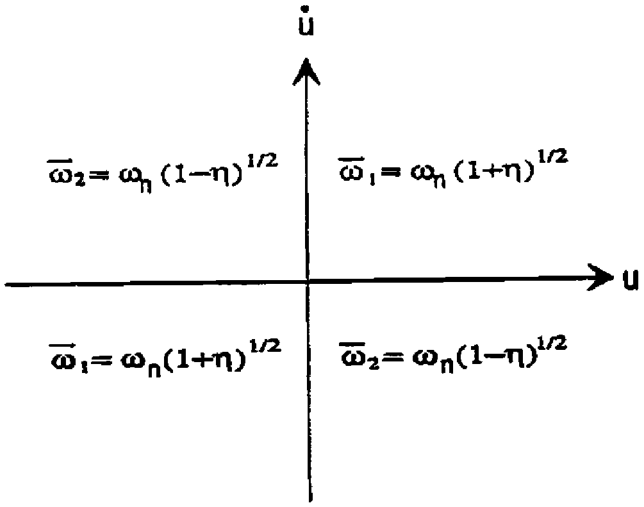

FIG.2 The dependence of the effective natural frequency for four quadrants in the phase plane

Because the effective frequency depends on which quadrant the state is, the hysteretic damping model is a time-varying system. Nevertheless, it is shown that the superposition of initial conditions is valid in view of Eq.(12); that is, the system with hysteretic damping as discussed herein is a linear system. According to Eq.(12), the trace on the phase plane can be shown as

$$u(t)^2 + \frac{\dot{u}(t)^2}{\bar{\omega}^2} = u_0^2 + \frac{\dot{u}_0^2}{\bar{\omega}^2} \tag{14}$$

Eq.(12) together with Eq.(14) is a combination of four quarter ellipses, each in each quadrant as shown in FIG.3, and the ratio of the intercepts between the two axes will be $\bar{\omega}_1$, or $\bar{\omega}_2$. The intercept on the positive u-axis represents the maximum or minimum responses in a cycle, so the ratio of two successive positive u intercepts is the ratio of decrement. The ratio of decrement δ can be derived as

$$\delta = \frac{A_2}{A_1} = \left(\frac{\bar{\omega}_2}{\bar{\omega}_1}\right)^2 = \frac{1-\eta}{1+\eta} \tag{15}$$

The time to travel through the 1st or 3rd quadrants once is T_1, and the time to travel through the 2nd or 4th quadrants once is T_2 as shown in FIG.3.

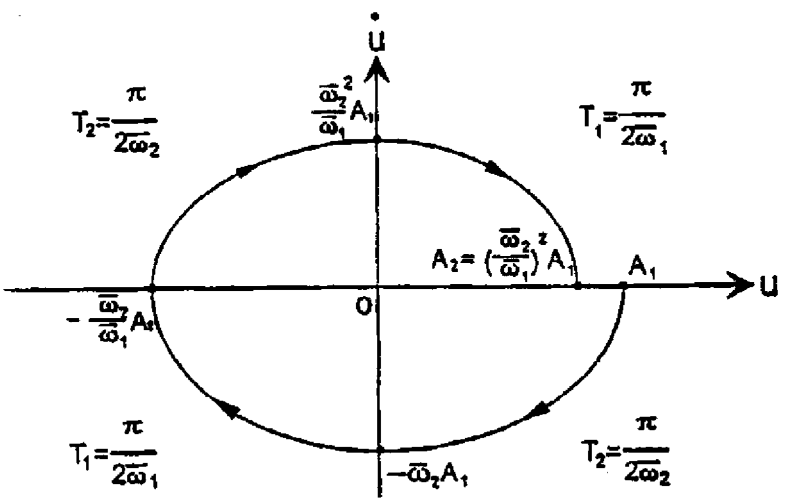

FIG.3 Typical trace of four elliptical curves in one cycle

Combining the four travelling times, the damped period, T_d, is

$$T_d = \frac{\pi}{\omega_n}\left(\frac{1}{\sqrt{1+\eta}} + \frac{1}{\sqrt{1-\eta}}\right) \tag{16}$$

The damped natural frequency ω_d is

$$\omega_d = 2\omega_n\left(\frac{\sqrt{1-\eta^2}}{\sqrt{1+\eta}+\sqrt{1-\eta}}\right) \tag{17}$$

The envelope function of each maximum (or minimum) response is found to be

$$Z(t) = A_1\left(\frac{1-\eta}{1+\eta}\right)^{\frac{t-t_1}{T_d}} \tag{18}$$

where A_1 is the maximum or minimum response occuring at time t_1. In order to be a stable system, the loss factor is larger than zero. Therefore, the ratio of decrement is lower than one, the envelope function will decrease gradually to zero as t increases to infinity.

Examples

In this section, two free vibration problems, initial disturbance of displacement and initial velocity disturbance are considered. The natural frequency, ω_n, of the SDOF system is 50 rad per second, and the loss factor η is 0.1.

The responses of initial displacement disturbance are shown in FIG.4, FIG.5 and FIG.6. FIG.4 shows that the displacement oscillation decreases gradually as time elapses, the dashed line representing the envelope function. FIG.5 shows that the velocity oscillation decreases with time. Fig.6 shows that the trace extends from the displacement axis to the velocity axis along an elliptical track in the 4th quadrant and then follows a smaller elliptical track in the 3rd quadrant. It also shows the same tendency in the 2nd and 1st quadrants. The tendencies of the other cycles are similar to that of the first cycle, and the final point of trace will focus on the origin of the phase plane. The responses of the initial velocity disturbance are shown in FIG.7, FIG.8, and FIG.9 and the tendency is similar to that shown in FIG.4, FIG.5 and FIG.6 except for the different starting state.

Concluding remarks

The closed-form solution for the free vibration of the hysteretic damping model is obtained in this paper. The hysteretic damping model reformulated in Eqs.(4) and (5) has been proved to be a linear time varying system. It has been found that each cycle contains four subcycles, and the explicit formulae for the decrement ratio and the damped natural frequency, which depend on the loss factor have also been obtained. The present note extends the applicability of the hysteretic damping model from the the frequency domain to the time domain. It can treat general loadings, harmonic or inharmonic, if the convolution integral is considered.

References

1. N. O. Myklestad, J. Appl. Mech. 19, 284(1952).
2. R. E. D. Bishop, J. Royal Aeronautical Society 59, 738(1955).
3. P. Lancaster, J. Royal Aeronautical Society 64, 229(1960).
4. R. E. D. Bishop and W. G. Price, Random Vibration, Elsevier, Amsterdam(1986).
5. L. Meirovitch, Elements of Vibration Analysis, McGraw-Hill, Singapore(1986).
6. A. D. Nashif, D. I. G. Jones and J. P. Henderson, Vibration Damping, John Wiley & Sons(1984).
7. R. W. Clough and J. Penzien, Dynamics of Structures, McGraw-Hill, New York(1975).

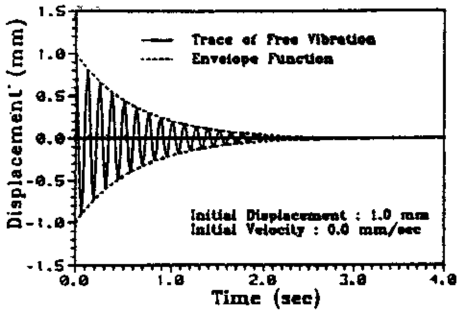

FIG.4 Displacement history for disturbance of initial displacement only

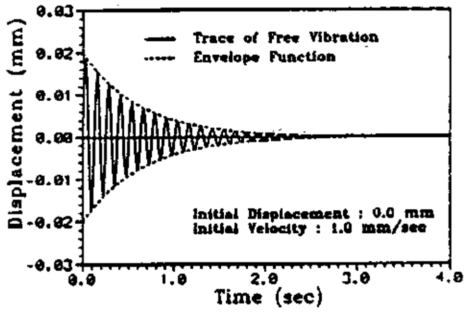

FIG.7 Displacement history for disturbance of initial velocity only

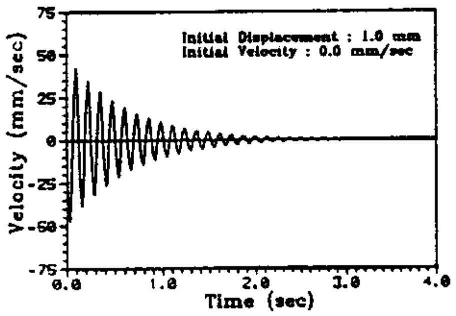

FIG.5 Velocity history for disturbance of initial displacement only

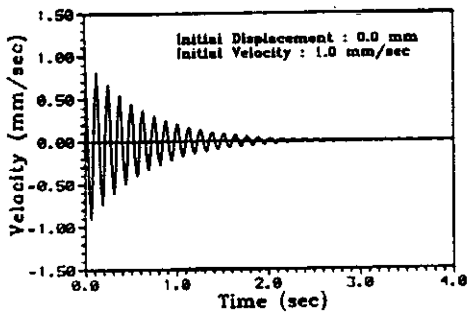

FIG.8 Velocity history for disturbance of initial velocity only

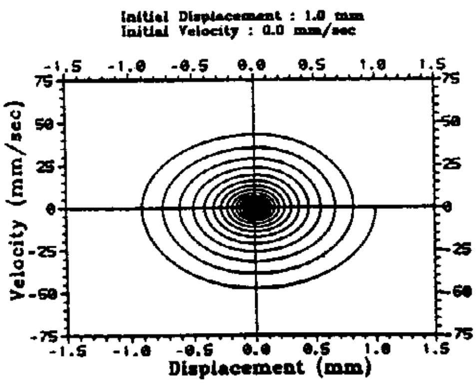

FIG.6 Phase plane for disturbance of initial displacement only

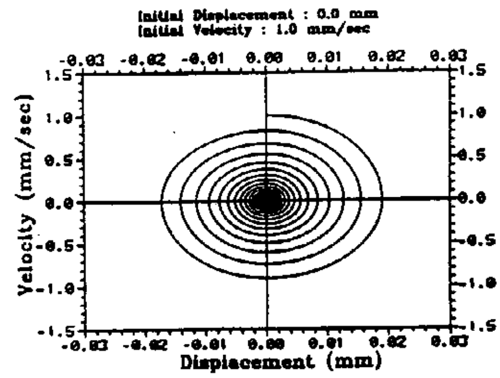

FIG.9 Phase plane for disturbance of initial velocity only