A SIMPLE MODEL TO PREDICT CRITICAL RICOCHET ANGLE IN LONG-ROD PENETRATION

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ABSTRACT

The ricochet of eroding long rods, from steel targets, is investigated by a series of three-dimensional numerical simulations in explicit finite element code. These are compared with the predictions of our analytical model and experimental results for ricochet. This approach is different than the rigid body treatment by A. Tate. Also it uses a new penetration velocity equation to predict critical ricochet angle. Critical ricochet angles were calculated for various impact velocities and strengths of the target plates in these approaches. It was predicted that critical ricochet angle increases with decreasing impact velocities and that higher ricochet angles were expected if higher strength targets were employed. New model’s results show a better agreement with numerical results and experiments than Tate and Rosenberg models.

Keywords : Critical ricochet angle, Long-rod, Analytical model, Numerical simulation.

1. INTRODUCTION

The phenomenon of ricochet is an important consideration in ballistics. While a large fraction of laboratory-generated ballistic data is gathered under idealized impact-conditions of normal incidence, the physically probable reality is that virtually all ballistic impacts on the battlefield occur at some nontrivial level of impact obliquity (as measured from the target normal). At low to moderate obliquity levels, the ballistic effect of the obliquity may only be that of an increased line-of-sight target thickness of the target. However, depending upon the material properties of the rod and target, the geometry of the rod, the impact velocity, and yaw, there will exist a critical angle of obliquity at and beyond which ricochet occurs.

Ricochet has been a long studied phenomenon. An excellent review of work in the area is provided as a chapter in Goldsmith’s review paper on projectile impact [1]. Much of the research in ricochet centers is on compact projectiles or on rigid projectiles. A much smaller fraction of the work is focused on medium- and long-rod ricochet, where a different phenomenology often manifests itself.

Radiographic evidence and hydrocode simulation of long-rod ricochet was presented by Jonas and Zukas [2] in 1978. With experimental radiography and corroborating hydrocode simulation, they demonstrate how the rod can form at the impact site (stationary with respect to the target) and deflect the rod from a rigid target surface. Jonas and Zukas was to demonstrate a hydro code modeling capability, no analytical modeling or analysis was offered.

In 1979, Tate [3] developed an early model for the ricochet of rods, in which the cylindrical rod (of square cross-section) responds as a rigid body away from the eroding tip. Tate’s model allows the local erosive deformation of the rod in the immediate vicinity of the impact. The asymmetric forces acting on this deforming rod tip are evaluated to ascertain their capacity to induce a rotation sufficient to bring about ricochet during the limited time before the rod tip becomes fully engaged in the target. While the model permits local deformation at the rod’s tip, affecting the line of action of the interaction force, ricochet is judged to occur only if the remainder of the rod is adequately rotated, in a rigid fashion about its center of gravity, so as to produce a net linear velocity in the rod tip parallel to the target surface. Tate’s approach produces an analytical expression for the target obliquity angle \( \theta_{\text{crit}} \) beyond which ricochet is predicted to occur. The expression for it is given as

\[
\tan^2 \beta > \frac{2 \rho_p V^2}{3 Y_p} \left( \frac{L^2 + D^2}{L D} \right) \left( \frac{V}{V-U} \right)
\]

where \( \rho_p \) is the rod’s density, \( V \) is the striking velocity of the rod, \( Y_p \) is the yield strength of the rod, \( L \) and \( D \) are the rod’s length and diameter, respectively, and \( U \) is the
initial penetration velocity of the rod into the target. Note that the need for the penetration velocity \( U \) as input to the model requires an auxiliary calculation from a ballistic penetration model, such that \( U \), in addition to the rod variables already listed, becomes a function of the target resistance and the target density.

One of the drawbacks of the Tate ricochet model is its failure to predict ricochet for the rigid-rod (i.e., \( U = V \)) scenario, because of the method used to calculate the line of action of the interaction force. Further, because of the model’s requirement to ricochet by way of rigid rod rotation, ricochet becomes increasingly improbable as the rod length is increased.

Jonas and Zukas [2], Senf et al. [4] showed in 1981 how the ricocheting rod can form a deformable shape at the impact site to deflect the rod from a rigid target surface. Such observations provided additional evidence that, even in the absence of erosion, a rigid rod assumption does not necessarily hold during the ricochet process. Like Jonas and Zukas, the intention of Senf et al. was to demonstrate hydrocode modeling capability, and so no additional analysis was offered.

Reid et al. [5] began to address, with analysis, the notion of the plastic hinge, traveling down the rod’s length, but stationary with respect to the target surface. In their analysis, they simplified the problem to consider only the transverse bending forces, and were able to analogize the problem to one of a transverse impact at the free end of a cantilever beam.

Johnson et al. [6] studied ricochet interaction of plasticine (modeling clay) rods and targets, upon predictions from Tate’s ricochet work. Johnson et al. limited their testing to impact obliquities above 75°. Photographic records of ricochet exhibited rod behavior similar to that reported by Jonas and Zukas [2], Senf et al. [4].

By 1983, several studies had recorded the ricochet problem and its results. There was a significant increase in the number of studies examining the impact of long rods on eroding targets, with new and additional materials tested. The impact velocities were raised to around 1400 m/sec, and impact obliquities were restricted to below 75°. One of the studies examined a long rod equipped with a square section of the rod is square, in order to simplify the section of the rod is square, in order to simplify the geometric “initial conditions” of the remaining rod and not upon the wholly remaining rigid rod as Tate assumed. Thus, it would appear that for the initial tip of rod is deflected, for example, the geometric “initial conditions” of the remaining rod and target obliquity which it is derived, are no longer in harmony with the methodology. This is in assuming that the interaction force’s line of action is calculated.

More recent studies of the ricochet phenomenon focused on spherical projectiles [9], rigid projectiles [10], computational methods [11], or else touch on ricochet peripherally as part of a larger examination [12,13]. None of these studies offer additional analytical modeling insight into the phenomenology of medium- and long-rod ricochet.

2. MODEL CONSTRUCT

We adopt Rosenberg’s arguments concerning the source of the asymmetric force acting on the tip of the long rod as it engages the oblique target. This interaction is shown in Fig. 1. which is taken from Rosenberg’s work [8].

As in [8] we assume that the tip of the rod is eroding according to Rosenberg’s theory [8], and that the cross section of the rod is square, in order to simplify the analysis. Thus the same relations can be derived for the angle \( \psi \) (see Fig. 1) and the length (S) of the eroding tip [8].

\[ S \cdot \sin \psi = \frac{tg^2 \beta \cdot \rho_s V^2}{R_i} \left( \frac{V + U}{V - U} \right) \] (2)

where \( R_i \) is the target resistance. Like Tate’s model, the Rosenberg model’s need for the penetration velocity \( U \) as input demands a further calculation that requires, in addition to those variables listed, knowledge of the rod strength \( Y_p \) and the target density. The model was shown to have good prediction ability for \( L/D = 10 \) tungsten (WA) rods launched against rolled homogeneous armor targets at striking velocities between 600 and 1400 m/sec and target obliquities between 55° and 75°.

The interaction methodology of Rosenberg et al. however, calculates the force interaction based upon a virgin, long-rod striking an erodible (but as yet undeformed) target. Thus it would seem that the methodology should only offer a prediction as to what happens to the initial portion of material against the target. Once the initial portion is deflected, for example, the asymmetric force acting on the tip of the ricocheting rod. However, the key point of departure for Rosenberg et al. is in assuming that the interaction force acts only upon the mass actively engaging the target, and not upon the wholly remaining rigid rod as Tate assumed. Thus, in the Rosenberg model, the interaction force acts to linearly deflect rod-tip material in the transverse direction, rather than acting to apply a rotational moment upon the rigid rod, as in the Tate ricochet model.
The force \( f \) which acts on the eroding surface is responsible for the bending of the rod and for its ricochet. Rosenberg assumed that the magnitude of this force depends on the pressure which the target exerts on the projectile. Thus, used the strength parameter \( R_T \) for the target and wrote for the force \( f \):

\[
f = R_T \cdot S \cdot D
\]  

(4)

where \( S \cdot D \) is the area of the eroding surface of the rod. Only the vertical component of \( f \) is of interest \( f \sin \psi \) and the time is needed for the whole tip of the rod to reach the target is \( t_m = \frac{D \cdot \tan \beta}{V} \). Thus, the vertical impulse imparted by the asymmetrical force \( f \) to the tip of the rod is:

\[
I = \int_0^t f \cdot \sin \psi \cdot dt = \frac{R_T \cdot D^2 \cdot (V - \tan \beta)}{2V}
\]

(5)

The relevant mass, on which the vertical impulse acts, is smaller than the mass of the tip \( \rho \cdot \frac{D^2}{2} \cdot \tan \beta \) by the amount of the eroded mass which is equal to \( \rho \cdot \frac{V \cdot U}{2} \cdot \tan \beta \). Thus, the remaining mass of the rod tip is:

\[
m = m_t - m = \rho \cdot \frac{D^2}{2} \cdot \tan \beta - \frac{V \cdot U}{2} \cdot \tan \beta
\]

(6)

Using momentum conservation we get from Eqs. (3) and (4):

\[
\rho_s D^2 \cdot \rho \cdot \tan \beta \cdot \frac{V + U}{2V} \cdot V_T = \frac{R_T \cdot D^2}{2} \cdot \frac{V - U}{V^2} \cdot \tan \beta
\]

(7)

Where \( V_T \) is the vertical velocity imparted by the asymmetrical force \( f \) to the tip of the rod (Fig. 2). Form Eq. (7) we get:

\[
V_T = \frac{R_T \cdot (V - U)}{\rho_s \cdot V^2} \cdot V_T \cdot \tan \beta
\]

(8)

In spite of Rosenberg method we use a new approach for penetration velocity as below:

First step considered here is the development of a relationship between erosion rate, \( l \), and the instantaneous velocity of the un-deformed portion of the rod \( v \). For a cylindrical penetrator of length \( L \) and diameter \( D \), the greater the instantaneous velocity of the rod, the higher the erosion rate. Therefore, it seems appropriate to assume that the rate, \( l \), is proportional to this velocity.

\[
l = \alpha \cdot v
\]

(9)

where \( \alpha \) is one of the incompressibility of material properties. \( \alpha \) is an erosion rate and can be presented as \( l/v \). As it is not dependent to projectile or target geometry, this is a useful parameter to investigate the long-rod penetration phenomena. In addition, changing the projectile geometry, the parameter \( \alpha \) is simplified in hypervelocity region.

Using the above assumption along with kinematic relationships about long-rod penetration:

\[
l = (u - v)
\]

(10)

results in:

\[
u = v (1 - \alpha)
\]

(11)

Equation (11) is a relation between \( u \) and \( v \) if the dependency of \( \alpha \) on material properties is established.

At this point it is appropriate to attempt to determine what type of a relationship should exist between \( u \) and \( v \). The objective is to develop a mathematical relationship based on the physics of the problem as follows:

1. At hyper velocities the relationship between \( u \) and \( v \) should be independent of the strength properties of the target and penetrator. (Classical jet penetration formula \( Z = L \frac{\rho_p}{\rho_T} \); Where \( \rho_p \) and \( \rho_T \) are projectile and target density that shown in Figs. 3 and 4).

2. At low velocities the relationship between \( u \) and \( v \) should depend on the strength properties of the impacting materials.
Fig. 3 Schematic of rod in penetration process (a) plastic portion X and undeformed portion L-X (b) penetration into target to a depth Z

Fig. 4 Schematic diagram of mass transfer from the undeformed to the plastic portion of the rod

3. Since \( l = -(v-u) \), the higher the difference between \( v \) and \( u \), the higher the erosion rate. For a given material, higher erosion rate corresponds to higher impact velocity. This translates into the fact that, at higher values of impact velocity \( v \), the difference between \( v \) and \( u \) will be higher. At low impact velocity, the erosion rate is expected to be low and the difference decreases.

4. In an impact situation, \( u \) is expected to depend on the ultimate yield strength of the target and penetrator. That is, \( u \) will be lower if the tensile strength of the target material is high (imposing more resistance to penetration) and it will be higher if the tensile strength of the projectile is high (all other conditions unchanged). This is because a high strength penetrator tends to behave more like a rigid body and consequently penetrate deeper into the target.

Considering the above generalities, at hypervelocity the relation between \( u \) and \( v \) takes the following form:

\[
u(1-\alpha) = \frac{v}{1+\sqrt{\frac{\rho_T}{\rho_P}}}\]  

Comparing these two equations we obtain:

\[
u(1-\alpha) = \frac{v}{1+\sqrt{\frac{\rho_T}{\rho_P}}}\]

From which an expression for \( \alpha \) can be obtained as

\[
\alpha = \frac{\lambda}{\lambda+1}
\]

where \( \lambda \) is the square root of the ratio of the density of the target to the density of the penetrator

\[
\lambda = \sqrt{\frac{\rho_T}{\rho_P}}
\]

Equation (15) is valid only at hypervelocity only and does not apply at low impact velocity. This term can be thought of as a limit as impact velocity becomes very high.

\[
\lim_{v \to \infty} \alpha = \frac{\lambda}{\lambda+1}
\]

It was already pointed out that at low velocity, strength parameters play a significant role and the relation between \( u \) and \( v \) should depend on these parameters. Thus, various expressions were proposed to account for the effect of strength parameters. For example, the following exponential relation was proposed:

\[
\alpha = \frac{\lambda}{1+\lambda} \left[ 1 - \left( \frac{\gamma}{1+\gamma} \right) e^{-\left( \frac{\rho_T}{\rho_P} \right)^{1/2}} \right]
\]

where \( \gamma \) is the ratio of the target ultimate strength to that of the penetrator

\[
\gamma = \frac{Y_T}{Y_P}
\]

The properties of this exponential expression are as follows:

1. At hypervelocity the exponential part of the expression for \( \alpha \) goes to zero and the relation between \( u \) and \( v \) becomes independent of the strength properties of the materials and reduces to the classical density law

\[
u = \frac{v}{1+\sqrt{\frac{\rho_T}{\rho_P}}}\]

2. At lower velocity the effect of the strength terms become pronounced as \( v \) decreases. The value of \( \alpha \) increases as the difference between \( v \) and \( u \) decreases corresponding to the fact that at low velocity the erosion rate becomes smaller. The relationship between \( u \) and \( v \) in this situation is
A full three-dimensional explicit finite element analysis with Lagrangian formulation was adopted to solve the problem of high-strain-rate deformation [14,15]. This has the form

\[ u = \nu \left[ 1 - \frac{\lambda}{1 + \lambda} - \left( 1 + \frac{\gamma}{1 + \gamma} \right) e^{-\left( \frac{\rho}{1 + \gamma} \right)^\frac{1}{2}} \right] \]  

Equation (21) corresponds to all the guidelines indicated previously and is considered to be a valid expression relating \( u \) and \( v \). This equation also indicates that \( u \) is always less than or equal to \( v \) as expected and the internal consistency of the problem is sustained.

Then we have:

\[ V_r = \frac{R_v}{\rho_s V_r^2} \left[ 2V - \frac{\lambda}{1 + \lambda} - 1 - \left( \frac{\gamma}{1 + \gamma} \right) e^{-\left( \frac{\rho}{1 + \gamma} \right)^\frac{1}{2}} \right] \]  

From Eqs. (8) and (21) we get:

\[ V_{tg} = V \cdot \tan \beta > \frac{V_r}{2} \]  

Figure 2 shows the two velocity vectors of the rod and one can easily show that ricochet occurs if

\[ \tan \beta > \frac{V_r}{V} \]  

Then we have:

\[ \frac{2V - \frac{\lambda}{1 + \lambda} - 1 - \left( \frac{\gamma}{1 + \gamma} \right) e^{-\left( \frac{\rho}{1 + \gamma} \right)^\frac{1}{2}}}{\frac{\rho}{1 + \gamma} - \left( \frac{\gamma}{1 + \gamma} \right) e^{-\left( \frac{\rho}{1 + \gamma} \right)^\frac{1}{2}}} \]  

This relation shows a better agreement with test data, because it has additional terms that account for the physics of the ricochet model.

\[ \sigma = (\sigma_0 + B(C - T)^n) \left[ 1 - \frac{T - T_m}{T_{c} - T_m} \right] \]  

In order to model a high-strain-rate mechanical response of the projectile and the target materials, a commonly used constitutive equation, the Johnson-Cook Eq. [18], was used as it is known to describe high-velocity mechanical response of a number of metals fairly well [19]. This has the form

\[ T_{c} = \frac{T - T_m}{T_{c} - T_m} \]  

Table 1 Material properties and constants for the Johnson-Cook model applied to the numerical model

<table>
<thead>
<tr>
<th>Property</th>
<th>WHA</th>
<th>RHA</th>
<th>S-7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shear modulus (GPa)</td>
<td>152.02</td>
<td>76.96</td>
<td>79.96</td>
</tr>
<tr>
<td>( \rho ) (kgm (^{-3} ))</td>
<td>17000</td>
<td>7840</td>
<td>7750</td>
</tr>
<tr>
<td>Specific heat (Kg (^{-1} ) K(^{-1} ))</td>
<td>134</td>
<td>477</td>
<td>477</td>
</tr>
<tr>
<td>( T_m ) (K)</td>
<td>1723</td>
<td>1809</td>
<td>1763</td>
</tr>
<tr>
<td>( \sigma_0 ) (MPa)</td>
<td>1410</td>
<td>1160</td>
<td>1539</td>
</tr>
<tr>
<td>( B ) (MPa)</td>
<td>223.3</td>
<td>415.9</td>
<td>477</td>
</tr>
<tr>
<td>( n )</td>
<td>0.11</td>
<td>0.28</td>
<td>0.18</td>
</tr>
<tr>
<td>( C )</td>
<td>0.022</td>
<td>0.012</td>
<td>0.012</td>
</tr>
<tr>
<td>( m )</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
</tr>
</tbody>
</table>
The interaction between the projectile and the plate was simulated by a Lagrangian-Lagrangian contact algorithm based on a slave-grid/master segment concept. This algorithm checks eventual penetration of slave grids through master segments and applies constant forces to push them back. Erosion of the projectile and the target was simulated through a so-called adaptive contact algorithm [20], which automatically updates contact definition between the interacting deformable bodies upon elimination of the elements when pre-set level of plastic strains, determined by a separate depth of penetration (DOP) calibration, are reached.

4. EXPERIMENTAL

Experiments were carried out to verify the numerical results. These results were used from last papers. The experimental set-up shown in Fig. 6 was used in last papers consists of three witness blocks (38mm thick RHA class 4), an oblique target plate (6.25mm thick RHA class 4), a velocity-measuring device and a solid propellant gun. WHA projectiles with $L/D$ ratios of about 1000 and 1500ms$^{-1}$ were impacted at velocities of 1000ms$^{-1}$ and 1500ms$^{-1}$. The velocities of the projectiles were controlled by adjusting the amount of solid propellant charge. The relations between the amount of the charge and the projectile velocities were calibrated in a preparatory experiment.

![Fig. 6 Schematic illustration of the experimental set-up for oblique impact of a long-rod projectile on a steel target plate performed in this study](https://example.com/fig6)

5. RESULTS AND DISCUSSION

5.1 Post-Impact Behavior of the Projectile and the Target Plate

Numerical results are graphically shown in Figs. 7 ~ 9 in terms of the mesh deformation with the lapse of time to analyze the behavior of the WHA projectile and the RHA target with thickness comparable to the projectile diameter during the oblique impact. When the projectile impact velocity is 1000ms$^{-1}$ and the target oblique angle is $10^\circ$, as in the case shown in Figs. 7(a) ~ 7(h), the projectile initially bends on impact (Fig. 7(a)). Subsequently, a plastic hinge is formed which remains at the initial point of impact with respect to a fixed coordinate system (Eulerian) resulting in its relative backward motion along the x-direction of the coordinate system (Fig. 5) as the projectile progresses forward (Figs. 7(b) ~ 7(d)). In the case being considered ($\theta = 10^\circ$), where the oblique angle is lower than the critical ricochet angle, the target does not deform much and no significant erosion of the impacted surface is noticed whilst the front end (denoted as head hereinafter) of the projectile lifts from the target surface after sliding some distance and eventually the projectile bounces away (Figs. 7(e) ~ 7(h)). Such behavior is yielded due to the asymmetric reaction force exerted from the contact area to the projectile, which is reported proportional to the area of the contact, target strength and oblique angle [8].

When the oblique angle of the target plate is increased to 12° whilst keeping the impact velocity the same, the projectile shows somewhat different behavior. As shown in Figs. 8(a) ~ 8(d), it initially pushes the impacted area of the target inward following impact since the target plate is allowed.

Whilst the head of the projectile tends to bounce back from the target due to the reaction force exerted from the contact area, the tail part of the projectile (Figs. 8(c), 7(f)) tends to penetrate into the target due to its initial trajectory of the projectile. In this case, the front part of projectile (head) slides and slides on the target surface, whilst the rear part behind it forming a stretched crater in the target (Figs. 8(a), 8(b)). Indeed, the relatively thin projectile in impact due to the reaction force exerted from the contact area at the initial stage of the impact, its trail slams into the target inward following impact since the target plate is allowed. As shown in Figs. 8(a) ~ 8(d), the elongation of the projectile becomes so severe that it results in the fragmentation of the projectile.

In the case where the oblique angle is further increased to 14° beyond the critical angle, as can be seen in Figs. 9(a) ~ 9(c), the behavior of the projectile shown in Figs. 8(a) ~ 8(d), and the target is similar to the case of critical ricochet shown in Figs. 8(a) ~ 8(d).

![RETRACTED](https://example.com/retracted)

Understanding the physical nature of the above behavior of the projectile and the target can be supplemented by analyzing the changes in the projectile velocities after impact, as has also been performed for normal penetration in the literature [21]. For this purpose, post-impact changes in the horizontal (along the x-direction) and vertical (along the y-direction) velocities of head and tail of the projectile have been monitored during the numerical calculations and the results are plotted in Figs. 10 to 15. Before impact, the head and the tail move at the same initial velocity of 1000ms$^{-1}$ and there is no vertical velocity term. For the case with relatively low oblique angle, e.g., $\theta = 10^\circ$, as shown in Figs. 10 and 11, the horizontal velocities of...
Fig. 7  Numerical results showing the behavior of the WHA projectile and the RHA target when the oblique angle is 10° and the impact velocity is 1000m/s

Fig. 8  Numerical results showing the behavior of the WHA projectile and the RHA target when the oblique angle is 12° and the impact velocity is 1000m/s

Fig. 9  Numerical results showing the behavior of the WHA projectile and the RHA target when the oblique angle is 14° and the impact velocity is 1000m/s
the head and the tail of the projectile after impact are kept almost identical, implying no significant axial strain, which prevents the projectile segmentation. It can also be seen that the horizontal velocities did not decrease noticeably. From this, it is inferred that the projectile does not encounter any significant resistance to its motion along the flight trajectory and that the impact interaction of the projectile with the target does not cause any large-scale deformation of the target.

Whilst there were only slight changes in the horizontal velocities, vertical velocities of the head and the tail undergo noticeable changes during the impact process. As can be seen in Fig. 10, the vertical velocity of the head initially increases to about 300 ms$^{-1}$ and remains almost the same thereafter, which would be associated with sliding on the target surface and subsequent takeoff of the head shown in Figs. 7(a) ~ 7(h). On the other hand, the vertical velocity of the tail is almost 0 until
about 80μs and then increases to about 550ms\(^{-1}\) at 140μs. This indicates that the impact of the head on the target does not cause any yawing force in the rear part of the projectile which is beyond the plastic hinge mentioned above. Near-constant vertical tail velocity of 460ms\(^{-1}\) after about 160μs would indicate the takeoff of the tail as shown in Figs. 7(f) and 7(h).

However, where critical ricochet was achieved (θ = 12° for the case considered herein), as shown in Fig. 12, the decrease in the horizontal velocity of the head with respect to time is more pronounced than in the previous case, indicating that the progress of the head is hindered more. In particular, as shown in Fig. 13, the horizontal velocity of the tail decreases to almost 0 from about 140μs, producing a velocity difference between the head and the tail of about 750ms\(^{-1}\). Such a large velocity difference may cause large-scale deformation and therefore it would explain the stretching of the projectile shown in Fig. 8(g) followed by the segmentation of the projectile shown in Fig. 8(h). At the same time, a sudden drop in the horizontal velocity of the tail between 100 and 150μs is believed to be related to target cratering shown in Figs. 8(f) and 8(g), which could exert a high resistance to the advance of the tail. When critical ricochet is achieved, even though the impact crater is formed on the target, this does not lead to target perforation. This can be explained from the changes in the vertical velocities of the head and the tail shown in Figs. 12 and 13, where it can be seen that the head and the tail sequentially acquire positive, vertical velocity components. They begin to take off from the target plate at about 0 and 150μs, respectively, indicating no further penetration of the target.

A similar trend is obtained when the target oblique angle is further increased, e.g. θ = 14°, as shown in Figs. 14 and 15 whilst two apparent differences are noticed. First, the horizontal velocity of the head, once it is decreased to about 700ms\(^{-1}\) at about 120μs, remains nearly constant implying that the flight of the head portion is no longer hindered by the target thereafter, probably due to the earlier segmentation of the projectile. In the previous case shown in Fig. 12, the head portion was connected to the tail portion through the elongated portion until the later time step so that the tail, still staying in the impact crater in the target, delayed the propagation of the head, which is represented as continuously decreasing velocity. Second, the behavior of the tail after segmentation is completely different: the vertical velocity of the tail decreases to a negative value of about –180ms\(^{-1}\) from about 150ms\(^{-1}\), which is then maintained almost constant after about 180μs. This indicates that the fragmented tail is heading downward, which would be responsible for the perforation of the target shown in Fig. 9(h).

The ricochet behavior illustrated in Figs. 7(a) to 9(h) are also supported by the experimentation carried out herein. Figure 16 shows the shape of the target plate and the witness block in the ricochet experiments. In this figure, the deformed shape of the plate is apparent with an asymmetric elliptical perforation hole. Occurrence of ricochet is indicated by observing deformed and eroded surfaces of the target plate and penetration holes in the witness block. When a projectile impacted the target plate at a lower oblique angle, as shown in Figs. 16(a) and 16(c), the ricochet process resulted in a long surface groove formed by erosion, and a single penetration hole on the witness block. At an oblique angle slightly higher than the critical value, as can be seen in Figs. 16(b) and 16(d), the projectile broke into two parts, resulting in an apparent groove (crater) followed by a single penetration hole in the target resulting from initial erosion whilst two penetration holes are noticeable in the witness block, one over and the other below the white line in Figs. 16(b) and 16(d) where the edge of the target was located.
The post-impact behavior of the deformable projectile and the deformable target with finite thickness described so far in general agrees qualitatively with what has been observed and predicted in the previous works in which ricochet occurred at un-deformable (and sometimes rigid) target surfaces. However, as apparent in Figs. 7(a) to 9(h), the inward deformation of the target plate due to the finite thickness comparable to the projectile diameter is shown to assist the segmentation of the projectile, followed by the perforation of the target plate by the broken rear part of the projectile. Such phenomena could be responsible for the difficulty of obtaining ricochet from relatively thin plates.

Senf et al. [4] showed how the ricocheting rod can form a plastic hinge at the impact site to deflect the rod from a rigid target surface. These test results (see Fig. 17) are similar to the FEM solution results as shown in Figs. 7(a) to 9(h). But in FEM solution the target plate is not rigid and it delected more.

5.2 Critical Ricochet Angles

In accordance with the definition mentioned in the introduction, changes in the critical ricochet angles were derived by analyzing the numerical results graphically in the manner described in last section and were plotted as functions of impact velocities in Fig. 18 for the RHA target plate. The ricochet angle curves shown in Fig. 18 were obtained from curve-fitting the numerical results to a first-order exponential decay function. The fitted equations, their parameter values, and the statistical analysis of the fitted results are also reported in the figure. The numerical results are confirmed with experimental results as shown in Fig. 18. In Fig. 18, the hollow circle markers indicate perforation of the RHA target plate by the long-rod projectile whilst the solid star markers indicate critical ricochet of the projectile. It can be seen that there is good agreement between the two.

The new developed model results on the critical ricochet angles are also compared with experimental, numerical results and existing two-dimensional analytical models developed by Tate [3] and Rosenberg et al. [8], independently. The critical ricochet angles based on these models have been calculated for a WHA long-rod projectile and a RHA target as functions of impact velocities in Fig. 19. Shown are the corresponding numerical results for the Tate model in the figure that the Tate model overestimates the critical ricochet angle for impact velocities higher than 1170 ms$^{-1}$, and vice versa for lower velocities. The difference in the trend of the critical ricochet angles is due to the difference in the method of interest for the curves. On the other hand, the model developed by Rosenberg et al. shows a better agreement with the numerical results, though the former overestimates the critical ricochet angles at all impact velocities.

The results show a better agreement with the experimental results. As shown Fig. 19, the difference between the analytical model prediction result and the numerical result decrease up to 25%. The results presented in this model, shifts to the experimental results, because:

1. This method was more detailed parameters to calculate the penetration velocity such as projectile density in comparing with Rosenberg analytical method.

Fig. 18 Comparison of the numerically predicted critical ricochet angles for various velocities with the experimental results
2. This method uses parameter $\alpha$ in calculating the critical ricochet angle, where $\alpha$ is based on the experimented results. Therefore the results might be accurate. Therefore new analytical model can be used as a practically useful guideline to estimate ricochet angles rather than existing two-dimensional analytical models developed by Tate [6] and Rosenberg et al. [7] and [8].

About effects of the target strength we have:

Whilst the RHA has been widely used as a primary armor material over decades, in some cases, stronger material such as high hardness armor (HHA) has also been adopted, though its use is limited due to lower toughness. To investigate the effect of material strength on the ricochet angle, the Johnson-Cook material constant terms in the Johnson-Cook model for S-7 tool steel, which has static yield strength and hardness similar to HHA produced by Thyssen Krupp AG, were taken from the literature [17] and applied to the numerical model. The ricochet angles calculated for S-7 tool steel were plotted as a function of impact velocity in Fig. 20. It can be seen that a higher ricochet angle is predicted for a given impact velocity if the target strength is increased.

5.3 Summary and Conclusions

Ricochet of a WHA long-rod projectile impacting on oblique steel target plates with finite thickness was investigated numerically using a full, three-dimensional, explicit finite element method with the predictions of our analytical model. Effects of the impact velocities of the projectiles and the hardness of the plates on the critical ricochet angle were considered.

It was predicted in the numerical analysis that the projectile and the target behave in three different ways depending on the oblique angle of the target. For a relatively low oblique angle, the impacted projectile bent and slid on the target surface to bounce away with very little velocity drop whilst no significant deformation of the target was predicted. With increasing oblique angle, the projectile impacted at an angle but the target deformed substantially. The tail portion of the target behind the plastic hinge appeared to play the role of pulling the projectile back onto the target surface so as to increase the ricochet angle.

Critical ricochet angles were also derived from the numerical analysis and new model and from existing two-dimensional analytical models developed by Tate [6] and Rosenberg et al. [8]. As the other need many simulations in each velocity to obtain the critical ricochet angles, and also any change in geometry can lead to a new simulation, then this process is time-consuming subsequently, presenting a new analytical formulation might be of great importance to find the critical ricochet angle.

In order to calculate the critical ricochet angle it has been only two analytical model (Tate and Rosenberg models). As these two models usually overestimate the results comparing with test experimental setup; however, the others compare their results with these two analytical methods. As the presented method predicts the results more accurately and the errors are reduced into 25%. Then it can be a new analytical model to reach the better agreements. Therefore new analytical model can be used as a useful method to predict critical ricochet angle in metal cases.
When the target hardness was considered the numerical results predicted that a higher ricochet angle can be obtained by employing harder target materials for a given impact velocity, which was appreciable at lower velocities in particular.

**NOMENCLATURE**

- $\beta$: Critical ricochet angle
- $\rho_T$: Target's density
- $\rho_p$: Projectile's density
- $V$: Striking velocity of the rod
- $L$: Projectile length
- $D$: Projectile diameter
- $U_{p}$: Projectile penetration velocity
- $Y_p$: Projectile resistance
- $R_T$: Target resistance
- $S$: Length of the eroding tip of the projectile
- $\psi$: Rotations of the projectile eroding surface relative to the un-deformed projectile edge
- $x$: Projection of $S$ in vertical direction ($S.Cos\psi$)
- $t$: Time
- $f$: The force acts on the projectile
- $m, m_1, m_2$: Residual mass off the projectile, Projectile tip mass, Projectile eroded mass
- $V_T$: Vertical velocity
- $i$: Projectile erosion rate
- $v$: Tail velocity (Instantaneous velocity of the un-deformed portion of the projectile)
- $\alpha$: Coefficient ($\alpha = -\frac{i}{v}$)
- $\lambda$: Coefficient ($\lambda = \sqrt{\frac{\rho_p}{\rho_T}}$)
- $\gamma$: Coefficient ($\gamma = \sqrt{\frac{\rho_p}{\rho_T}}$)
- $Y_T$: Target ultimate strength
- $\sigma_0, \sigma_0$: Static and dynamic yield strengths of target, respectively
- $\dot{\varepsilon}_p$: Strain rate
- $B, C, n, m$: Material constants in Johnson-Cook relation
- $T, T_r, T_m$: Temperature, reference temperature, melting temperature

**REFERENCES**


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