Applications of dual boundary integral equations to exterior acoustic problems

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ABSTRACT

This paper presents the mechanism why the irregular frequencies are imbedded in the exterior acoustics using the dual BEM. The relation between the matrices of influence coefficients for the interior and exterior acoustic problems is examined. Also, the irregular (fictitious) frequencies embedded in the singular or hypersingular integral equations are discussed, respectively. It is found that the irregular values depend on the kernels in the integral representation for the solution. A two-dimensional dual BEM program for the exterior acoustic problems was developed. Numerical experiments using the program are conducted to check the validity in comparison with the theoretical proof of the independence of boundary conditions which have been shown by Chen using the degenerate kernels. Both the radiation and scattering problems are considered. Two cases, including the Dirichlet and Neumann boundary conditions, show that the singular integral equation results in the fictitious frequencies which are associated with the eigenfrequencies of the interior Dirichlet problem, while the hypersingular integral equation results in the fictitious frequencies which are associated with the interior Neumann problem. Burton and Miller approach is employed to deal with the problem of fictitious frequencies.

Keywords: dual BEM, radiation, fictitious frequency and exterior acoustic problem

Introduction

The fictitious frequencies, or so called irregular frequencies in BEM, have been studied by mathematicians [1, 2, 3] and boundary element researchers [4] for a long time. For a continuous system, Chen [5, 6] proved analytically using the dual series model that the positions where the fictitious frequencies depend on the kernel in the integral representation for the solution. The types of boundary condition can not change the positions where fictitious frequency occurs once the integral formulation is chosen. Later, Chen and Kuo [7] applied the theory of circulant to understand the occuring mechanism of irregular frequencies in a discrete system by considering a circular example. However, no numerical examples using the dual BEM were provided in the two papers [6, 7].

In this paper, a dual BEM program was developed to study the fictitious frequencies numerically. The positions of fictitious frequencies for the exterior problems using the singular integral equation (UT) or the hypersingular integral equation (LM) will be examined. Four numerical examples of non-uniform radiation problems and scattering problems subject to the Dirichlet and Neumann boundary conditions, will be illustrated to show the mechanism of fictitious frequencies. Numerical results using the program will be verified in comparision with the analytical solutions and DtN results [11, 12, 13]. It is shown that the integral formulation, either singular or hypersingular equation, has different fictitious frequencies. However, the positions of irregular frequencies are independent of the types of the boundary condition, once the method, either the UT or the LM method, is adopted. To circument the problem of numerical instability near the fictitious frequency, the Burton and Miller method will be employed by combining the dual integral equations with an imaginary constant. Based on the theoretical proof for a continuous system [6], analytical derivation for a discrete system using circulants [7] and the present numerical study using the dual BEM, agreement among them can be made.

Dual integral formulation for two-dimensional radiation and scattering problems

The governing equation for an exterior acoustic problem is the Helmholtz equation as follows:

$$(\nabla^2 + k^2)u(x_1, x_2) = 0, \quad (x_1, x_2) \in D,$$

where ∇^2 is the Laplacian operator, D is the domain of the cavity and k is the wave number, which is angular frequency over the speed of sound. The boundary conditions can be either the Neumann or Dirichlet type. Based on the dual formulation, the dual equations for the boundary points are

$$\pi u(x) = C.P.V. \int_B T(s,x)u(s)dB(s) - R.P.V. \int_B U(s,x)t(s)dB(s), \quad x \in B \tag{1}$$

$$\pi t(x) = H.P.V. \int_B M(s,x)u(s)dB(s) - C.P.V. \int_B L(s,x)t(s)dB(s), \quad x \in B \tag{2}$$

where $C.P.V.$, $R.P.V.$ and $H.P.V.$ denote the Cauchy principal value, the Riemann principal value and the Hadamard principal value, $t(s) = \frac{\partial u(s)}{\partial n_s}$, B denotes the boundary enclosing D and the explicit forms of the four kernels, U, T, L and M, can be found in [5, 8, 9].

Dual BEM formulation for two-dimensional radiation and scattering problems

The linear algebraic equations for an interior problem discretized from the dual boundary integral equations can be written as

$$[T^i_{pq}]\{u_q\} = [U^i_{pq}]\{t_q\} \tag{3}$$

$$[M^i_{pq}]\{u_q\} = [L^i_{pq}]\{t_q\}, \tag{4}$$

where the superscript "i" denotes the interior problem, $\{u_q\}$ and $\{t_q\}$ are the boundary potential and flux, and the subscripts p and q correspond to the labels of the collocation element and integration element, respectively. The influence coefficients of the four square matrices $[U], [T], [L]$ and $[M]$ can be represented as

$$U^i_{pq} = R.P.V. \int_{B_q} U(s_q, x_p)dB(s_q) \tag{5}$$

$$T^i_{pq} = -\pi\delta_{pq} + C.P.V. \int_{B_q} T(s_q, x_p)dB(s_q) \tag{6}$$

$$L^i_{pq} = \pi\delta_{pq} + C.P.V. \int_{B_q} L(s_q, x_p)dB(s_q) \tag{7}$$

$$M^i_{pq} = H.P.V. \int_{B_q} M(s_q, x_p)dB(s_q), \tag{8}$$

where B_q denotes the q^{th} element and $\delta_{pq} = 1$ if $p = q$; otherwise it is zero. The detail to determine the influence coefficients can be found in [9]. For the exterior problem, we have

$$[T^e_{pq}]\{u_q\} = [U^e_{pq}]\{t_q\} \tag{9}$$

$$[M^e_{pq}]\{u_q\} = [L^e_{pq}]\{t_q\}. \tag{10}$$

where the superscript "e" denotes the exterior problem. According to the dependence of the outnormal vectors in these four kernel functions for the interior and exterior problems, their relationship can be easily found as shown below [10]:

$$U^i_{pq} = U^e_{pq} \tag{11}$$

$$M^i_{pq} = M^e_{pq} \tag{12}$$

$$T^i_{pq} = \begin{cases} -T^e_{pq}, & if\ p \neq q, \\ T^e_{pq}, & if\ p = q \end{cases} \tag{13}$$

$$L^i_{pq} = \begin{cases} -L^e_{pq}, & if\ p \neq q, \\ L^e_{pq}, & if\ p = q. \end{cases} \tag{14}$$

Based on the relations in Eqs.(11) $\sim$ (14), the dual BEM program can be easily extended to solve for exterior problems.

In order to avoid the problem of fictitious frequency, the Burton and Miller formulation [1] is employed by combining the dual equations as follows,

$$\{[T^e_{pq}] + \frac{i}{k}[M^e_{pq}]\}\{u_q\} = \{[U^e_{pq}] + \frac{i}{k}[L^e_{pq}]\}\{t_q\} \tag{15}$$

where $i^2 = -1$.

Numerical examples

Case 1: Nonuniform radiation problem (Dirichlet condition)

For the first example, a non-uniform radiation problem from a sector of a cylinder [11] is considered. The model has a constant value on an arc ($-\alpha < \theta < \alpha$) and vanishing elsewhere. Two points of potential discontinuity in the boundary data can be found. The governing equation and boundary condition are shown in Fig.1. The normalized analytical solution to this cylinder problem of a radius a is

$$u(r,\theta) = \frac{2}{\pi}\sum_{n=0}^{\infty}{}' \frac{sin(n\alpha)}{n}\frac{H^{(1)}_n(kr)}{H^{(1)}_n(ka)}cos(n\theta), \tag{16}$$

where the symbol ' denotes that the first term ($n = 0$) is halved. We select $\alpha = 20^o$. Fig.2 shows the contour plots for the real part of the analytical and numerical solutions. The analytical solution is obtained by using 20 terms series representations. Sixty-three elements are adopted in the dual BEM mesh. The positions where the irregular values occur can be found in Fig.3 for the solution $t(a, 0)$ versus k by using either the UT or the LM equation only. It is found that irregular values occur at J^m_n, the mth zeros of $J_n(ka)$ for the UT formulation, while the LM formulation has the irregular values of J'^m_n, the mth zeros of $J'_n(ka) = 0$. The zeros for the Bessel functions and their derivatives are shown in Table 1. Also, the Burton and Miller formulation is employed to avoid the numerical resonace and the UT and LM results agree well except at the irregular wave numbers as shown in Fig.3. The performance of the dual BEM in comparison with the analytical solution and the DtN results [11] is quite good.

Case 2: Nonuniform radiation problem (Neumann condition)

In order to clarify how the irregular frequencies depend on the types of boundary conditions, the second example with the Neumann boundary condition is designed in Fig.4. The analytical solution is

$$u(r,\theta) = \frac{2}{\pi}\sum_{n=0}^{\infty}{}' \frac{-1}{k}\frac{sin(n\alpha)}{n}\frac{H_n^{(1)}(kr)}{H_n^{(1)\prime}(ka)}cos(n\theta). \tag{17}$$

The contour plots for the real-part solutions are shown in Fig.5. It indicates that numerical results agree well with the analytical solution. Also, it is interesting to find that the irregular frequencies in Fig.6 occurs at the same positions in comparison with those of Fig.3. This confirms the conclusion in [6, 7] that the irregular frequencies depend on the integral formulation (UT or LM method) instead of the types of boundary conditions (Dirichlet or Neumann).

Case 3: Scattering problem (Dirichlet condition)

In order to check the validity of the program for scattering problem, example 3 is considered. The incident wave is plane wave and the object is a soft cylinder as shown in Fig.7. The analytical solution for the scattering field is

$$u(r,\theta) = -\frac{J_0(ka)}{H_0^{(1)}(ka)}H_0^{(1)}(kr) - 2\sum_{n=1}^{\infty} i^n \frac{J_n(ka)}{H_n^{(1)}(ka)}H_n^{(1)}(kr)cos(n\theta), \tag{18}$$

Fig.8 shows the contour plots for the real-part solutions. The positions where the irregular values occur can be found in Fig.9 for the solution $t(a,0)$ versus k by using either the UT or the LM equation only. It is found that irregular values occur at J_n^m, the mth zeros of $J_n(ka)$ for the UT formulation, while the LM formulation has the irregular values of $J_n'^m$, the mth zeros of $J_n'(ka) = 0$. In comparing Fig.9 with Figs.3 and 6, it indicates that the fictitious eigenvalues are dominated by the method, instead of boundary condition and problem types. Also, the Burton and Miller formulation is employed to avoid the numerical resonace and the UT and LM results agree well except at the irregular wave numbers as shown in Fig.9. The performance of the dual BEM in comparison with the analytical solution and the DtN results [12] is quite good.

Case 4: Scattering problem (Neumann condition)

By changing the soft cylinder in Example 3 into a rigid one as shown in Fig.10, we have the analytical solution

$$u(r,\theta) = -\frac{J_0'(ka)}{H_0^{(1)'}(ka)}H_0^{(1)}(kr) - 2\sum_{n=1}^{\infty} i^n \frac{J_n'(ka)}{H_n^{(1)'}(ka)}H_n^{(1)}(kr)cos(n\theta) \tag{19}$$

Fig.11 shows the contour plots for the real-part solutions. The positions where the irregular values occur can be found in Fig.12 for the solution $u(a,0)$ versus k by using either the UT or the LM equation only. The performance of the dual BEM in comparison with the analytical solution and the DtN results [13] is quite good.

Concluding remarks

The mechanism why fictitious frequencies occur in the dual BEM has been examined by considering non-uniform radiation and scattering problems of a cylinder. It is found that the irregular values depend on the integral formulation, the UT or the LM equation, instead of the types of boundary condition. Also, the radiation and scattering problems have the same fictitious values once the method is chosen. The examples show that the first UT equation results in fictitious frequencies at the zeros of $J_n(ka) = 0$, which are associated with the interior acoustic frequencies of essential homogeneous boundary conditions, while the second LM equation produces fictitious frequencies at the zeros of $J_n'(ka) = 0$, which are associated with the interior eigenfrequency of natural homogeneous boundary conditions. The numerical results using the dual BEM program agree very well with the analytical solutions and DtN results. Burton and Miller method was successfully employed to deal with the problem of fictitious frequency.

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對偶邊界積分方程在外域聲場問題之應用

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摘要

本文探討對偶邊界元素法之虛擬頻率產生的機制, 並建立聲場上內域問題及外域問題中影響係數矩陣之間的關係。同時個別討論了隱藏於奇異積分方程及超強奇異積分方程中的不規則 (或稱之虛擬) 頻率，並發現不規則頻率的產生與積分方程表示式中的核函數有關。而陳以可分解核函數從理論及數值實驗上比較並證明不規則值的產生與邊界條件無關。本研究以外域聲場輻射與散射問題為例，並考慮 Dirichlet 及 Neumann 邊界條件得到當以奇異積分方程求解時產生的虛擬特徵值會對應到內域的 Dirichlet 問題之特徵值，當以超強奇異積分方程求解時產生的虛擬特徵值會對應到內域的 Neumann 問題的特徵值，而與邊界條件無關。為了克服此數值問題，本文採用 Burton 與 Miller 法予以克服虛擬頻率。

關鍵字: 對偶邊界元素法、虛擬頻率與外域聲場問題。

Table 1 Zeros of the Bessel functions for $J_n(k)$ and $J'_n(k)$

J_n^m	$m=1$	$m=2$	$m=3$	$m=4$	$m=5$
$J_0(k)=0, n=0$	2.4048	5.5201	8.6537	11.7915	14.9309
$J_1(k)=0, n=1$	3.83171	7.01559	10.17346	13.3237	16.4706
$J_2(k)=0, n=2$	5.1356	8.4172	11.6198	14.7960	17.9598
$J_3(k)=0, n=3$	6.38016	9.76102	13.0152	16.22346	19.40941
$J_4(k)=0, n=4$	7.58834	11.0647	14.3725	17.6160	20.8269
$J_5(k)=0, n=5$	8.77148	12.3386	15.7002	18.9801	22.2178
$J_n'^m$	$m=1$	$m=2$	$m=3$	$m=4$	$m=5$
$J'_0(k)=0, n=0$	0	3.83171	7.01559	10.17346	13.3237
$J'_1(k)=0, n=1$	1.84118	5.33144	8.53632	11.70600	14.8636
$J'_2(k)=0, n=2$	3.05424	6.70713	9.96947	13.17037	16.3475
$J'_3(k)=0, n=3$	4.20119	8.01524	11.3459	14.5858	17.7887
$J'_4(k)=0, n=4$	5.31755	9.2824	12.6819	15.9641	19.1960
$J'_5(k)=0, n=5$	6.41562	10.5199	13.9872	17.3128	20.5755

J_n^m and $J_n'^m$ are the mth zeros of the Bessel functions, $J_n(k)$ and $J'_n(k)$, respectively.

Figure captions

Figure 1. The nonununiform radiation problem (Dirichlet condition) for a cylinder.

Figure 2. The contour plot for the real-part solutions (analytical solution: dashed line, numerical results: solid line).

Figure 3. The positions of irregular values using different methods.

Figure 4. The nonununiform radiation problem (Neumann condition) for a cylinder.

Figure 5. The contour plot for the real-part solutions (analytical solution: dashed line, numerical result: solid line).

Figure 6. The positions of irregular values using different methods.

Figure 7. The scattering problem (Dirichlet condition) for a cylinder.

Figure 8. The contour plot for the real-part solutions (analytical solution: dashed line, numerical result: solid line).

Figure 9. The positions of irregular values using different methods.

Figure10. The scattering problem (Neumann condition) for a cylinder.

Figure 11. The contour plot for the real-part solutions (analytical solution: dashed line, numerical results: solid line).

Figure 12. The positions of irregular values using different methods.

Table captions

Table 1 The zeros for the Bessel functions, $J_n(ka)$ and their derivatives, $J_n'(ka)$.

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