On the rank deficiency of BEM/BIE using degenerate kernels

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Summary of final report

In the past decades, the boundary integral equation (BIE) as well as the boundary element method (BEM) attracts mathematicians and engineers, respectively. However, the mathematical degeneracy in the BEM/BIEM may appear in four aspects: degenerate boundary, degenerate scale, spurious eigenvalue and fictitious frequency. The commercial codes of BEM may result in ghost solutions which mislead engineers. Mathematicians may be interested in this topic while engineers may be not capable to interpret the numerical outcomes. The degenerate kernel is a powerful tool to explain the mechanism of the degeneracy in the BEM/BIEM. Although a degenerate kernel plays an important role in the theory of integral equations and gives a natural approximation, the use in engineering problems seems to have taken a back seat to other methods such as quadrature and collocation. In this regard, we demonstrate the power how the degenerate kernel can explain the rank-deficiency mechanism of degenerate boundary, degenerate scale and fictitious frequency.

In the first part, we proposed two ways to understand the rank deficiency in the continuous system (BIE) and the discrete system (BEM). The infinite dimensional D.O.F. for the continuous system can be reduced to finite dimensional space using the generalized Fourier coordinates. In other words, the influence matrix can be determined by using the generalized coordinates of Fourier series. On the other hand, the discrete system in the BEM for the circular case can be analytically studied thanks to the spectral property of circulants. The influence matrix can be reconstructed by using the eigen systems (eigenvalue and eigenvector). The property of the second-order tensor for the influence matrix under different observers are examined. The equivalence of the influence matrix derived by using the generalized coordinates and the circulants are proved. In order to match the two results, the odd number of degrees of freedom is adopted since the number of Fourier bases is also odd.

Why a rank-deficiency influence matrix due to a degenerate boundary in symmetric and anti-symmetric cases may yield acceptable results is explained by using the Fredholm alternative theorem in the second part. To verify the validity of the formulation, simple cases of anti-plane shear containing a crack or a rigid-line inclusion are given to demonstrate. It is shown that the singular BIE can solve the symmetric problems containing the degenerate boundary, while the hypersingular BIE can deal with the anti-symmetric problem. A new and easier approach is provided by adding the

symmetric or anti-symmetric boundary constraint to replace the dependent equations. The role of enforcement of symmetric or anti-symmetric constraint after replacing the dependent equations is also examined. The method can solve symmetric or anti-symmetric problems although the dual BEM can solve all problems suffering degenerate boundary.

In the third part, the J-integral is our concern. The J-integral of the slant crack and the rigid-line inclusion under the anti-plane shear are analytically derived by using the degenerate kernel in the elliptical coordinates. The path independence of the J-integral is analytically demonstrated by using the elliptical path. Positive and negative J-integrals are analytically derived by using the degenerate kernel and numerically implemented by using the dual BEM for the crack and the rigid-line inclusion, respectively. It is interesting to find that the J-integral is path independent but not an invariant in different observer systems as only one component of the vector of the first order tensor. The tensor properties are also examined analytically and numerically.

In the fourth part, the mechanism of the fictitious frequency is explained. Regarding the treatment for the fictitious frequency in the indirect BEM, we propose an approach by adding extra sources. We add some fundamental solutions with unknown source strength in the representation of the field to complete the base of solution space. It is different from the null-field point of the CHIEF. The null-field point provides the extra constraint equation in the direct BIE, but the present approach supplies the deficient base due to the fictitious frequency. Coincidentally, the failure points exist in the present method as well as the CHIEF approach. Finally, we analytically derive the locations of possible failure source points by using the degenerate kernel.