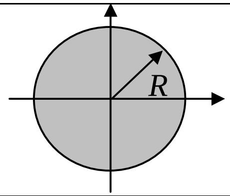
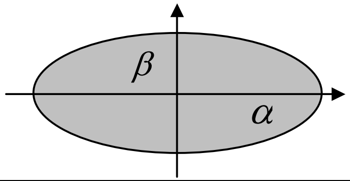
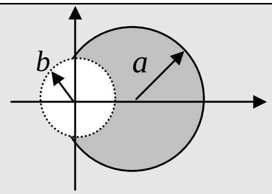
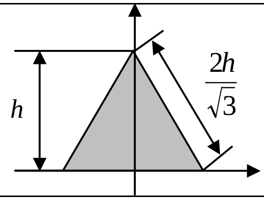
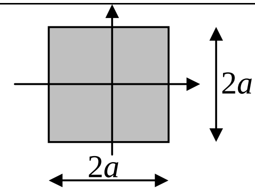
Table 2-1 Determination of the degenerate scale for the two-dimensional Laplace problems using integral formulation

Cross Section					
Normal scale	$R=2.0$	$\alpha=3.0,\ \ \beta=1.0$	$a=2.0,\ \ b=\frac{2}{3}a$	$h=3.0$	$a=1.0$
Equation of boundary	$x_1^2+x_2^2=R^2$	$\frac{x_1^2}{\alpha^2}+\frac{x_2^2}{\beta^2}=1$	$(r^2-b^2)(1-\frac{2a}{r}\cos\theta)=0$	$x_2=0$ $x_2-h-\sqrt{3}x_1=0$ $x_2-h+\sqrt{3}x_1=0$	$x_1-a=0,\ \ x_2-a=0$ $x_1+a=0,\ \ x_2+a=0$
Prandtl function, Φ ($\nabla^2\Phi=-2$, x on B)	$-\frac{1}{2}(x_1^2+x_2^2-R^2)$	$\frac{-\alpha^2\beta^2}{\alpha^2+\beta^2}(\frac{x_1^2}{\alpha^2}+\frac{x_2^2}{\beta^2}-1)$	$\frac{1}{2}[b^2-r^2+2a(r^2-b^2)\frac{\cos\theta}{r}]$	$\frac{1}{2h}(h^2-3x_1^2-2hx_2+x_2^2)$	N.A.
Complex function ***F*(*z*)=*w*+*iv***	0	$\frac{1}{2}\frac{(\alpha^2-\beta^2)}{(\alpha^2+\beta^2)}iz^2$	$-ia(z-\frac{b}{z})+\frac{1}{2}ib^2$	$-\frac{1}{2h}(z^3-ihz^2+h^2z)$	$-i\frac{z^2}{2}+i\frac{a^2}{2}$ $-i\frac{8a^2}{\pi^3}\sum_{n=0}^{\infty}\frac{(-1)^n\cos(\lambda_n z)}{(2n+1)^3\cosh(\frac{\lambda_n a}{2})}$
Warping function w $\nabla^2 w=0$	0	$\frac{\beta^2-\alpha^2}{\beta^2+\alpha^2}x_1x_2$	$-a(r+\frac{b^2}{r})\sin\theta$	$-\frac{1}{2h}(x_1^3-3x_1x_2^2+2hx_1x_2^2+h^2x_1)$	x_1x_2 $-\frac{8a^2}{\pi^3}\sum_{n=0}^{\infty}\frac{(-1)^n\sinh(\lambda_n x_2)\sin(\lambda_n x_1)}{(2n+1)^3\cosh(\frac{\lambda_n a}{2})}$
Torsional rigidity	$G\frac{\pi}{2}R^4$	$G\frac{\pi\alpha^3\beta^3}{\alpha^2+\beta^2}$	$2Ga^4k_2$	$G\frac{\sqrt{3}}{45}h^4$	Gk_1a^4
Reference equation	$u(x)=\int_B U(s,x)\psi_1(s)dB(s)$, where $u(x)=1$, x on B, $[U]\{\psi\}=\{1\}$.				
$\Gamma=\int_B\psi_1(s)dB(s)$	1.4480 $(\frac{1}{\ln(2)})$	1.4509 $(\frac{1}{\ln(2)})$	1.5539 (N.A.)	2.6972 (N.A.)	6.1530 (6.1538)
Expansion ratio $d=e^{-\frac{1}{\Gamma}}$	0.5020 (0.5)	0.5019 (0.5)	0.5254 (N.A.)	0.6902 (N.A.)	0.8499 (0.85)
Degenerate scale	R=1.0040 (1.0)	$\alpha+\beta$ = 2.0058 (2.0)	a=1.0508 (N.A.)	h=2.0700 (N.A.)	a=0.8499 (0.85)

Note: Data in parentheses are exact solutions.

Data marked in the shadow area are derived by using the polar coordinate.