行政院國家科學委員會補助專題研究計畫■期中進度報告

（以退化核求解拉普拉斯、赫姆茲與雙諧和方程式之系統性解法(1/3)）

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A systematic approach for solving Laplace, Helmholtz and biharmonic equations using degenerate kernels

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Abstract In this project, a systematic approach is proposed to deal with engineering problems containing circular boundaries. The mathematical tools, degenerate kernels and Fourier series, are utilized in the null-field integral formulation. The kernel function is expanded to the degenerate form and the boundary density is expressed into Fourier series. By collocating the null-field points on the real boundary, the singularity is novelly avoided. Five gains of well-posed model, singularity free, boundary-layer effect free , exponential convergence and mesh-free approach are achieved. By matching the boundary condition, a linear algebraic system is obtained. After obtaining the unknown Fourier coefficients, the solution can be obtained by using the integral representation. This systematic approach can be applied to solve the Laplace problems. Finally, several examples are demonstrated to show the validity of present formulation.
Key words: degenerate kernels, Fourier series, null-field, mesh-free, linear algebraic system

摘要：本計劃提出一系統性之方法來求解含圓型邊界的工程問題。在零場積分方程中搭配退化核與傅立葉級數展開。其中核函數展開成退化核的形式，把邊界密度用傅立葉級數展開，可把點佈在真實邊界上，且又可避開奇異積分，此乃是本計劃的最大創意。我們的方法有五個優點：良態的模式、不需計算奇異積分、無邊界層效應、指數收斂與無網格法。代入邊界條件後，可得到線性代數系統。解得傅立葉係數後，其解可以表示成積分表示式。這種系統性解法可以應用在拉普拉斯問題上。本計畫最後提供幾個算例來驗證我們方法的正確性。
關鍵字：退化核、傅立葉級數、零場積分方程、無網格、線性代數系統。

Introduction

Engineering analysis can be formulated as mathematical models of the boundary value problems. In order to solve the boundary value problems, researchers and engineers have paid more attention on the development of boundary integral equation method (BIEM), boundary element method (BEM) and meshless method than domain type methods, finite element method (FEM) and finite difference method (FDM). Among various numerical methods, BEM is one of the most popular numerical approaches for solving boundary value problems. Although BEM has been involved as an alternative numerical method for solving engineering problems, five critical issues are of concern.

(1) Treatment of singularity and hypersingularity

It is well known that BEM are based on the use of fundamental solutions to solve partial differential equations. These solutions are two-point functions which are singular as the source and field points coincide. Most of the efforts have been focused on the singular boundary integral equation for problems with ordinary boundaries. In the past, several regularizations for hypersingularity were offered to handle it in direct and indirect ways. In the present approach, we employed the degenerate kernel to represent the two-point fundamental solution for problems with circular boundaries. The singularity and hypersingularity disappeared in boundary integral equation after describing the potential into two parts. The idea of changing real boundary to fictitious boundary (fictitious BEM) or putting the observation point outside the domain (null-field approach) can remove the singular and hypersingular integrals. However, they result in an ill-posed matrix which will be elaborated on later.

(2) Boundary-layer effect

Boundary-layer effect in BEM has received attention in the recent years. In real applications, data near boundary can be smoothened since maximum principle always exists for potential problems. Nevertheless, it also deserves study to know how to manipulate the nearly singular integrals in applied mathematics. Many regularization techniques can be found in the literature. How to eliminate the boundary-layer effect in BEM is vital for researchers.

(3) Convergence rate

Undoubtedly, BEM is very popular for boundary value problems with general geometries since it requires discretization on the boundary only. Regarding to constant, linear and quadratic elements, the discretization scheme does not take the special geometry into consideration. It leads to the slow convergence rate. For example, Fourier series is suitable for boundary densities on circular boundaries while the spherical harmonic function is always employed to approximate the boundary density on surface of sphere. Although previous researchers have employed the Fourier series expansion, no one has ever introduced the degenerate kernel in boundary integral equations to tackle their problems. Mathematicians have proved that the exponential convergence instead of the algebraic convergence in the BEM can be achieved by using the degenerate kernel and Fourier expansion.

(4) Ill-posed model

As mentioned previously in the first issue, to avoid directly calculating the singular and hypersingular integrals by using null-field approach or fictitious BEM yields an ill-condition system. The influence matrix is not diagonally dominated and needs preconditioning. To approach the fictitious boundary to the real boundary or to move the null-field point to the real boundary can make the system well-posed. However, singularity appears in the meantime. We may wonder is it possible to push the null-field point on the real boundary but free of facing the singular or hypersingular integrals. The answer is yes and can be found in this project.

(5) Mesh on boundary is still necessary.

To develop a BEM with several advantages, singularity free, the suppression of boundary-layer effect, exponential convergence, well-posed model and mesh-free is the main motivation of this project.

Engineering problems with circular boundaries are often encountered, *e.g.* missiles, aircraft, naval architecture, etc., either to reduce the weight of the whole structure or to increase the range of inspection as well as piping purposes. Analytical approach using bi-polar coordinate [1] was developed for two-holes problems. Complex variable techniques were also employed for the annular case. For a problem with several holes, many numerical methods, *e.g.* finite element method (FEM) and boundary element method (BEM), were resorted to solve. To develop a systematic approach for engineering problems with circular boundaries is not trivial.

Null-field integral equation approach is used widely for obtaining the numerical solutions to engineering problems. Various names, *e.g.* T-matrix method [2] and extended boundary condition method (EBCM) [3], have been coined. A crucial advantage of this method consists in the fact that the influence matrix can be computed easily. Although many works for acoustic and water wave problems have been done, we focus on the solid mechanics here.

In this project, we review the recent development of the null-field integral equation approach [4-10] for boundary value problems (BVPs) with circular boundaries. The key idea is the expansion of kernel functions and boundary densities in the null-field integral equations. Vector decomposition technique using the adaptive observer system is required for nonfocal cases. Applications to the Laplace problems are addressed. Not only interior problems but also exterior cases are solved. Several examples were demonstrated to see the validity of the new formulation.

Null-field integral equation approach for boundary value problems

Suppose there are N randomly distributed circular boundaries bounded to the domain D and enclosed with the boundary, B_k ($k = 0, 1, 2, \cdots, N$) as shown in Fig. 1. We define

$$B = \bigcup_{k=0}^{N} B_k \ . \tag{1}$$

In mathematical physics, boundary value problems can be modelled by the governing equation,

$$\mathcal{L}\, u(\mathrm{x}) = 0 \,, \quad \mathrm{x} \in D \,, \tag{2}$$

where $\mathcal{L}$ is be the Laplace operator, $u(\mathrm{x})$ is the potential function and D is the domain of interest. For the 2-D Laplace problem, the integral equation for the domain point can be derived from the third Green's identity, we have

$$2\pi u(\mathrm{x}) = \int_B T(\mathrm{s},\mathrm{x}) u(\mathrm{s}) dB(\mathrm{s}) - \int_B U(\mathrm{s},\mathrm{x}) t(\mathrm{s}) dB(\mathrm{s}), \ \mathrm{x} \in D, \tag{3}$$

$$2\pi \frac{\partial u(\mathrm{x})}{\partial \mathrm{n}_\mathrm{x}} = \int_B M(\mathrm{s},\mathrm{x}) u(\mathrm{s}) dB(\mathrm{s}) - \int_B L(\mathrm{s},\mathrm{x}) t(\mathrm{s}) dB(\mathrm{s}), \ \mathrm{x} \in D, \tag{4}$$

where s and x are the source and field points, respectively, $t = \partial u / \partial \mathrm{n}$, B is the boundary, n_x denotes the outward normal vector at the field point x and the kernel function $U(\mathrm{s},\mathrm{x})$, is the fundamental solution, and the other kernel functions, $T(\mathrm{s},\mathrm{x})$, $L(\mathrm{s},\mathrm{x})$ and $M(\mathrm{s},\mathrm{x})$, are defined in the dual boundary integral method (BIEM) [6].
By moving the field point to the boundary, the Eq. (3) and (4) reduce to

$$\pi u(\mathrm{x}) = C.P.V.\int_B T(\mathrm{s},\mathrm{x}) u(\mathrm{s}) dB(\mathrm{s}) - R.P.V.\int_B U(\mathrm{s},\mathrm{x}) t(\mathrm{s}) dB(\mathrm{s}), \ \mathrm{x} \in B, \tag{5}$$

$$\pi \frac{\partial u(\mathrm{x})}{\partial \mathrm{n}_\mathrm{x}} = H.P.V.\int_B M(\mathrm{s},\mathrm{x}) u(\mathrm{s}) dB(\mathrm{s}) - C.P.V.\int_B L(\mathrm{s},\mathrm{x}) t(\mathrm{s}) dB(\mathrm{s}), \ \mathrm{x} \in B, \tag{6}$$

where $C.P.V.$, $R.P.V.$ and $H.P.V.$ denote the Cauchy principal value, Riemann principal value and Hadamard principal value, respectively. By collocating the field point x outside the domain (including boundary), the null-field integral equations yield

$$0 = \int_B T(\mathrm{s},\mathrm{x}) u(\mathrm{s}) dB(\mathrm{s}) - \int_B U(\mathrm{s},\mathrm{x}) t(\mathrm{s}) dB(\mathrm{s}), \ \mathrm{x} \in D^c \cup B, \tag{7}$$

$$0 = \int_B M(\mathrm{s},\mathrm{x}) u(\mathrm{s}) dB(\mathrm{s}) - \int_B L(\mathrm{s},\mathrm{x}) t(\mathrm{s}) dB(\mathrm{s}), \ \mathrm{x} \in D^c \cup B, \tag{8}$$

where D^c is the complementary domain. The x point in Eq. (7) and (8) can be in the D^c or exactly on the real boundary if appropriate degenerate kernels are expressed. The set of x is closed.

Expansions of the fundamental solution and boundary density

Instead of directly calculating the $C.P.V.$, $R.P.V.$ and $H.P.V.$ in Eq.(5) and (6), we obtain the linear algebraic system from the null-field integral equations of Eq.(7) and (8) through the kernel expansion.

Based on the separable property, the kernel function $U(\mathrm{s},\mathrm{x})$ can be expanded into the separable form by dividing the source and field point:

$$U(\mathrm{s},\mathrm{x}) = \begin{cases} U^i(\mathrm{s},\mathrm{x}) = \sum_{j=1}^{\infty} A_j(\mathrm{s}) B_j(\mathrm{x}), \ |\mathrm{s}| \ge |\mathrm{x}|, \\ U^e(\mathrm{s},\mathrm{x}) = \sum_{j=1}^{\infty} A_j(\mathrm{x}) B_j(\mathrm{s}), \ |\mathrm{x}| > |\mathrm{s}|, \end{cases} \tag{9}$$

where the $A(\mathrm{x})$ and $B(\mathrm{x})$ can be found for the Laplace operator and the superscripts "i" and "e" denote the interior ($|\mathrm{s}| \ge |\mathrm{x}|$) and exterior ($|\mathrm{x}| > |\mathrm{s}|$) cases, respectively. To classify the interior and exterior regions, Fig. 2 shows for one, two and three dimensional cases. For the degenerate form of T, L and M kernels, they can be derived according to their definitions.

We apply the Fourier series expansions to approximate the potential u and its normal derivative t on the B_k circular boundary

$$u(\mathrm{s}_k) = a_0^k + \sum_{n=1}^{m} (a_n^k \cos n\theta_k + b_n^k \sin n\theta_k), \ \mathrm{s}_k \in B_k, \ k = 0, 1, 2, \cdots, N, \tag{10}$$

$$t(\mathrm{s}_k) = p_0^k + \sum_{n=1}^{m} (p_n^k \cos n\theta_k + q_n^k \sin n\theta_k), \ \mathrm{s}_k \in B_k, \ k = 0, 1, 2, \cdots, N, \tag{11}$$

where a_n^k, b_n^k, p_n^k and q_n^k ($n = 0, 1, 2, \cdots$) are the Fourier coefficients and θ_k is the polar angle measured with respect to the x-direction.

After collocating the null-field points in the null-field integral equation of Eq. (7), the boundary integrals through all the circular contours are required. It is worth noting that the origin of the observer system is located on the center of the corresponding circle under integration to entirely utilize the geometry of circular boundary for the expansion of degenerate kernels and boundary densities. Figure 1 shows the boundary integration for the circular boundaries in the adaptive observer system.

By collocating the null-field point x_k on the kth circular boundary for Eq. (7) in Fig. 1, we have

$$0 = \sum_{k=0}^{N} \int_{B_k} T(\mathrm{s}_k, \mathrm{x}_j) u_k(\mathrm{s}) dB_k(\mathrm{s}) - \sum_{k=0}^{N} \int_{B_k} U(\mathrm{s}_k, \mathrm{x}_j) t_k(\mathrm{s}) dB_k(\mathrm{s}), \ \mathrm{x} \in D^c, \tag{12}$$

where N is the number of circular boundaries including the outer boundary and the inner boundaries. Therefore, a linear algebraic system is obtained

$$[\mathbf{U}]\{\mathbf{t}\} = [\mathbf{T}]\{\mathbf{u}\}, \tag{13}$$

where $[\mathbf{U}]$ and $[\mathbf{T}]$ are the influence matrices with a dimension of $(N+1)(2m+1)$ by $(N+1)(2m+1)$, $\{\mathbf{u}\}$ and $\{\mathbf{t}\}$ denote the column vectors of Fourier coefficients with a dimension of $(N+1)(2m+1)$ by 1 in which m indicates the truncated terms of Fourier series. Then, the resulted linear algebraic system is obtained. After the boundary unknowns are solved, the field potential can be easily obtained according to Eq. (3).

Illustrative examples

Case 1: Infinite medium with two circular holes under the anti-plane shear (Laplace equation)
A hole centered at the origin of radius a_1 and the other hole of radius $a_2 = 2a_1$ centered on x axis at $a_1 + a_2 + d$ are considered where d denotes the nearest distance between the holes. In order to be compared with the Honein *et al.*'s results [7] obtained by using the Möbius transformation, the stress along the boundary of radius a_1 is shown in Fig. 3 and good agreement is made.

Case 2: A circular bar with three circular holes under torsion (Laplace equation)
A circular bar with three equal circular holes removed is under torque at the end. The contour plot of the axial displacement is shown in Fig. 4. Good agreement is made after comparing with the Caulk's data [9]. Table 1 shows the comparison of the torsional rigidities G of three cases with different geometries of circular holes. The present solutions show improvement over Ling's results [8] in every case. The discrepancy in the second example in Table 1 may ascribe to the Ling's lengthy calculation in error as pointed out by Caulk [9].

Case 3: A circular beam with two circular holes under bending (Laplace equation)
Naghdi [10] and Bird and Steele [11] both calculated the stress concentration for the four equal-sized circular holes problem under bending. Bird and Steele [11] stated that the deviation by Naghdi's data is 11%. The grounds for this discrepancy were not identified in their project. Our numerical results are more agreeable to the Naghdi's data as shown in Fig. 5. For the two equal-sized problems under bending, the stress concentration for $d / a_1 = 0.125$ is shown in Fig. 6. Our numerical results are well compared with the Bird and Steele's data [11].

Conclusions

A semi-analytical approach was proposed for solving BVPs with circular boundaries. Some results for Laplace problems were presented. Although the BIE for the boundary point was employed, we need not to face the problems of $C.P.V.$ and $H.P.V.$ after introducing the degenerate kernel. Not only the singularity is transformed to the series sum but also the boundary-layer effect is eliminated. In order to verify the formulation, applications to the Laplace (the first year project) problems were done. Five gains of well-posed model, singularity free, boundary-layer effect free, exponential convergence and mesh-free approach were achieved. Extension to other shapes, *e.g.* ellipse, as well as three dimensional problems is straightforward once the corresponding degenerate kernel is available.

References

1. Lebedev NN, Skalskaya IP and Uflyand YS, 1979, "Worked problem in applied mathematics," *Dover Publications*, New York.
2. Waterman PC, 1965, "Matrix formulation of electromagnetic scattering," *Proc. IEEE* Vol. 53, pp. 805-812.
3. Doicu A and Wriedt T, 1997, "Extended boundary condition method with multiple sources located in the complex plane," *Optics Commun.*, Vol. 139 , pp. 85-91.
4. Chen CT, 2005, "Null-field integral equation approach for Helmholtz (interior and exterior acoustic) problem with circular boundaries," Master Thesis, *Department of Harbor and River Engineering*, National Taiwan Ocean University, Taiwan.
5. Chen JT, Hsiao, CC and Leu SY, 2006, "Null-field integral equation approach for plate problems with circular holes," *ASME J. Appl. Mech.*, Vol. 73, pp. 679-693.

6. Chen JT, 2005, “Null filed integral equation approach for boundary vale problems with circular boundaries,” Keynote lecture of ICCES 2005, India.
7. Honein E, Honein T, Herrmann G, 1992, “On two circular inclusions in harmonic problems” *Quarterly of Applied Mathematics*, Vol. 50, pp. 479-499.
8. Ling CB, 1947, “Torsion of a circular tube with longitudinal circular holes,” *Quarterly of Applied Mathematics*, Vol. 5, pp. 168-181.
9. Caulk DA, 1983, “Analysis of elastic torsion in a bar with circular holes by a special boundary integral method,” *ASME Journal of Applied Mechanics*, Vol. 50, pp. 101-108.
10. Naghdi AK, 1991, “Bending of a Perforated Circular Cylindrical Cantilever,” *International Journal of Solids and Structures*. Vol. 28, pp. 739-749.
11. Bird MD and Steele CR, 1992, “A Solution Procedure for Laplace’s Equation on Multiply Connected Circular Domains.” *ASME Journal of Applied Mechanics*, Vol. 59, pp. 398-404.

Self-evaluation

1. According to our proposal, we have succeeded in employing null-field formulation to solve Laplace problems with circular boundaries in the first year.
2. A keynote lecture of ICCES 2005 was delivered in India.
3. Four SCI papers appear as shown below：

   (1). J. T. Chen, W. C. Shen and A. C. Wu, 2006, Null-field integral equations for stress field around circular holes under anti-plane shear, Engineering Analysis with Boundary Elements, Vol. 30, pp. 205-217.

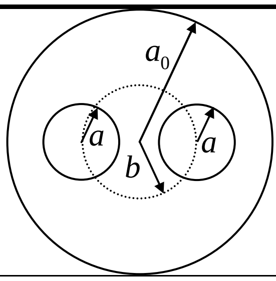
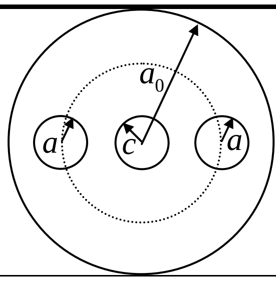
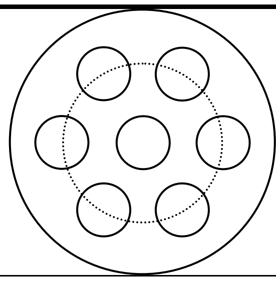
   (2). J. T. Chen, W. C. Shen and P. Y. Chen, 2006, Analysis of Circular Torsion Bar with Circular Hole Using Null-field Approach, Computer Modeling in Engineering & Science, Vol. 12, No. 2, pp. 109-119.

   (3). J. T. Chen and P. Y. Chen, 2006, Bending of a perforated circular cylindrical cantilever using null-field integral formulation. Journal of Mechanics, Accepted.

   (4). J. T. Chen and W. C. Shen, 2007, Degenerate scale for multiply connected Laplace problems, Mechanics Research Communications, Vol.34, pp.69-77.

4. For more details, please visit our web site of *http://ind.ntou.edu.tw/~msvlab/*.

Table 1 Torsional rigidity in Ling's examples

Case		 $a/a_0=2/7, b/a_0=3/7$	 $c/a_0=1/5, a/a_0=1/5,$ $b/a_0=3/5$	 $c/a_0=1/5, a/a_0=1/5,$ $b/a_0=3/5$
$\frac{2G}{(\mu\pi a_0^4)}$	Caulk (First-order approximate)	0.8739	0.8741	0.7261
	Caulk (BIE formulation)	0.8713	0.8732	0.7261
	Ling's results	0.8809	0.8093	0.7305
	Present method ($m=10$)	0.8712	0.8732	0.7244

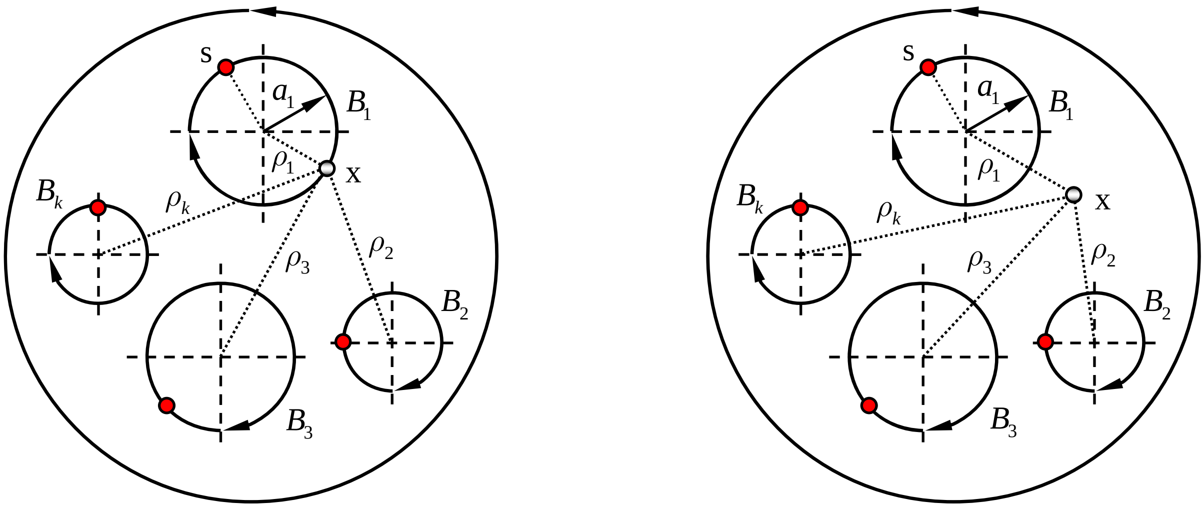


Fig. 1. Sketch of null-field and domain points in conjunction with the adaptive observer system (left: collocation on the boundary point, right: collocation on the interior point).

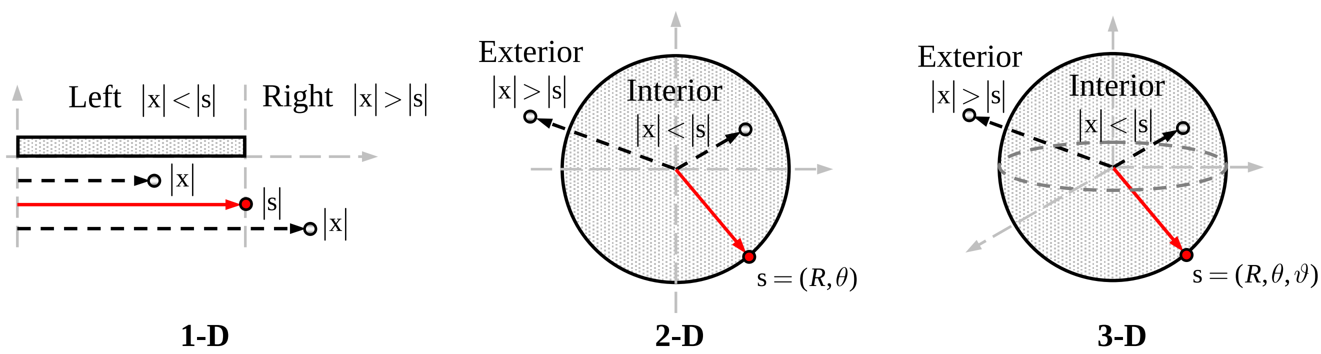


Fig. 2 The degenerate kernel for the one, two and three dimensional problems.

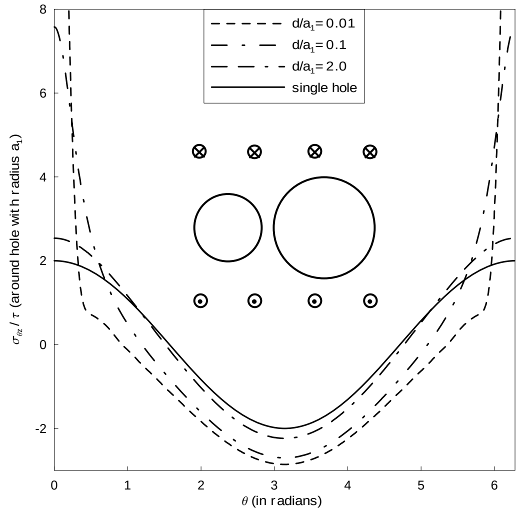


Fig. 3 Stresses around the hole of radius a_1. **(Laplace equation)**

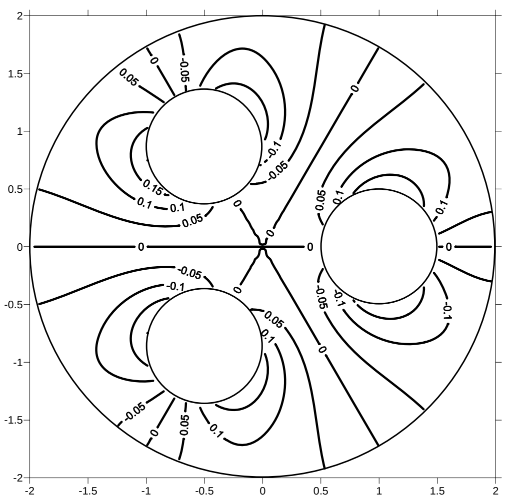
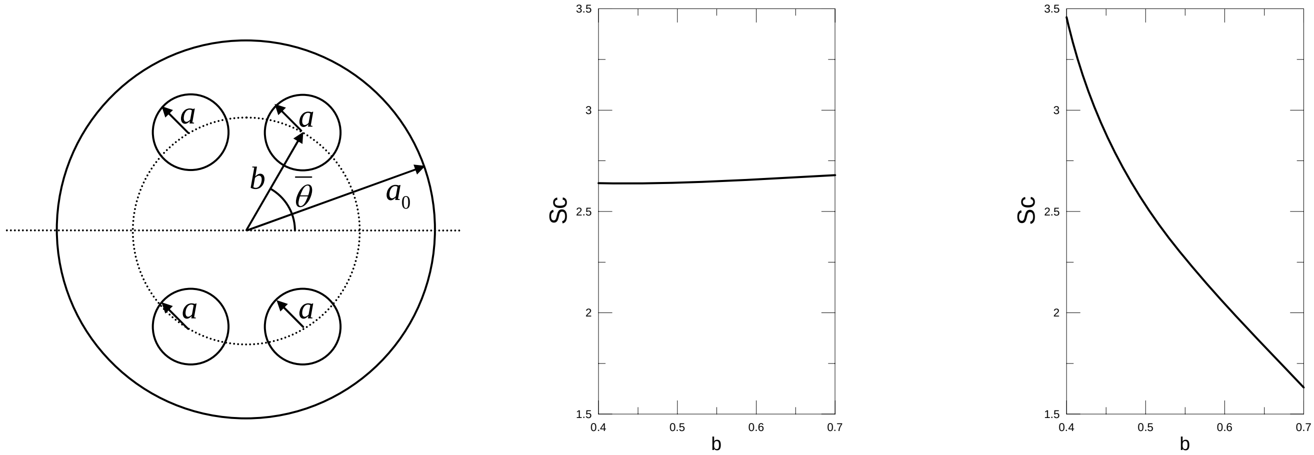


Fig. 4 Displacement for the weakened circular bar by three holes. **(Laplace equation)**



$\bar{\theta} = \pi/8$ (Present method) $\bar{\theta} = 3\pi/8$ (Present method)

Fig. 5 Stress concentration versus b for $a = 0.12$ and $a_0 = 1.0$.**(Laplace equation)**

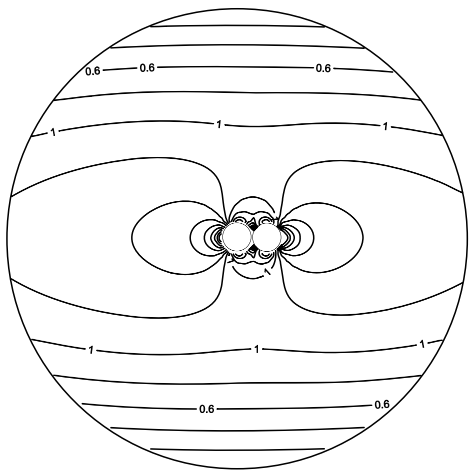


Fig. 6. Contour of stress concentration for $d/a_1 = 0.125$. **(Laplace equation)**

國科會補助出席國際會議報告

94 年 12 月 13 日

報告人姓名	陳正宗	服務機關名稱（請註明系所）及職稱	國立臺灣海洋大學河海工程系特聘教授
會議期間及地點	自 2005 年 12 月 1 日至 2005 年 12 月 6 日(印度 Chennai）	本部核定補助文號	年 月 日 台（ ）文二字第 號
會議名稱	（中文）計算及實驗工程與科學國際會議 （英文）ICCES2005 International Conference on Computational & Experimental Engineering and Sciences		
發表論文題目	（中文）含圓形邊界值問題零場積分方程解法 （英文）Null-field integral equation approach for boundary value problems with circular boundaries		
報告內容應包括下列各項： 一、參加會議經過 二、與會心得 三、建議 四、攜回資料名稱及內容 五、其它			

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印度清奈學術(ICCES 2005) 之旅

陳正宗

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一、 參加會議經過

本次會議為 ICCES2005（International Conference on Computational & Experimental Engineering and Sciences)，於 12 月 1 日至 12 月 6 日在印度清奈（Chennai）印度理工學院馬德拉斯分校 (Indian Institute of Technology-IIT, Madras)舉行。此系列會議自 1986 第一屆在東京舉行，1992 年在香港開會。當時筆者尚是博士研究生，主辦人 Prof. Atluri 即邀請本人發表論文並擔任會議分段主持人。記得當時申請教育部補助，未獲通過。後經 MSC 公司支助方能成行。對於 Atulri 教授提攜後進之情至今難忘。唯未能於 ICES92 一睹大師風彩實為憾事(因 Atluri 教授個人健康因素未能參與)。自 ICCES 2002, 2003, 2004 連續 Atluri 教授均以 Keynote lectures 邀請，個人均因故未能成行。本次總算如願以償。大會以工程、實驗及科學計算方法為主題，六天會議分別在 IIT 的 IC&SR 及 Management Building 進行，與會人數大約七百多人，是一個大型的國際學術會議，其中不乏國際知名學者，如 Beskos, Achenbach、Hutchinson、Atluri、Ohno、Mow、Tong、Chong 等人參與並作專題演講。這次學術會議，台灣共有七位教授參與。來自中國大陸約二十餘位。

本次會議同時為創始人 Atluri 慶祝六十大壽，主辦單位非常投入主辦這次國際會議，並邀請印度總統 Kalam 博士在開幕盛會致詞。可以看出印度總統對本次會議極高的期許，並希望以科學及技術提昇印度整體經濟生活水準。會議議程相當緊湊，除了白天的論文發表外，每晚均安排文化響宴。連續一星期早出晚歸，相當辛苦，不過也過得相當充實。除在學術專業與各國學者交流外，對異國風情文化亦略有體會與感受。

二、 與會心得

首先，感謝國科會補助出國參加國際研討會，使申請人能有機會參與這次的 ICCES2005 國際會議。一償自 ICES 1992 至今十餘年再度參與的宿願。本次係受大會邀請以 Keynote Lecture 進行演講:

含圓形邊界值問題零場積分方程解法(Null-field integral equation approach for boundary value problems with circular boundaries)，並擔任 Session 的主持人。此報告部份的研究成果已被 EABE, CMES 與 ASME-JAM 期刊接受發表。

筆者在過去二十年(1985-2005)有關邊界積分方程的研究，大陸與國外學者均相當關注，並感到高度興趣。在此方面研究成果已受國際計算力學同行的注意。文獻已超過三百多篇論文引用我們在台灣點點滴滴的工作成果。對一個土博士而言，筆者相當加珍惜這豐碩的果實。期望在海大的研究團隊能持續作出國際水平的研究成果。這次與會人數有七百多人，每日均安排 Plenary lecture 後，則分十幾場進行論文發表。討論之熱烈，可想而知。除了會議進行時的討論外，連中場 Coffee Break 的時間，也看到多位學者不斷熱烈討論，是筆者參加國多次內外研討會見過最熱絡的一次。

在會議上，可看到許多學者所提出的計算方法，在各個不同領域上的應用，讓眼界放寬。對於尋求開闢新研究領域，增進國際學術研究的互動關係，均有助益並避免在國內閉門造車。所謂知彼知此，方能百戰百勝。學術是沒有國界的，不會因為你是工程背景就被限制住，只能做實務，學數學的人就只能去發展理論。所謂 There is nothing more practical than the right theory. Whether the theory is right or not depends on the experiment.

對於一個傑出研究者而言，理論、計算與實驗缺一不可。作為一個工學院的教授而言，希望是一個會使用數學方法或技巧解決工程問題的工程師(mathematical engineer)，又是一個具有物理與工程觀念的數學家(engineering mathematician)。對於英語的學習，深深體認，來自不同國家的學者，皆使用共通的語言(英語)來溝通，對於英語的掌握，對一個土博士而言更是要去好好面對，早日走出國際，跟上潮流。

三、建議

本次研討會給我的感覺是學術交流與國際視野開拓的重要性。國際間，針對各種專業領域不乏會有知名學者。在交流的過程中，可從問答之間，感受高手過招之樂；亦可在私下討論時，了解每位學者間所關注的焦點，能使我們對整體研究趨勢有些了解，有助我們掌握新的研究方向。因此，教育部或國科會往後應盡量補助國內年輕學者或博士生，早日參與國際學術會議，開拓其國際視野並邁向國際；同時，也希望能多多補助支持國內大專院校，承辦一些大型國際會議，使無法獲得出國補助的學生及國內年輕老師，也能參與國際會議，增加與國外學者進行交流與見習的機會，亦可提升台灣在國際上的知名度。

在此國家政治外交的困境下，學術突圍不失為一好棋。唯一遺憾的是，本次雖有多位國內學者以 Keynote lecture 發表，然 Plenary lecture 則無。這意味著我們尚待努力的空間仍大。

四、攜回資料名稱及內容

攜回的主要資料，除了該會議的詳細議程外，以及一本大會論文(CD-ROM)。

---India-chen.doc（J T Chen）